OZANAM's
Compleat Course
OF THE
MATHEMATICKS,
Volume the Fourth.

Advertiſement.

At the *Hand* and *Pen* in *Barbican*, are Taught, *viz.* Writing in all Hands, Merchant's Accounts, Book-keeping, Algebra, Geometry, Meaſuring, Surveying, Gauging, Mechanicks, Fortification, Gunery, Navigation, Dialling, and other Parts of Mathematicks; alſo the Uſe of the Globes and Maps, after a Natural, Eaſy and Conciſe Method, without Burthen to the Memory.

By *Robert Arnold.*

Cursus Mathematicus:

OR, A

Compleat COURSE

OF THE

MATHEMATICKS.

Vol. IV.

CONTAINING

MECHANICKS,

AND

PERSPECTIVE.

Written in *French* by Mr. *OZANAM*,
Professor of the *Mathematicks* at *Paris*.

Done into *English*, and Amended in Several Places, by *J. T. Desaguliers* of *Hart-Hall*, *OXON.*

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To the Right Honourable

Philip Lord *Glammiſs.*

My LORD,

THE many Favours I have receiv'd from Your LORDSHIP, deſerve a much better Return than I am able to make; however give me leave to hope, that this humble Acknowledgment, ſince it is the beſt I can offer, will be accepted as a ſmall Inſtance of the great Reſpect and Honour I have for Your LORDSHIP's Perſon, and of the juſt Senſe I retain of thoſe Kindneſſes which I cannot repay. For indeed, the chief Advantage I propoſe to my Self by the Publication of this Book, is to let the World

World know, how much I ſtand indebted to Your LORDSHIP'S Goodneſs.

Your early Inclination to Mathematical Studies, and the conſiderable Progreſs You have made in thoſe Sciences, encourage me to think that a Work of this nature will not be altogether unwelcome to Your Peruſal: And as Your Value for any Part of Learning encreaſes in proportion to the Uſefulneſs of it; ſo I thought this Volume the fitteſt to offer Your LORDSHIP; becauſe it contains the Practical Part of *Mathematicks*, and treats of the Nature of thoſe Things which Your Strong Reaſon perceives at one View, without the Tedιouſneſs of a Demonſtration. Your LORDSHIP has already been pleas'd to Countenance my Endeavours, towards the promoting of Mathematical Learning, by Honouring me with Your Preſence at

at a *Course of Natural Experiments*, and with a Goodneſs peculiar to great Souls, approving and recommending my poor Performances that way.

Did Your LORDSHIP want the Merit which You are known to have by all that have the Honour of Your Acquaintance; yet, To be the Offſpring of an Illuſtrious and Truly Heroick Family, To be Deſcended from Ancient, from Honourable, from Royal, from * *Roman* Blood, is a diſtinguiſhing Advantage peculiar to Your LORDSHIP's Name: Or if You were deſtitute of this great Priviledge, and deriv'd no Honour from the Virtue of Your Anceſtors; ſuch is Your LORDSHIP's Perſonal Merit, as might juſtly be eſteem'd Equal to the moſt Noble Pedigree.

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* *See* Collier's *Dictionary under the Word* LYON.

But why do I pretend to offer at Your LORDSHIP's Character any farther than relates to my own Obligations? I must leave that Task to the Management of more Skilful Writers, who can never want a fit Subject for their Pens, while Your LORDSHIP continues, as You have hitherto done, to adorn Your high Quality with Candour, Learning, and Virtue, the proper Graces and Ornaments of True Nobility. I am,

My LORD,

Your LORDSHIP's

Most Humble and

Obedient Servant,

J. T. DESAGULIERS.

THE

PREFACE.

THE greateſt part of the Lovers of Mathematicks are won to that Science by its ſenſible Beauties only, they are taken by the Wonders that it works, and delighted with its admirable *Phænomena;* They are willing to know what they have admir'd, to perform thoſe Things which at firſt they cou'd not account for; and take pleaſure in ſurprizing others, as themſelves have been ſurpriz'd. *Mechanicks* and *Perſpective*, which chiefly reſpect Senſible Objects, and enter, as it were, into the very Secrets of Nature, may be look'd upon as the Fruits of the Study of the other parts of Mathematicks; and if the Courſe of this Science may be compar'd to that of the Year, this Volume muſt be look'd upon as the *Autumn,* by reaſon of the Fruits that it offers, and the Pleaſures that it gives; whilſt the others are look'd upon as thoſe ſevere Seaſons, which we go thro' with uneaſineſs.

To obſerve the natural Order of Sciences, we cannot but treat of *Mechanicks*, juſt after we have taken notice of the Meaſures and Proportions to be obſerv'd in *Fortifications*. We ſhou'd in vain draw the moſt juſt Plan, if we did not put in Execution the Deſign which we have form'd: And it is not enough for the Mind to conceive the moſt exact Figures, and to expreſs them by a Draught upon the Ground, unleſs the Force of the Body be employ'd to build according to that Draught. Our

Art

Art is not Magical, it cannot command Spirits by Words and Figures, it acts only upon Bodies; we muſt make uſe of what is Material, Hard, and Heavy, to reſiſt when we are attack'd, or attack when we have a right ſo to do, in order to maintain the Authority of Princes, to whom God has given the Power to do themſelves Juſtice by the Force of Arms.

The Object of *Mechanicks* is whatſoever is *Heavy*, *Hard*, and *Difficultly mov'd out of its place*. To give and direct Motion belongs to Mechanicks; and as the Effects of this Science are viſible, they can neither be cenſur'd nor look'd upon as Fables or vain Imaginations, as was the Harmony of *Amphion*, who was ſaid to Build Cities by the Power of his Song.

Nothing is more Plain or Simple than the Principles of Mechanicks; the Firſt of its Engines is a *Leaver*, or mere Staff, which being applied to the greateſt Burthens, and help'd in a fit Place by ever ſo weak a Power, will by means of that ſmall Power ſhake the whole Maſs, and raiſe it in ſpight of all its reſiſtance. This Management of Force is what I ſhall treat of in the Firſt Part of this Fourth Volume, and it is the whole Secret of Mechanicks; a Science which ought not to be leſs eſteem'd by reaſon of this Name, which Uſe has given to thoſe Arts which Neceſſity has made Valuable, and which are often more Ingenious than ſuch as have only Pleaſure for their Object.

To Mechanicks we owe the Invention of *Watches* or Clocks with Wheels, and the new Diſcoveries that have been made concerning the Uſe of the *Nerves*, *Muſcles*, and *Veſſels*, the Wonderful *Circulation* of the Blood, the *Motion* of Animal Spirits, and the Manner of the Operation of the Senſes; all which things have been

made

made ſo plain, that the Animate Body has been acknowledg'd to be only an *Automaton* or Mechanical Engine. The New Phyſicians brought up in Mathematical Schools have gone greater Lengths, and by the help of Optical Inſtruments, have diſcover'd the Springs of Living Machines, abridg'd and folded up in their Seeds, whoſe Encreaſe is caus'd by Heat, which extends the Parts according to the Laws of Motion.

If we look into other Arts, we ſhall find them indebted to Mechanicks for their Chief Beauties: *Painting*, for Example, has borrow'd from Mechanicks the Proportion of the Attitudes or Poſtures which it gives to Animals; and by the Laws of this Science, *Natural Philoſophy* explains the Syſtems of the Motion of Celeſtial Bodies, the Impoſſibility of the Meeting of *Epicurus*'s Atoms, the wonderful Experiments of the Loadſtone, and the Effects which are produc'd in a *Vacuum.*

Such as wou'd deſpiſe *Perſpective*, which is the Second Part of this Volume, might ſay that it has been only invented for the Pleaſure of the Sight, a Pleaſure ſo much the more blameable as it is accompanied with Error, upon which it ſo much depends, that a Piece of Perſpective cannot be well drawn unleſs it deceives; for if it diſcovers the Truth it is reckon'd Courſe, and ſo ill-done, as not to bear looking at. The Two Painters, one of which deceiv'd the Birds, and the other his Adverſary, wou'd have been but little eſteem'd, if they had expos'd the Truth; that is, if the Firſt had expos'd to Sight true Grapes, and if the Second had cover'd his Picture with a true Curtain; but becauſe they took advantage from the Prejudice of Men and Brutes, the Memory of their Works has been tranſmitted to Poſterity. They'll

ſay,

ſay, can Perſpective be call'd an Art, ſince it Works only upon *Error*, which is its Matter as well as its End? After this rate Drunkenneſs and Folly might be honour'd, whoſe Viſions are yet more deceitful; and Potions might be eſteem'd, which cauſe ſuch Dreams as he deſigns who mixes the Ingredients for that purpoſe. Laſtly, they'll ſay, that they don't ſee what Perſpective is good for, except mere Pleaſure, and ſometimes a blameable Diverſion; ſince *Mountebanks* make uſe of it to abuſe the Credulous, and make Magical Superſtitions eſteem'd.

It is true, that Pleaſure is one of the Ends of Perſpective, and I ſhou'd be ſorry that it ſhou'd want that Charm which makes it ſo valuable; but having Charms, and flattering a Senſe, which ſeems only given us for innocent Pleaſures, cannot make it criminal. The *Proſpect of the Univerſe* ſeen at any Time, and in any Place, is an admirable Piece of Perſpective, which God has created to divert the Eyes, and repreſent part of his Greatneſs in the beautiful Diſpoſition of Viſible Objects: All the Rules of Perſpective are obſerv'd in it, the Diſtances are expreſs'd by the moſt confus'd or *Aerial* Colours, as they call 'em, and by the Magnitude of the Objects, which decreaſes as they are farther off, or ſeen under leſs Angles. Theſe are the Meaſures and Proportions which Perſpective ſtudies, imitates and obſerves. It cannot be juſtly blam'd, ſince it may alledge in its Defence, that it is an Imitation of the *Prototype*, or Original, which is drawn by God Almighty.

Who, from a very high and diſtant Place, wou'd not imagine Men to be no higher than Dwarfs, and Horſes no bigger than Sheep? Who wou'd not ſuppoſe all Colours to be Brown, when ſeen from a Place to which their Force cannot reach, ſo as to make 'em diſtinguiſhable? Theſe are not Errors,

but

but Neceſſities impos'd by the Laws of Nature: We can only ſee by the help of Light; and Light is communicated by Rays or Right-lines, which cannot be Parallel, or keep the ſame Diſtance from one another; becauſe as they are means of Sight, they muſt meet in Points in Men's Eyes, and there paint the Objects, and form Viſual Angles, whoſe Aperture makes us judge of the Greatneſs or Smallneſs of Objects, according as that Aperture is greater or leſs.

Perſpective then does not cauſe *Error*, but imitates *Truth*, which makes us every where Senſible of our Weakneſs. It ſuppoſes our Sight apt to loſe the Diſtinction of Objects, as they are more diſtant from us; and if it repreſents that Weakneſs, it is a Truth and not an Error. The Fallacy of Dreams is not to be compar'd with the Beauties of Perſpective, becauſe thoſe Dreams have no Rules; but Perſpective has moſt certain and invariable ones; Dreams only diſturb and deceive, whereas Perſpective ſtudies Diſtinction and obſerves it exactly, to mark Lengths and Diſtances, and hinder us from believing that all that we ſee is equally near to, or diſtant from us.

To do Perſpective juſtice, we ought to ſay that its true End is to diſcover Error and correct it, by ſhewing that the different Repreſentation of Objects are founded upon certain Rules of Nature, and that whatever ſurprizes ſuch Men as are curious of extraordinary things, has nothing Supernatural; that the moſt ſurprizing Sights depend upon the Obſervation of Certain Meaſures; and that the meaneſt of Men may perform, by help of the Rules of this Science, all that Jugglers attribute to Art-Magick, and pretend to do by the Aſſiſtance of Infernal Spirits.

Such

Such Repreſentations as have been publickly ſhewn, where Artiſts have play'd the Conjurers with Tricks of Perſpective, have more undeceiv'd the Vulgar and cur'd 'em of their fooliſh Eagerneſs to believe extraordinary Things, than what ever Means the Government cou'd have us'd to deſtroy ſo pernicious a Curioſity.

I ſhall not here treat of that Part of Perſpective which conſiders Colours, neither ſhall I give an account of thoſe Tricks of *Dioptricks* and *Catoptricks* which are only to divert the Eyes, and ſhew the admirable Variations of Colours and Appearances: Such curious Performances are to be met with in my *Mathematical Recreations*. I ſhall only Speak here of the *Principles of Perſpective*; and the certain Rules which it has eſtabliſh'd to diſcern the Effects of the Diſtances, and to take the Heights and Shortnings of near or diſtant Objects; to teach Painters the Perfection of their Art, the Height and Meaſures of the Figures, inward Parts of a Building, and Pieces of Architecture; what Heights muſt be given to Statues, and what Slope to Roofs; and the Angle for the Point of Sight, that all may appear in its Proportion: To teach Architects and Engineers how to repreſent their Deſigns in a little Compaſs, by raiſing one part of their Draughts and leaving the other in Plan: And laſtly, to give Rules to Gold-Smiths, Embroiderers, Enamellers, Silk-Weavers, Joyners, Carvers and Plaiſterers, and all ſuch as are any ways concern'd with Drawing or Painting.

To

To the READER.

H*Aving in ſeveral places render'd the French Word* Mouvement (*Motion*) *by the Engliſh Word* Velocity, *we thought it not improper to give here the Diſtinction between* Velocity *and* Motion; *becauſe the French Author has confounded them, uſing the Word* Mouvement *indifferently for both.*

Motion (Local, *we mean, as it muſt be underſtood throughout the Whole Book*) *is that Force or Energy by which a Body changes its place: And Velocity is the Space which a Moving Body goes thro' in ſuch a determinate Time; as, for Example, if the Body E moves 4 Inches downwards in One Second of Time, and the Body D moves 2 Inches upwards in the ſame Time; E will have Four Degrees of Velocity, and D Two. Now the Quantity of Motion in a Body is found by multiplying the Weight and the Velocity into one another; as if E ſhou'd weigh Two Pounds and D Four; 4 the Velocity of E, Multiplied by 2 its Weight, will give 8 for the Degrees or Quantity of Motion in E; and 2 the Velocity of D, multiplied by 4 its Weight, will likewiſe give 8 for the Quantity of Motion in D.* Plate 1. Fig. 8.

An

An Explanation of the Characters:

+ Signifies *Plus* or more. − Sig. Less. = Sig. Equal to : :: : Sig. Proportion, as thus:
A : B :: C : D. Sig. As A is to B, so C is to D. *Q. E. D.* Sig. *Quod erat demonstrandum*; or, Which was to be demonstrated. *Q. E. I. & D. Quod erat inveniendum & demonstrandum*: or, Which was to be found out and demonstrated.

ERRATA in the *Mechanicks*.

PAGE 63. Line 5. for *Square* read *Squares*. Pag. 118. Over-against lin. 40. read in the Margin *Fig.* 90. Pag. 122. Over-against lin. 33. read in the Margin *Plate* 20. *Fig.* 98. Pag. 155. Over-against lin. 39. read in the Margin *Plate* 23. *Fig.* 122. Pag. 173. Over-against lin. 10. read *Fig.* 136. Pag. 180. lin. 41. instead of *cause*, r. *because*. Pag. 183. instead of *Plate* 25. *Fig.* 132. r. *Plate* 38. *Fig.* 146. Over-against lin. 19. r. *Fig.* 147.

In the *Perspective*.

PAGE 6. line 37. for AB AC, read AB, BC. Lin. *ult.* for FH, read FO. Pag. 7. lin. 1. for FH, r. FO. Lin. 12. GLHM, r. GNM. Lin. 20. INKO, r. IKL. Lin. 25. ENFH, r. ENFO. GLHM, r. GNH. Lin. 30. ENFH, r. ENFO. INKO, r. ILK. Pag. 15. lin. 13. PG, r. FG.

The

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A

A
TREATISE
OF
MECHANICKS.

MECHANICKS are a Science that teaches to move heavy Bodies with eaſe and conveniency, by the means of Engines; whence it has alſo the Name of *Moving Powers*. It is call'd alſo in *French*, *Ingenieuſe*, becauſe it Wittily contrives Ingenious Machines; ſome of which Move of themſelves, Run, Jump, and Fly; and others raiſe and carry prodigious Burthens, and have ſtrange and ſurprizing Effects. It is alſo call'd *Staticks*, becauſe it examines, not only the Properties of Weight, and Local Motion; but alſo the Centers of Gravity, the *Æquilibrium*, and the Deſcent of Natural Bodies.

Nevertheleſs we ſhall here conſider *Staticks* as a part of *Mechanicks*, which we ſhall divide into Two Books; the Firſt ſhall treat of *Simple* and *Compound Engines*, and the Second of *Staticks*. We ſhall add a Third Book, which will give the Principles of *Hydroſtaticks*.

DEFINITIONS.

I.

MOtion, in General, is the change of a Thing; and when that change is made in the ſubſtance of the Thing, it is call'd *Generation*, or *Corruption*, which belongs to Phyſicks: But when it happens to the quantity of the Thing, it is call'd *Encreaſe*, or *Diminution*, which belongs to Geometry: And laſtly, when it is made in reſpect to Place, it is call'd *Local*

Motion, which belongs to Mechanicks; and which consequently we ſhall in a particular manner conſider here. Thus we ſhall ſay, that *Local Motion* is the change of Place, or the continual Paſſage of one Body that moves from one Place to another, whether in the Whole, or only in its Parts; that Motion may be *Equal*, which is that by which the *Mobile*, or a Body which moves, goes thro' Equal Spaces in Equal Times; as the motion of Celeſtial Bodies, which being Circular, ought to receive no alteration, becauſe it is made round a Center, which is equally diſtant: And *Unequal*, that which encreaſeth continually, when it is not interrupted, as the Motion of Terreſtrial Bodies; which is not Uniform, when they fall freely towards the Center of the Earth, as Experience ſhews every Day.

Galilæus calls that Unequal Motion, which is the Natural Motion of heavy Bodies, a *Motion uniformly accelerated*; becauſe Experience ſhew'd him that the Body, which moves by falling freely downward, acquires in Equal Times from its firſt falling, Equal Degrees of Velocity; that is to ſay, that dividing the time it takes up in falling, into equal parts, which we ſhall call *Moments*, the Velocity of the *Mobile*, or moving Body, at the ſecond Moment is double of what it was the firſt, which is reckon'd from the beginning of its fall; and that likewiſe the Velocity, which it acquires the third Moment, is Triple of that which it had the firſt, and the Velocity of the fourth Moment Four Times that of the firſt, and ſo on; which agrees exactly with the Experiments that have been made.

Whence it follows, that the Spaces which the moving Body has gone thro', are in a duplicate *Ratio*, or as the Squares of the Moments, and of the Velocities, becauſe the Moments and Velocities are ſuppos'd to encreaſe equally; and that the Spaces which are gone through, are in a *Ratio* made up of that of the Moments, and that of the Velocities: That is, that if a heavy Body is, for Example, 10 Minutes a falling, and that the firſt Minute (which we call a Moment) the Body deſcends a League, the ſecond Minute it will have deſcended Four Leagues; and the third Nine Leagues, and ſo on, counting from the point of reſt according to the Squares of the Natural Numbers, 1, 4, 9, 16, 25, *&c.* ſo that at the tenth Minute the Space that it ſhall have gone thro' will be of an Hundred Leagues.

Whence one may eaſily conclude, that each Moment, or Minute, the Spaces gone thro', grow one above another, according to the Series of the uneven Numbers 3, 5, 7, 9, *&c.* which are the differences of the Squares 1, 4, 9, 16, 25, *&c.* ſo that if a Body deſcends One League the firſt Minute, from the

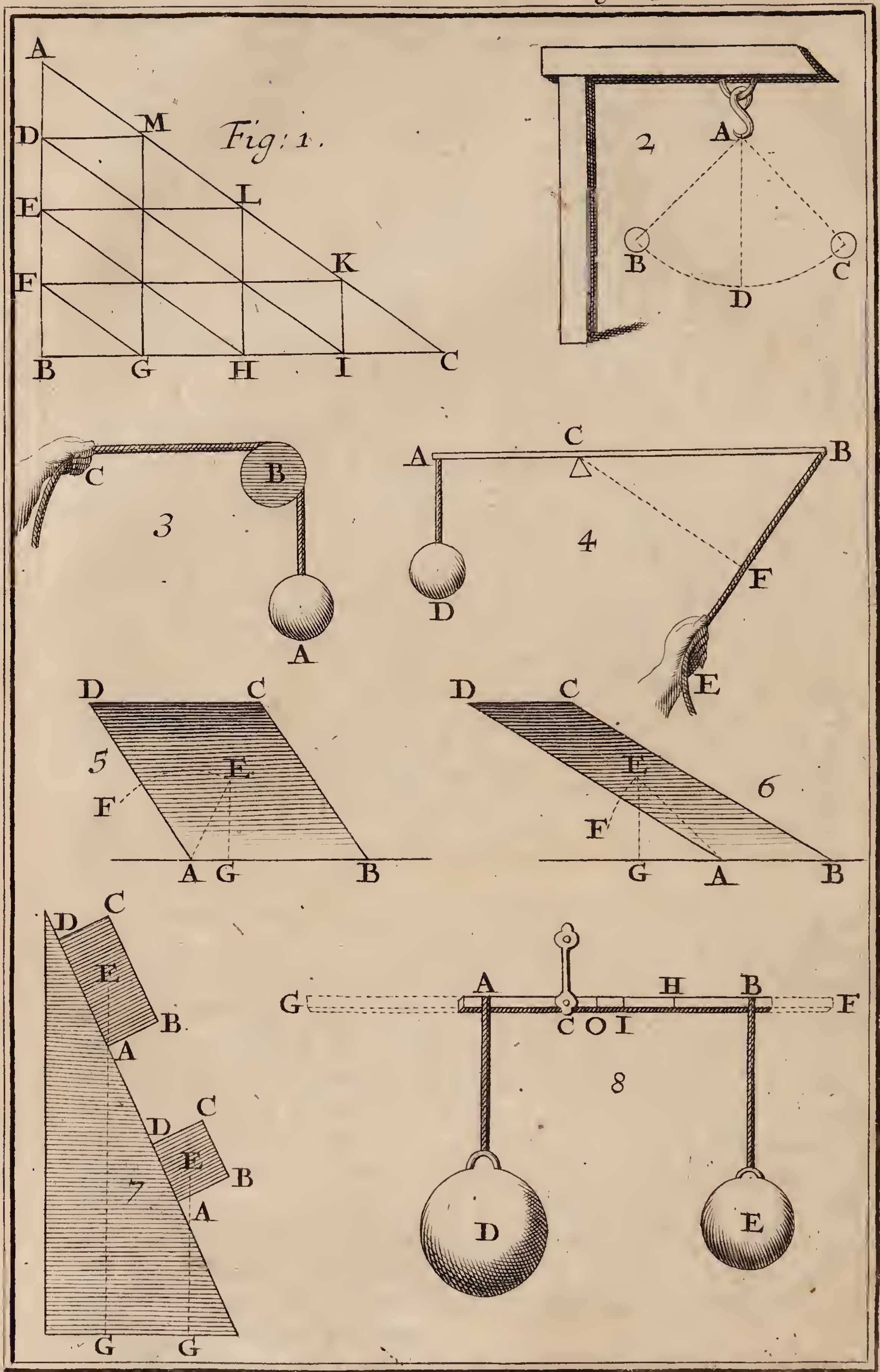
Fig: 1.
A
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the first to the second Minute it will have descended Three Leagues, or Three times as far as it did the first: And the Space which it will go thro' from the second to the third, will be of Five Leagues, or Five times as far as it descended the first, &c. *Plate* 1. *Fig.* 1.

Because, (*by the Fourth Proposition of the Sixth Book of* Euclid) similar Triangles are to one another as the Squares of their homologous sides, one may consider the Spaces gone thro' in equal Moments as similar Triangles, and the Moments and Velocities as the homologous sides of those Triangles. This will be better understood by the Triangle ABC, which we take for the Space gone thro' by the moving Body; which, for Example, has fallen in Four Minutes, whose Measure shall be the side AB, the base BC representing the Velocity, which the Body has acquir'd in falling.

If you divide the Time AB, and the Velocity BC, each into Four equal parts, because the time of the fall has been suppos'd of Four Minutes, each of those parts AD, DE, EF. FB, of the line AB, will represent One Minute, or One Moment, and each of the parts BG, GH, HI, IC, of the line BC will represent One Degree of Velocity, because we have suppos'd that the Velocities and the Moments encrease continually in the same Proportion. If you join the points of division of the Two sides AB, BC, by lines parallel to the third side AC, and that by the same points of division, parallel lines be drawn to the Two said sides AB, and BC, the Triangle ABC, will be divided into Sixteen little Triangles, equal to one another, and similar to ABC.

After this Construction and the Supposition that we have made, it will be easily known that the line AD representing the first Moment of the fall of a Body, the line DM, or BG its equal, will represent the Velocity acquir'd by a Body falling the first Moment, and the Triangle ADM will represent the Space, which that Body has gone thro' with One Degree of Velocity. It will likewise appear, that the line AE representing the second Moment of the fall of the said Body, the line EL, or BH its equal, will represent the Velocity which the Body has acquir'd in falling the second Moment, and the Triangle AEL will represent the Space that this Body has gone through, with Two Degrees of Velocity, which Space AEL is Four times as much as the first Space ADM, &c.

II.

Vibration is the Circular Motion of a Body, usually Spherical, as B, or C, which is call'd *Pendulum*; because it is suspended by an inflexible Thread AB, or AC, which *Fig.* 2.

Plate 1. *Fig.* 2. is tied tothe fix'd Point A, which point is call'd the *Center of reciprocal Motion*, because the *Pendulum* moves about that point A to return to D its lowest point, which is the point of rest, when it has been taken from it; moving first on one side, then on the other side of that point D by Arches of a Circle, as BDC; which is call'd *simple Vibration*, when the Weight has mov'd from B to C, to distinguish it from *compound Vibration*, which is the Arch BDC redoubled, describ'd by the reciprocal Motion of the Weight, when from B it has mov'd to C, and from C it has gone back almost to the same Point B, from whence its Motion began. The length AB, or AC, of the inflexible Thread, reckoning it from A the Center of Motion to the Center of the *Pendulum*, is call'd the *Length of the Pendulum.*

All the Vibrations of the same *Pendulum*, whether great or small, are perform'd very near in the same space of Time, that is, a *Pendulum* is about as long a coming back from C to B, as it was in going from B to C. But the *Pendulums* of different lengths have an unequal number of Vibrations in an equal time, because a longer *Pendulum* takes up more time in in its Vibrations than a shorter: And it has been found by several Experiments, as we have already said in our Geometry, that the lengths of Two *Pendulums* are reciprocally proportionable to the squares of the numbers of their Vibrations in an equal time; that is, the length of the first *Pendulum* is to that of the second, as the square of the number of the Vibrations of that second in such a time, is to the square of the number of the Vibrations of the first in the same time.

It is observ'd in liquid Bodies, as in Water, that they have another circular Motion, call'd the *Motion of Undulation*, which happens when a heavy Body is thrown into the Water, which makes the parts of the Water turn in a Circle; and this motion is call'd *Undulation.*

III.

Heaviness, which is also call'd *Weight*, and *Gravity*, is the natural Inclination which is in heavy Bodies to move, when they are not born up, and fall towards the Center of the Earth; which for that reason is call'd *Centrum gravium*, or the *Center of heavy Bodies.*

The *Specifick Gravity* of a Body, is that which proceeds from the natural density of the parts of its matter, which makes one Body weigh more than another of the same bulk. Thus we know that the Specifick Gravity of Water is greater than that of Oil, that the Specifick Gravity of Gold is greater than that of Silver, *&c.*

But

But the *Absolute Weight* of a heavy Body, is the force which it has to descend freely in a fluid *Medium*; when it touches nothing else but the parts of that *Medium*; as the absolute Weight of a Stone which is in the Air, is the force which it has to descend freely, when it touches nothing else but the parts of the Air.

Lastly, we call the *Relative Weight* of a heavy Body, which the Latins call *Momentum*, and the Greeks ῥοπὴ, the Force which such a Body has to descend when it touches something else besides the parts of the *Medium*, as when it bears on an inclin'd Plain, or the end of a Leaver, or of a Balance; where it often happens that the Body in question becomes a Counterpoise to a greater Weight, as it is nearer or farther from the Center of Motion; and this is call'd *Æquilibrium*. It is plain, that the Absolute is greater than the Relative Gravity of a Body, which is compounded of the Absolute Gravity, and the distance from the fix'd Point; which makes the Body act with more or less Facility, according as 'tis more or less distant from the fix'd Point.

IV.

A *Power*, is whatever can move a heavy Body, whence it is also call'd the *Moving Force*. Thus Weight is a Power in reference to the heavy Body, which it may move; and that Power is call'd *Inanimate Power*, to distinguish it from *Animate Power*, which is the Power of an Animal.

We make an estimate of the *Quantity of a Power*, by the Quantity of the Weight of the Body which it sustains, drawing it or pushing it upwards merely in the same line in which it endeavours to descend. Thus we may say, that a Power is *double* or *triple* another Power, when it does sustain twice or thrice as much as the other Power.

V.

The *Center of Motion* of a heavy Body, or the *fix'd Point*, which the Latins call *Ansa* or *Fulcrum*, and the Greeks ὑπομόχλιον, or *Propping Point*, is that by which a Body is held, and about which it may be mov'd. This Point in a Balance is that which it hangs by, and in a Leaver that whereon the Engine rests.

VI.

The *Center of Gravity* of a heavy Body, is a Point by which a Body being ſuſpended, all its parts, which are about that Point, will balance one another, and reciprocally hinder one another from falling; ſo that it will remain in any given poſition.

It is evident, that the Center of Gravity wou'd joyn it ſelf to the *Centrum Gravium*, if the Body cou'd deſcend thither; and that that Center of Gravity, in a regular and homogeneous Body, is the ſame with its *Center of Magnitude*, which is a point in that Body as far diſtant as it can be from all its extremities.

A *Body* is call'd *Homogeneous*, whoſe matter is uniform, and every where of the ſame weight, and *Heterogeneous*, whoſe matter is of different weight in different parts. It is plain, that a liquid Body has of it ſelf no Center of Gravity, becauſe its parts are not fix'd to one another, but are in a continual motion, as Water, and whatever is call'd Liquor.

This holds in Fluids, tho' a fluid Body is not altogether the ſame as a liquid Body; for a *Liquid Body* is that which being in ſufficient quantity flows continually, and ſpreads it ſelf below the Air till its upper ſurface is level, or in a horizontal poſition: and a *Fluid* is ſuch a Body as is eaſily paſs'd thro', and whoſe ſeparated parts joyn again immediately, as Air, Flame, Water, Oil, Mercury and other Liquors.

VII.

The *Line of Direction* of a heavy Body, or of a Power, is the Right-line in which that Body, or that Power endeavours to move. In a heavy Body, 'tis the Right-line in which it endeavours to deſcend; and in a Power, the Right-line by which that Power draws or puſhes a Weight to ſuſtain it, or move it.

Plate I. Fig. 3. As if the Weight A hanging from the point B, by the Thread AB, by its Weight endeavours to deſcend along the line AB, which is its Line of Direction; but if the Thread AB, going over a Pulley B, is continu'd towards C, where there is a Power which hinders the Weight A from deſcending, by drawing it in the line BC, that line BC is the Line of Direction of the Power in C.

VIII.

VIII.

The *Application of a Power to a Leaver*, is the Angle *Fig. 4.* which the Line of Direction of that Power makes with the Leaver. Let AB be a Leaver, whose fix'd Point is C, and let a Power in E sustain the Weight D, suspended at the end A, by the Thread AD, in such manner that the Right-line BE be the Line of Direction of that Power; and the Angle ABE, which that Line of Direction BE makes with the Leaver AB, is the Application of the Power to that Leaver AB. We shall hereafter demonstrate, that a Power being applied at right Angles, is capable of a greater force than if it was applied at oblique Angles; because in such a case it comes nearer to the fix'd Point, as you will see in the following Definition.

IX.

The *distance of a Power*, or of *a Weight*, is a Line drawn perpendicular from the fix'd Point of any Engine upon the Line of Direction. As if the right-line BE be the Line of Direction of the Power at E; its Perpendicular CF, which comes from C, the fix'd Point of the Leaver AB, will be the distance of the Power; as if that Power was in F, which distance wou'd be equal to the line BC, if the Line of Direction BE was Perpendicular to it. Wherefore the distance of the Weight D, whose Line of Direction AD, is perpendicular to the Leaver AB, will be the part AC, as if the Weight was at A.

X.

The *Center of Percussion*, is that Point by which a Body in its Motion strikes, with its greatest Energy, another Body which opposes its Motion. 'Tis evident that the Center of Percussion is in respect to the Velocities, what the Center of Gravity is in respect to Weight.

SUPPOSITIONS.

I.

ALthough the Superficies of the Earth be Gibbous, or Convex, yet we will suppose a small part of it Plain, or Flat; because our Senses make us think it so, and that

Suppoſition cannot lead us into Error, in reſpect of Engines, becauſe of the ſmall Extent of even the greateſt Engine, when compar'd with the whole Surface of the Earth.

II.

When heavy Bodies fall freely, they tend to the *Centrum Gravium*, or to the Center of the Earth, by Right-lines perpendicular to its Surface, and conſequently parallel to one another.

This Suppoſition is a conſequence from the foregoing one, tho' in ſtrictneſs it be as falſe as that, it being certain that Right-lines, which meet in a point, cannot be Parallel. Nevertheleſs there is no danger in ſuppoſing them ſuch, becauſe the Bodies which we compare, are ſo near one another, and the concourſe of the Lines of Direction ſo far from us, that they may paſs for Parallel without any ſenſible Error.

Whence it follows, that Two oppoſite Walls of a ſquare Chamber, exactly built according to the Rule and Plumb-line are parallel,(tho' in a Mathematical ſtrictneſs one may ſay, that they are farther diſtant at the top than at bottom) becauſe the difference is too ſmall to be diſcern'd by our Senſes.

III.

The Bodies of the greateſt ſpecifick Gravity, when not hinder'd, come nearer to the Earth, than thoſe whoſe ſpecifick Gravity is leſs. So we ſee by Experience, that Wood, Oil, Wax, and ſeveral other Bodies that are of a leſs ſpecifick Gravity than Water, ſwim upon that Water; and if by force they are retain'd at the bottom of the Water, they will riſe again above the ſame Water as ſoon as they are left at liberty. And Bodies of a greater ſpecifick Gravity than Water, as Stone, and Metals, ſink to the bottom of it.

IV.

All the parts of a hard Body are at reſt, and join'd to one another. A *Hard Body* is a Body that cannot be eaſily paſs'd thro', and whoſe parts being divided when it is paſs'd thro', do not come together again; which diſtinguiſhes it from a Fluid, as Water, which is eaſily paſs'd thro', and whoſe parts being divided (as with a Stick, for Example) immediately join again when you take out the Stick.

V.

V.

The weight of a Body bears it ſelf upon what-ever ſuſtains it. Experience ſhews, for Example, that if any one bears up a Bucket of Water hanging at a Rope, he feels all the weight of the Bucket, the Water, and all the Rope as one Weight: And that whoever holds a Stick by one end bears all the weight of that Stick.

VI.

Though all the Engines, which are us'd in Mechanicks to raiſe Bodies of a vaſt Weight, are very imperfect, it being impoſſible to give 'em all the juſtneſs and exactneſs which the Theory requires; nevertheleſs we ſhall in the ſequel, ſuppoſe 'em without Imperfection; that by means of that Suppoſition we may draw juſt Conſequences, and foreſee well enough what Engines will do, by Reaſonings drawn from the preceding Suppoſitions, and following Axioms. So that we ſhall ſuppoſe Bodies perfectly Hard, and perfectly Smooth, and of a homogeneous Matter: Lines perfectly Streight, without Weight, Thickneſs, or Flexibility, unleſs when we make expreſs Mention of it: Cords extremely Pliable, *&c.*.

AXIOMS.

I.

THE *Center of Gravity* is an indiviſible Point; that is, a heavy Body cannot have Two different Centers of Gravity: And as we have already ſaid, in Regular and Homogeneous Heavy Bodies, the Center of Gravity is the ſame with the Center of Magnitude.

II.

The different Weights of Bodies Homogeneous and of the ſame Matter, are to one another as the Maſs or Solidity of thoſe Bodies. As if a cubick Foot of a certain homogeneous Matter weighs, for Example, One Pound; Two cubick Feet of the ſame Matter will weigh Two Pounds.

Hence a way is found to know the Weight of a Homogeneous Body by its Solidity known in cubick Feet or Inches; or its Solidity by knowing its Weight in Pounds or Ounces, having

having once found out, by means of a Balance, the Weight of a homogeneous Body of the same Matter; and its Solidity by Geometry: After which the Solidity of a propos'd Body of known Weight, or the Weight of a Body of known Solidity, may be easily found by the Rule of Three direct.

III.

The Weight of a homogeneous Body is equally distributed in all its parts: And if that Weight were reduc'd to the Center of Gravity of that Body, it wou'd Move it but as it did before; for the Center of Gravity rules all. So when we have said, that heavy Bodies endeavour to descend in Right-lines, towards the Center of the Earth; this is to be understood in respect to their Centers of Gravity: And one may say that the Line of Direction of a Body that descends freely, is a Right-line drawn from the *Centrum Gravium* thro' the Center of Gravity of that Body.

IV.

A heavy Body always descends to the lowest place that it can go, when it does not meet with any heavy Body that opposes its descent, which is to be understood of its Center of Gravity, where the chief endeavour to descend is made; so that to the end that a Body may Move, its Center of Gravity must descend; otherwise it cannot Move.

Plate 1. *Fig.* 5. So one may see that the inclin'd Body ABCD, which is set upon a horizontal Plain, cannot fall towards the part D, which it inclines to, because its Center of Gravity E wou'd rise; as one may know in describing from the point A, as Center, the Arch EF, which is the same, as E the Center of Gravity, wou'd make about the point A, if the Body ABCD cou'd fall; because one part of that Arch rises above the point E.

Fig. 6. But one may see that the inclin'd Body ABCD, which leans upon a horizontal Plain, ought of necessity to fall towards the part D, to which it inclines; because its Center of Gravity E may descend; as it will appear by drawing, as before, from the point A, thro' the point E the Arch EF, which is the same that will be describ'd by the Center of Gravity E, about the point A, when the Body ABCD shall fall; because all the points of that Arch fall below the point E, as it is easy to demonstrate.

So that one may see, that to have a Body remain firm upon any thing that supports it and is not inclin'd, the Line of Direction

rection must fall necessarily in some part of the Foot, or Base of the said Body; which will of necessity fall down, if its Line of Direction falls any where out of that Base, as in the Sixth *Figure.* *Plate* 1. *Fig.* 6.

Whence it follows, that by how much less the Base of a Body is, even tho' it shou'd not be inclin'd, so much more easily it will move out of its place; because the least Change is capable to make the Line of Direction go out of its Foot: Which is the reason that a Bowl rolls easily upon a Plain, and that a Needle can't stand upon its Point.

It follows also, that the wider the Base of a Body is, the easier it will support itself, because then the Line of Direction can't go out of the Base without a greater Change. No wonder then, that inclin'd Tow'rs, as that of *Bolonia*, and that of the *Escaliers*, which seem to threaten their ruin, don't fall.

One may also easily see, that if the Plain, which sustains the Body ABCD, be inclin'd, that Body will Slide when its Line of Direction does fall into any part of its Base AD: And that it will Roll, when its Line of Direction EG shall fall out of the said Base AD, as it will happen to the Body ABCD, which is below the other mark'd with the same Letters. *Fig.* 7.

Whence it follows that a Bowl set upon an inclin'd Plain, as the Roof of a House, will roll incessantly till it come to the lowest place, because its Line of Direction not being perpendicular to the Plain, since it is perpendicular to the Horizon, cannot pass thro' its Foot where it touches the Plain, it being almost an indivisible Point.

We *naturally observe this Law of Mechanicks* upon all occasions, to keep our selves from Falling; as when we have a mind to Rise from a Seat, we bend our Body so as the Line of Direction of our Bodies may pass thro' our Feet, upon which we bear our selves when we begin to rise.

Painters and Carvers ought to observe this Law; that is, they ought not to give their Figures such *Postures* as they cannot naturally have.

Other Animals also naturally observe the same Law to Stand, and keep themselves from Falling.

It follows also that the Body B, or C, which hangs from the point A, will remain at Rest, when its Line of Direction passes thro' that point A; because it will be at the lowest place D, from whence if the Body be remov'd, it will come back thither by its own Gravity, because then its Center of Gravity may descend; but it won't stop there till after a certain number of Vibrations caus'd by the Velocity that it will acquire in endeavouring to go to it, which will cause it to go beyond *Fig.* 2.

the

the ſaid Point and riſe again by a *violent Motion*; that is, by a Motion which is given it contrary to its Nature.

What we have ſaid of the Center of Gravity of a heavy Body, ought alſo to be underſtood of the *common Center of Gravity* of Two heavy Bodies, which is the point of a Leaver, or of a Balance, about which the Two Bodies faſten'd to that Leaver, or that Balance, remain in *Æquilibrio*; becauſe thoſe Two may be conſider'd as One Weight, whoſe particular Center of Gravity is the ſame as the common Center of Gravity of thoſe Two ſeparate Bodies.

That is to ſay, that as heavy Bodies move but to deſcend, and that they always deſcend as much as they can, whether they do it by Inclination, or by any extrinſick Cauſe; if the deſcent of Two Bodies one to another is hindred, they will put themſelves in ſuch a Poſition as the Center of Gravity may want the leaſt Motion to end their deſcent.

V.

Two equal Weights that are faſten'd by their Centers of Gravity at each end of a Balance, ſuſpended by the middle, that is, whoſe fix'd Point is preciſely in the middle of thoſe Two Weights, are in *Æquilibrio*; becauſe being equal, there is no reaſon why one ſhould deſcend ſooner than the other.

VI.

If a Power can ſuſtain a Weight by the means of an Engine; a Power greater, as little as can be imagin'd, will overpoiſe or cauſe the ſaid Weight to move.

VII.

If Two Weights hanging at ſuch Diſtances from the fix'd Point, are in *Æquilibrio*, Two other Weights equal to them Two, put in their room, will be alſo in *Æquilibrio*.

VIII.

A Weight equal in Gravity to a Body, hanging from the Center of Gravity of that Body, will have the ſame effect as the Weight of that Body, which in this caſe ought to be look'd upon as nothing.

This

This Axiom is equivalent to the Third; for to hang from the Center of Gravity of a Body, a Weight equal to that Body, is the ſame thing as to reduce all the Weight to the Center of Gravity.

IX.

The Weight, or Power, which puſhes or draws a certain Point of its Line of Direction, puſhes or draws after the ſame manner, all the other Points which are in the ſame Line of Direction.

As if a Power applied in E, ſhou'd ſuſtain the Weight D, drawing by the Line of Direction EB, that Power will draw after the ſame manner all the Points of the ſame Line EB. *Plate* 1. *Fig.* 4.

Whence it follows, that the effect of the Power won't at all be chang'd; if inſtead of placing it in E, we ſhould place it in F, or in any other Point of the ſame Line of Direction FB.

It follows alſo, that the Weight D weighs as much near the Ground, as when it is farther off; becauſe we attribute no Gravity to the Rope AD, which ſuſtains it.

And that a Power applied at oblique Angles, has leſs force than that applied at right Angles, and which conſequently is farther off from the fix'd Point, which encreaſes its force, as it is plain, by *Prop.* 1. of the Balance, which we ſhall ſpeak of in the following Book.

The First Book.

OF Simple *and* Compound *ENGINES.*

WE call *Engine*, what-ever Motion may be caus'd or hindred with: And *Simple Engine*, (which is properly call'd *Organon*, or *Instrument*) that which is made only of One Piece, as a Leaver.

Simple Engines are usually Six, (*viz.*) the *Balance*, the *Leaver*, the *Pulley*, the *Axle in a Wheel*, the *Wedge*, and the *Screw*, which we shall treat of in their order in the following Chapters.

CHAPTER I.

Of the Balance.

THE *Balance* is a streight inflexible Rod, without Weight, moveable about a fix'd Point, and laden at each side of the fix'd Point, with one or many Weights, which are fasten'd to it by their proper Centers of Gravity.

A *Balance* is said to be *Horizontal*, when it's parallel to the Horizon: And *Inclin'd*, when it leans to the Horizon more at one end than the other. The fix'd Point divides the Balance into Two Parts, which are call'd the Two *Brachia*, and which, taken together, make the *Beam* of the Balance.

PROPOSITION I.

THEOREM.

If Two Weights, tied to the Ends of a Horizontal Balance, are to one another reciprocally as their distance from the fix'd Point, they shall hang in Æquilibrio.

I Say, that if from the Two ends A, B, of the Horizontal Balance AB, whose fix'd Point is C, there hang the Two Weights D,E, of which the first D: is to the second E:: reciprocally as the distance BC of that second: to the distance AC of the first; these Two Weights D,E, shall be in *Æquilibrio* about the point C, so that That point C shall be their common Center of Gravity. *Plate* 1. *Fig.* 8.

PREPARATION.

Lengthen the *Brachium* AC of the Balance towards G, so that the line AG be equal to the other *Brachium* BC, and likewise the *Brachium* BC towards F, so that the line BF be equal to the other *Brachium* AC: And then the fix'd Point C will be precisely in the middle of the Two points F, G; that is, the Two parts CF, CG, shall be equal to one another; so that if the Line FG be considered as a homogeneous Cylinder, the fix'd Point C, which is its Center of Magnitude, by *Def.* 6. will be its Center of Gravity, by *Ax.* 1. Transfer again AG to AH, or which is all one, BF to BH; because if from the Two equal lines AH, BC, the common line CH be taken, the line AC will remain, or BF equal to BH. Therefore considering the Two lines GH, FH, as Two homogeneous Cylinders of equal Bases join'd at the point H, their Centers of Magnitude A,B, will also be their Centers of Gravity, by *Ax.* 1.

DEMONSTRATION.

Because by *Supp.* the Weight D: is to the Weight E:: as BC: is to AC, or as AH: is to BH, or as the double GH: is to the double FH, and that by 14. 12. the Cylinder IH: is also to the Cylinder FH:: as the length GH: is to the length FH, the Weight D: will be to the Weight E:: as the Cylinder GH: to the Cylinder FH; so one may attribute to the Cylinder GH all the weight of the *Pondus* D, which hangs from its Center of Gravity A, and to the Cylinder FH all the weight of

Plate 1. *Fig.* 8. of the *Pondus* E, which hangs from its Center of Gravity B, which will make no alteration, by *Ax.* 3. and 9. And as the point C is the common Center of Gravity of the Two Cylinders GH, FH, or the particular Center of Gravity of the One Cylinder GF, it will also be the common Center of Gravity of the Two Weights D, E; so that these Two Weights D, E, must remain in *Æquilibrio* about the fix'd Point C. [*Q. E. D.*]

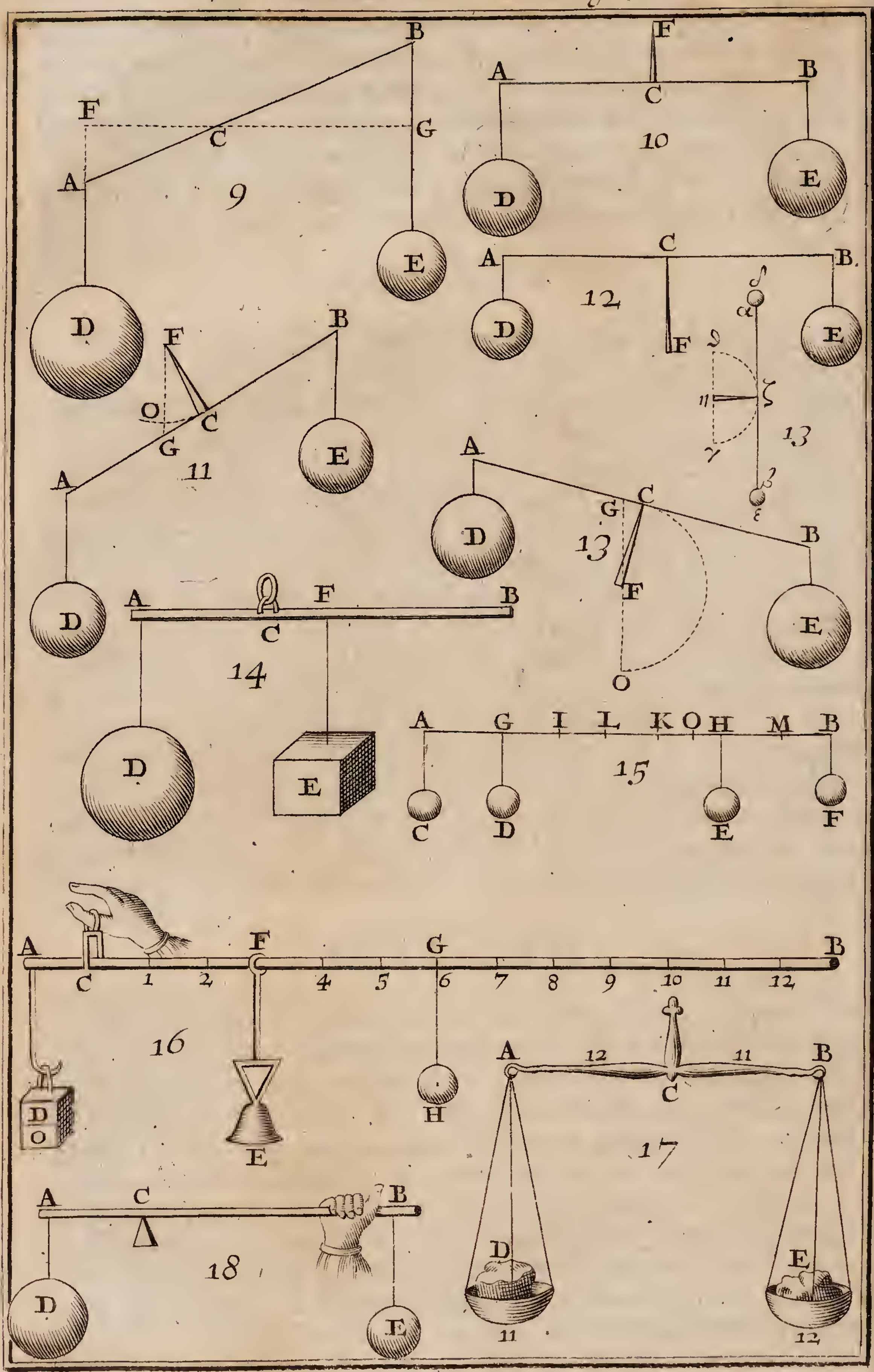
COROLLARY.

'Tis plain from this Proposition, that if the Weights D, and E, are equal to one another, and their distances AC, BC, likewise equal to one another; the Two Weights D, E, will also be in *Æquilibrio* about the fix'd Point C: And that if the same Weights D, E, are unequal, the least E must be as much more distant from the fix'd Point C, as the biggest D; that is, as much as to say, the distance BC: must be as much greater than the distance AC:: as the Weight D: is greater than the Weight E; so that if that Weight D be, for Example, double the Weight E, the distance BC also must be double the distance AC, that the least Weight E may equiponderate with the greatest D. Whence it is easy to conclude, that if the distance BC be ever so little encreas'd, that Weight E which answers to that distance will sink lower; and likewise if the distance AC of the Weight D be ever so little encreas'd, that Weight D will sink: Or else without changing the distance AC, BC, if either of the Weights D, or E be ever so little increas'd it will overpoise and throw up the other.

SCHOLIUM.

The inverse Proposition is also true, (*viz.*) that *if the Weights D, E, are in* Æquilibrio *about the fix'd Point C, they will be to one another in a reciprocal* Ratio *to their distances BC, AC,* because otherways One of these Two Weights wou'd overpoise, (*viz.*) that which shou'd have a greater *Ratio* to the other, than the distance of that other to the distance of the first, as we have just observ'd in the foregoing Corollary.

Plate 2. *Fig.* 9. We have also suppos'd in the Proposition, that the Balance AB was Horizontal; nevertheless the Proposition will be as true in an inclin'd Balance, because by *Supp.* 2. the Lines of Direction of the Two Weights D, E, which hang freely from the Two points A, B, being parallel to one another, the Two Weights D, E, act upon the inclin'd Balance AB, as upon the Horizontal one FG, because of the Two similar Triangles ACF, BCG, where one may know that the Two distances CF, CG, are proportional to the Two lines AC, BC,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BC, which conſequently may be taken for the true diſtances of the Weights D,E, from the fix'd Point C; it being certain by *Ax.* 9. that the Weights D,E, which are faſten'd to the Two ends A,B, of the inclin'd Balance AB, have the ſame effect as if they were faſten'd to the ends F, G, of the horizontal Balance FG, whoſe fix'd Point is the ſame point C.

PROPOSITION II.

THEOREM.

If a Balance having its Center of Motion above, and at each end a Weight equally diſtant from the fix'd Point, is in a horizontal Poſition, it will remain in that Situation; but if its Poſition be alter'd by inclining it on either ſide, it will return to its firſt Situation.*

**That is, Above the Center of Gravity.*

I Say firſt, that if the Balance AB, (which has at its ends A, B, the equal Weights D,E, which draw by equal diſtances AC, BC, and has its Center of Motion upwards at the point F, which anſwers perpendicularly to C the middle point, which is the common Center of the Two equal Weights D,E, ſo that the line CF be inflexible,) be Horizontal, it will remain in that Situation; that is, it will be at reſt when ſuſpended by the point F. *Fig.* 10.

DEMONSTRATION.

Since the line AB is parallel to the Horizon, by *Supp.* its perpendicular CF will alſo be perpendicular to the Horizon, and conſequently will be the Line of Direction of the Quantity made up of the Two weights D,E, which is ſuſtain'd by the point F, ſo by *Ax.* 9. it is as if the Balance was ſuſpended by the point C, which is the middle of AB, in which caſe by *Prop.* 1. the Two Weights D,E, muſt remain in *Æquilibrio. Q. E. D.*

I ſay in the ſecond place, that if the Balance AB, be ſo inclin'd, that one of its ends, as A, ſink lower than the other end B, this Balance AB, as ſoon as it is at liberty will return of it ſelf to its firſt Poſition, which was Horizontal. *Fig:* 11.

DEMONSTRATION.

Becauſe the line AB is inclin'd to the Horizon, its perpendicular CF will alſo be inclin'd to the Horizon, and conſequently decline from the line FG, which is perpendicular to

Plate 2. *Fig.* 11. the Horizon, and which is the Line of Direction of the Quantity made up of the Two Weights D, E, which is sustain'd by the point F; whence by *Ax.* 9. it is as if it was sustain'd by the point G, from which the Weight E being farther distant than the Weight D, has more force to descend than the Weight D, by *Coroll. Prop.* 1. and therefore ought to cause the inclin'd Balance AB to return to its horizontal Position. *Q. E. D.*

For a farther proof of this, one may add, that the Weight E ought to descend, and the line CF place it self, in the line FG, because the point C, which is the common Center of Gravity of the Two equal Weights D, E, may in that manner descend, as it will appear by describing from the point F, where the Balance is suspended, thro' the point C, the Arch CO, which is the same as C the common Center of Gravity will describe, when the inclin'd Balance AB returns to its horizontal Position, and the line FC will fit it self to the Line of Direction FG, and that C the common Center of Gravity will descend as low as it can, to wit, to the point O, *&c.*

PROPOSITION III.

THEOREM.

This Theorem is false. See the Note at the bottom of the next Page.

If a Balance having its Center of Motion below, and at each end equal Weights equally distant from the fix'd Point, is in a horizontal Position, it will remain in it: But if it be inclin'd ever so little on either side, it will continue to move on that side till it has acquir'd a Position perpendicular to the Horizon.

Fig. 12. I Say first, that if the Balance AB, (which bears at its ends A, B, the equal Weights D, E, drawing by the equal distances AC, BC, and which has its Center of Motion below at the point F, which answers perpendicularly to the middle point C, so that the line CF be inflexible,) be Horizontal, it will remain in that Position.

DEMONSTRATION.

Because by *Supp.* the line AB, is parallel to the Horizon, its perpendicular FC will also be perpendicular to the Horizon, and consequently become the Line of Direction of the Quantity compounded of the Two Weights D, E, which is sustain'd

ſuſtain'd by the point F, on which all that compound Weight bears: So by *Ax.* 9. it is as if the Balance AB was ſuſpended by the point C middle of AB, in which caſe, by *Prop.* 1. the Two equal Weights D,E, muſt remain in *Æquilibrio*, that is, the Balance muſt be at reſt. *Q. E. D.*

I ſay in the ſecond place, that if the Balance AB be ever ſo little inclin'd, for Example, toward the Weight E, that Weight E and the whole Balance will continue to incline about the Center of Motion F, till it has taken a Situation perpendicular to the Horizon. * *Fig.* 13.

DEMONSTRATION.

Becauſe the line AB is inclin'd to the Horizon, its perpendicular CF will be alſo inclin'd to the Horizon, and conſequently decline from the line FG, which being perpendicular to the Horizon, and paſſing thro' the Center of Motion F, is the Line of Direction of the Quantity compounded of the Two Weights D,E, which bears upon the point F, whence by *Ax.* 9. 'tis juſt as if it was ſuſtain'd by the point G, from which the Weight E being farther diſtant than the

* *This Paragraph as well as the Theorem is falſe; for the Balance will not remain in the Poſition which is perpendicular to the Horizon; but turn round till AB the Balance becomes parallel to the Horizon, and the Center of Gravity C deſcends to O, the Center of Motion remaining the ſame at F; tho' by the alteration of the Balance it is above the Balance, as it was below it,&c. This may be prov'd from the known Laws of Mechanicks, and the Author's own Axioms.*

DEMONSTRATION.

Since by Def. 1. *the Two Weights* δ,ε, *fix'd at* α, β, *the End of the Balance* $\alpha\beta$ *have the ſame effect as if they hung from the ſaid Ends; as the Weights D, E, do from A,B; I ſay, that the Balance* $\alpha\beta$ *cannot remain in the perpendicular Poſition repreſented in the Figure, becauſe by* Ax. 4. *the Center of Gravity* ζ *muſt deſcend to* ϑ *the loweſt place that it can fall to.*

Thus the line $\kappa\zeta$ *will be again in the Line of Direction* $\gamma\vartheta$, *and the Balance will be again Horizontal, but below the Center of Motion, as it was above it, juſt as it is repreſented in Fig.* 10. Q.E.D.

The Theorem ſhou'd be thus expreſs'd, in the Words of the Ingenious Mr. *J. Keil* of *Chriſt-Church, Oxon. If the Center of Gravity be plac'd perpendicularly over the Center of Motion, it will remain in that Poſition; but if the Body be ever ſo little Inclin'd, the Center of Gravity will move, till it has plac'd itſelf under the Center of Motion in the ſame Prependicular.*

Weight D, must cause the Balance to incline more and more towards E, by *Coroll. Prop.* 1. and this must give the Balance AB a Position perpendicular to the Horizon. *Q. E. D.* *See the foregoing Note.*

For a farther proof of this, one may add that the Weight E must continue to descend, and the line CF to fall in with the Line of Direction FG, because the point C, which is the common Center of Gravity of the Two equal Weights D, E, may in that manner descend and come under the point F in C, which is the lowest place it can fall to, as one may know in describing from F, the point whereon the Balance bears, thro' the point C, the Arch CO, which is the same as will be describ'd by C the common Center of Gravity, when the inclin'd Balance AB will continue to move about the point F, to place it self in a Position perpendicular to the Horizon, *&c.* *

* *The Author shou'd have said, parallel to the Horizon See the last Note.*

PROPOSITION IV.

PROBLEM.

Knowing the Weight of Two heavy Bodies apply'd to the ends of a Balance of a known length, to find upon that Balance the common Center of Motion.

Plate 1. *Fig.* 8. LET us suppose that the Weight D, which hangs from the end A of the Balance AB, whose length is 24 Inches, be of 12 Pounds, and that the Weight E, which is fasten'd to the other end B, be of 6 Pounds. To find the common Center of Gravity of these Two Weights, or the fix'd Point, from which the Balance AB laden with the Two Weights A, B, being suspended, these Two Weights will be in *Æquilibrio*; seek to these Three Numbers 18,6,24, which are the sum of the Two Weights D, E, of the Weight E, and of the Balance AB, a fourth Number proportional, which will be 8 Inches for the part AC. If then AC be taken of 8 Inches, the fix'd Point C will be found, about which the Two Weights D,E, will remain in *Æquilibrio*.

DEMONSTRATION.

Because, *by the Construction*, we have this Analogy, D + E : E : : AB : AC, dividing we shall also have this other Analogy, D : E : : BC : AC, which shews by *Prop.* 1. that the Two Weights D, E, must remain in *Æquilibrio* about the point C. *Q. E. D.*

SCHO-

SCHOLIUM.

Tho' we have attributed no Weight to the Balance AB, yet 'tis impossible but that it should have some, which causes the foregoing Operation not to be true in strictness; for tho' the Weights D, E, are in a reciprocal *Ratio* of their distances AC, BC, and the point C consequently the Center of Gravity of the Quantity compounded of these Two Weights, yet the weight of the Balance AB is not comprehended in it; wherefore the *Æquilibrium* will be hinder'd, and the Balance will overpoise on the side of E the least Weight: For supposing the Balance AB to weigh, for Example, Three Pounds, in which case the *Brachium* AC will weigh One, and the other *Brachium* BC Two, or double the weight of the first, because the Weight D has been suppos'd to be double the Weight E, having reckon'd D 12 Pounds, and E 6; it will be known that the Quantity made up of the Weight D and the *Brachium* AC, is of 13 Pounds, and that the Quantity made up of the Weight E and the *Brachium* BC, is of 8 Pounds, and that the *Ratio* of 13 to 8 being less than that of BC to AC, the point C cannot be the common Center of Gravity of the Quantity made up of the Two Weights D, E, and the Balance AB. *Plate* 1. *Fig.* 8.

Therefore to solve more exactly the Problem propos'd, which exactness must always be observ'd when the Gravity of the Balance AB will be considerable, we must suppose a Weight equal to the Two D, E, to hang from C the common Center of Gravity, which by *Ax.* 3. will not alter the effect of the Two Weights D, E; and that from the middle point I, which is the Center of Gravity of the Balance AB, there hangs another Weight equal to that of the Balance; and considering CI as a Balance laden with its Two Weights at its ends CI, find out as has been taught, O the common Center of Gravity of these Two Weights, *&c.*

PROPOSITION V.

PROBLEM.

Knowing the Length and Weight of a Balance which has at one of its ends a Body of known Weight, to find the fix'd Point, about which the Weight of the Balance and the Weight of the Body shall remain in Æquilibrio.

SUpposing the Balance AB to weigh 16 Ounces, and to have 12 Inches in length: To find upon this Balance the point C, from which the Balance being suspended, by the *Plate* 2. *Fig.* 14

 help

Plate 2. *Fig.* 14. help of its own Gravity, ſhall keep in *Æqilibrio* the heavy Body D, which hangs from its end A, and is ſuppos'd to weigh 8 Ounces: To theſe Three Numbers 24: 16: 6: (which are the Sum of the Weight of the Body and the Bolance, the particular Weight of the Balance, and half of its length,) find out a fourth Proportional, which will be 4 Inches for the part AC. If then we take the part AC of 4 Inches, we ſhall have the point C, from which the Balance AC hanging, its Weight will be in *Æquilibrio* with the Body D.

DEMONSTRATION.

Imagine the Body E of equal weight with the Balance AB, to hang from F the middle Point, or Center of the ſaid Balance, which I ſuppoſe uniform, and equally heavy in all its parts; which will cauſe no alteration in the poſition by *Ax.* 3. Imagine alſo AF as a Balance without Weight, and laden with the Two heavy Bodies D, E, applied to its ends; it will appear that ſince by *Conſtr.* we have this Analogy, D + E : E : : AF : AC, by diviſion we ſhall have this, D : E : : CF : AC, which will ſhew by *Prop.* 1. that the point C is the common Center of the Two Bodies D,E, and conſequently that taking away the Body E and reſtoring to the Balance AB its Weight, it will be the common Center of Gravity of the Body D, and the Balance AB. *Q. E. D. & I.*

PROPOSITION VI.

PROBLEM.

Several Bodies of known Weight being applied to a Balance, to find upon that Balance the common Center of Gravity of all thoſe Bodies.

Plate 2. *Fig.* 15. TO find upon the Balance AB, to which we ſhall attribute no Weight, the Center of Gravity of the Quantity made up of the Four Bodies C,D,E,F, of known Weight; find by *Prop.* 4. upon the Balance AB, I the common Center of Gravity of the Two Bodies C,F, and upon the Balance GH, K the common Center of Gravity of the Two other Bodies D, E; and laſtly, upon the Balance IK, L the common Center of Gravity of a heavy Body applied at I, and equal to the Two C,F, and of another Body applied at K, and equal to the Two D,E; and that point L ſhall be the point about which the Four Bodies C,D,E,F, will be in *Æquilibrio.*

DE-

DEMONSTRATION.

Reducing the Two Bodies C,F, to I their common Center of Gravity, and likewise the Two Bodies D, E, to K their common Center, they will act upon the Balance IK, as upon the Balance AB, by *Ax.* 3 : And as they must be in *Æquilibrio* about the point L, in the Balance IK, because they are in a reciprocal *Ratio* of their distances LI, LK ; they will also remain in *Æquilibrio* about the said point L in the Balance AB. *Q. E. D. & I.* *Plate* 2. *Fig.* 15.

SCHOLIUM.

If there was besides these, another heavy Body, hanging from some other point of the Balance AB, as from the point M, the Weight of the Four Bodies C,D,E,F, must be reduc'd to their common Center of Gravity L, by imagining that from the point L there hangs a Body equal in Weight to the Four C, D, E, F, and the Balance LM must be divided in a point, as O, in such manner that the said *aggregate* Body applied in L, be to the Body hanging at M, as the distance OM is to the distance OL ; and that point O will be the fix'd Point requir'd.

PROPOSITION VII.

PROBLEM.

Two Bodies being given, the heaviest of which hangs at one of the ends of a Balance of known Length and Weight, and given fix'd Point; to hang that of least Weight in such manner, that being assisted by the Weight of the Balance, it may keep the heaviest Body in Æquilibrio *about the fix'd Point.*

LET us suppose the Balance AB to weigh 2 Ounces, and be 14 Inches long: Let us again suppose that the Body DO, which is applied to its end A, distant (for Example) an Inch from the fix'd Point C, weighs 15 Ounces ; to find the point F, where the Body E, which weighs (for Example) an Ounce, being applied and assisted by the Gravity of the Balance AB, shall keep the other Weight DO in *Æquilibrio* about the Center of Motion C ; divide the Balance AB, into Two equal parts at the point G, which by *Ax.* 1. will be its Center of Gravity ; and suppose the Body H of the same *Plate* 2. *Fig.* 16.

Plate 2. *Fig.* 16. Weight with the Balance AB, that is, of Two Ounces Weight, to hang from the said G. Then to these Three Numbers 1: 6:: 2: which are the distance AC, the distance CG, and the Body H, find a fourth Proportional, which will be 12 Ounces for the part O of the Body DO; wherefore the other part D will be 3 Ounces. Lastly, to these Three other Numbers 1: 3:: 1: (which are the Body E, the part D, and the distance AC,) find a fourth Proportional, which will be 3 Inches for the distance CF. If then the Body E be applied to the point F, which is 3 Inches distant from the fix'd Point C, this Body E shall keep the Body DO in *Æquilibrio* about C the Center of Motion.

DEMONSTRATION.

Since by *Constr.* the distance AC: is to the distance CG:: as the Body H, or the Weight of the Balance AB: is to the Body O: and that the Body E: is to the Body D:: as the distance AC; is to the distance CF; it follows, by *Prop.* 1. that the point C is the common Center of Gravity of the Two Bodies H,O, in the Balance AG, and of the Two Bodies ED, in the Balance AF. Whence it is easy to conclude, that it is also the common Center of Gravity of the Quantity made up of the Two Bodies D,O, (or of the Body DO alone) and the Two Bodies E, H: And that thus the point F is found, from which the Body E being suspended, and assisted by the Gravity of the Balance AB, keeps the Body DO in *Æquilibrio* about the fix'd Point C. *Q. E. D & I.*

COROLLARY.

By this Operation is found the manner how to divide the *Statera Romana*, or *Steel-yard*, call'd by the Greeks Φάλαγξ, which may be done in this manner:

Prepare a long Rod of Wood, or any other solid matter, as Iron, of an equal thickness all over, and equally heavy in all its parts, as AB, and find its just Weight with a common Pair of Scales. At its end A fasten a Hook, to hold whatever is to be weigh'd, and pretty near that end mark C for the Center of Motion, or for the fix'd Point. Apply beyond that point C, the Weight E moveable by means of its Ring F which Weight is call'd the *Counterpoise*, and must be of known Weight reckoning the Ring with it. Lastly, according to the former Operation, mark the points 1, 2, 3, 4, *&c.* where the Counterpoise E being applied successively, shall keep in *Æquilibrio* a Body of One Pound, a Body of Two Pounds, one of Three, one of Four Pound Weight, *&c.* which are suppos'd to be hang'd at A, one after another.

SCHO-

SCHOLIUM.

Tho' this Method be good in the Theory, I ſhou'd not be willing to rely upon it, by reaſon of the Irregularity that is uſually in the Matter: wherefore in the Practice it will be better, mechanically to mark thoſe points of Diviſion, by holding the Balance in a horizontal Poſition, and moving the Weight E from C towards B, to the Points, 1,2,3,4, *&c.* till it is equiponderant to One, Two, Three, or Four Pounds, and ſo on till the longeſt *Brachium* BC be full of different marks, and then the *Steel-yard* will be finiſh'd, which will be fit to weigh vaſt Burthens, whereas a common Pair of Scales can only weigh ſmall things. *Plate 2. Fig. 16.*

PROPOSITION VIII.

PROBLEM.

To make a Deceitful Balance, which being Empty, and alſo being laden with unequal Weights, ſhall remain in Æquilibrio.

MAKE one *Brachium* of the Balance, as AC, a little longer than the other *Brachium* BC, a Twelfth part, for Example; ſo that AC: may be to BC :: as Twelve: is to Eleven; and reciprocally let the Scale E, which belongs to the ſhorteſt *Brachium* be alſo a Twelfth part heavier than the Scale at D, which belongs to the longeſt *Brachium*, ſo that the Weight of E: be to that of D:: alſo as 12: to 11, to the end that theſe Two Scales being Empty, and their Weight being in a Reciprocal *Ratio* of their Diſtances AC, BC, may remain in *Æquilibrio* about the fix'd Point, *by Prop.* 1. *Plate 2. Fig. 17.*

If in theſe Scales Weights be put, which bear the ſame *Ratio* as 12 to 11, ſo that the leaſt Weight be in the lighteſt Scale, and the greateſt in the heavieſt Scale, theſe Scales fill'd with their Weights will be Quantities, of the ſame *Ratio* as 12 to 11, and conſequently of a reciprocal *Ratio* to their diſtances AC, BC, whence *by Prop.* 1. they will alſo be in *Æquilibrio* about the Center of Motion C.

Thus we ſhall have a falſe Balance, which being Empty ſhall hang in *Æquilibrio*, and being laden with unequal Weights after the manner aforeſaid, ſhall alſo hang in *Æquilibrio*: But to find out the Deceit, one muſt only change the Weights from one Scale to the other; becauſe if the Balance be falſe, it will then loſe its *Æquilibrium*; for the Weights help'd

help'd by the ponderosities of their Scales, will be no longer in the reciprocal *Ratio* of their Distances.

CHAPTER II.

Of the Leaver.

Plate 1. *Fig.* 4. THE *Leaver* is a kind of Balance, or Rod, as AB, which instead of being suspended from, rests upon a point as * C, which we have call'd the *fix'd Point*, or *Center of Motion*, having the Weight on one side, and the Power on the other. It has been so call'd, because it serves to bear, and raise Burthens with facility, and so much the more easily as the Power is more distant from, or the Weight nearer to the fix'd Point, as we shall demonstrate, after we have describ'd Two or Three sorts of Leavers of common use.

* *In a Leaver, this point is commonly call'd the Fulcrum or Fulcimen.*

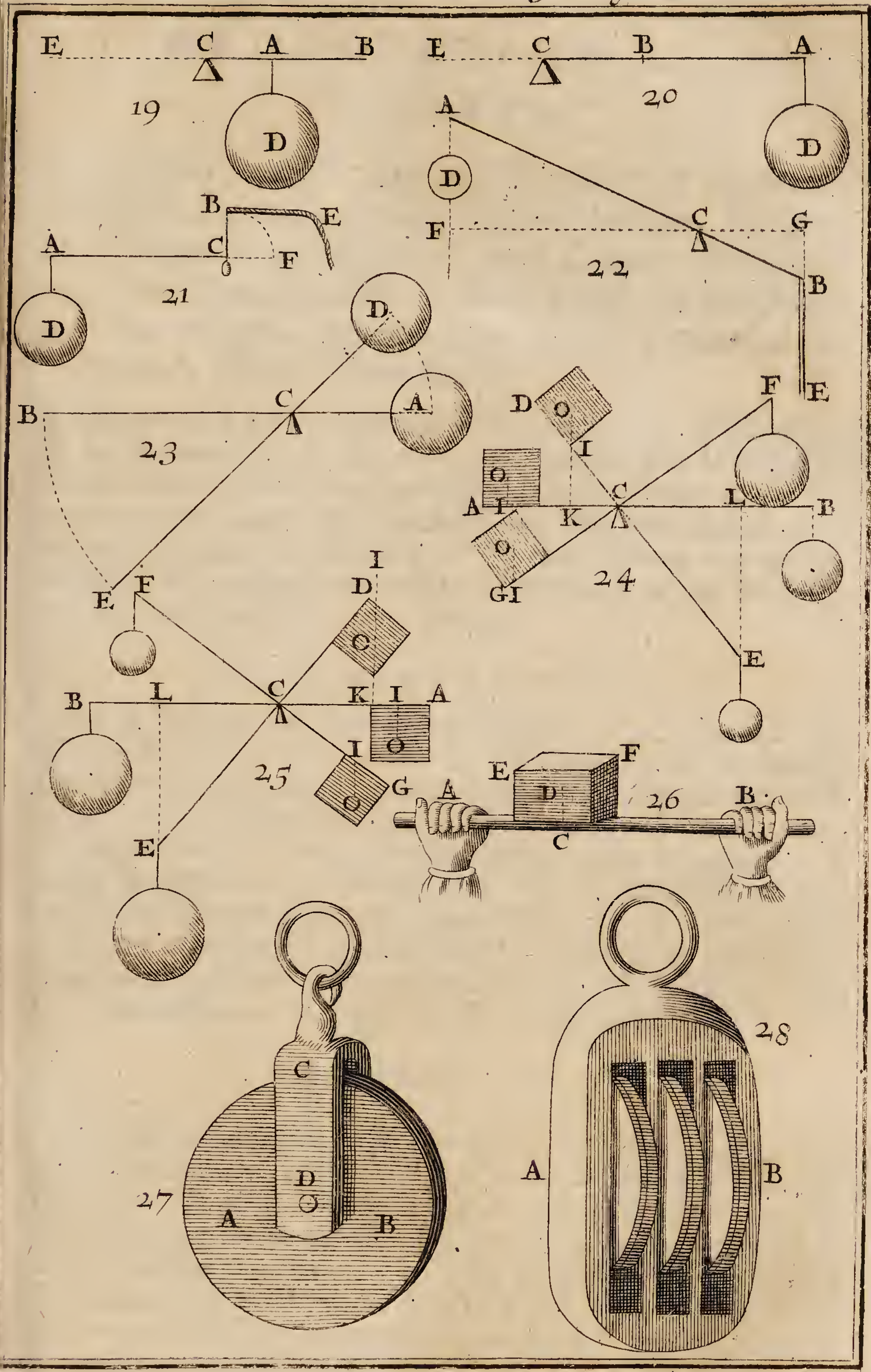
The *Leaver of the First Kind*, is that wherein the Prop, or *Plate* 2. *Fig.* 18. the fix'd Point C, is between the Weight suspended at the end A, and the Power applied at the other end B. It is plain that *Scissors, Pinchers, Snuffers*, &c. are Leavers of the First Kind.

The *Leaver of the Second Kind* is that wherein the fix'd *Plate* 3. *Fig.* 19. Point C is at one end, and the Power applied at the other end B, the Weight D being suspended at the point A between the ends; that is, between the Power and the fix'd Point. It is plain that the Oars and Rudder of a Boat are Leavers of the Second Kind; as also such Cutting Knives as are fix'd at one end, as those us'd by Druggists for cutting aromatick Wood and Roots, by Bakers for cutting their Bread, and Last-makers for cutting Wood: And likewise Doors, whose Hinges are as the fix'd Point.

The *Leaver of the Third Kind* is that whose fix'd Point C is *Fig.* 20. at one end, and the Weight D suspended at A the other end, the Power being applied at the point B, between the ends; that is, between the Weight and the *Fulcrum*. It is plain that a Ladder which is lifted by the middle in order to rear it against a Wall, is a Leaver of the Third Kind.

There is yet a Fourth Kind of Leaver, call'd the *Bended* *Fig.* 21. *Leaver*, whose use will be shew'd in the sequel: as ACB, so call'd, from its being bent at the fix'd Point C. It appears plainly that such a Leaver is of the First Kind, because the Weight D hangs at its end A, and the Power is applied at its other end B, where it draws by the Line of Direction BE. A Hammer to draw out a Nail is a Bended Leaver.

PRO-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PROPOSITION I.

THEOREM.

If a Power, whose Line of Direction is perpendicular to a Leaver parallel to the Horizon, bears up a Weight by the means of that Leaver, the Ratio *of that Power will be to the Weight, as the* Ratio *of the Weights distance from the fix'd Point to the Power's distance from the fix'd Point.*

I Say that if a Power applied in B, whose Line of Direction is perpendicular to the Horizon, sustains the Weight D, whose Center of Gravity corresponds to the point A, by the means of the Leaver AB parallel to the Horizon, whose fix'd Point or Center of Motion is C, that Power : is to the Weight D : : reciprocally as AC the Distance of the Weight : is to BC the Distance of the Power. *Plate 2. Fig. 18.*

DEMONSTRATION.

For if instead of the Power applied at B, you fix the Weight E, which shall keep the Weight D in *Æquilibrio* about the fix'd Point C, it will appear that That Leaver AB, which is of the First Kind, is only a horizontal Balance, concerning which we have demonstrated that the Weight E : is to the Weight D : : as the distance AC : to the distance BC : and as the Weight E has the same effect as the Power applied at B, and the same Line of Direction BE, it is necessarily equal to that Power, which consequently will be the Weight D, as the distance AC, to the distance BC. *Q.E.D.*

SCHOLIUM.

The Demonstration will be after the same manner in a bended Leaver, as ACB ; but one must suppose the Line of Direction BE of the Power to be parallel to the Horizon, or perpendicular to the Bended *Brachium* BC, which in this case will represent the distance from the Power to the fix'd Point C, because I suppose the Angle ACB is a right Angle ; for if the *Brachium* AC be lengthned beyond the *Fulcrum* C, towards F, in such manner that CF be equal to CB, and that the Power be applied at F to act downwards, it will have the same effect in F * as in B, because of the equal distances CB, CF, *&c.* *Plate 3. Fig. 21.*

* *That is, acting at B towards E in a Direction parallel to the Horizon ; as it must act at F, in a Direction perpendicular to the Horizon.*

The

Plate 3. *Fig.* 19, 20. The Demonstration will also be the same in a Leaver of the Second and of the Third Kind; for if in either case you lengthen the Leaver beyond the *Fulcrum* C, towards E, so that the line CE be equal to the distance BC of the Power, and that instead of applying the Power at B, it be applied at E, it will at E have the same effect as at B; and, as the Power in E: is to the Weight D :: as the distance AC: to to the distance CE, in the Leaver ECA, which is of the First Kind; the same Power at B: will be also to the Weight D :: as the distance AC :: to the distance CE; or BC its equal.

COROLLARY.

From this Proposition, we shall draw the same Consequences as have been drawn from the First Proposition concerning the Balance; namely, that the farther the Power is distant from the fix'd Point C, the more Force it will proportionably have; so that if the Power that is in B, be remov'd from the *Fulcrum* C, twice the length BC, it will need but half the Force which it wanted at B, to support the Weight D. That is, if the Weight E (of an Hundred Pounds, for Example) being apply'd at B is able to sustain the Weight D at the distance BC, a Weight of 50 Pounds only will be able to sustain the same Weight D, at a distance which is double that of BC. (*Plate* 2. *Fig.* 18.)

Plate 3. *Fig* 21. Whence it is easy to conclude, that as in the Leaver of the First Kind, and as in the Bended Leaver, which is also of the First Kind, AC the distance of the Weight may be greater or less than BC the distance of the Power; so the Power may be greater or less than the Weight; in such manner that it will be Equal to it when the two Distances AC, BC, happen to be equal, as in the Balance.

Fig. 19. Now, as in using the Leaver of the Second Kind, the Distance AC of the Weight D, must necessarily be less than BC the Distance of the Power at B, so the Weight must of necessity be greater than the Power; that is, the Power has more Force than a Weight equal to it, and a Little Power can sustain or raise a Greater Weight; and as on the contrary, in using the Leaver of the Third Kind, the Distance AC of the Weight D, must of necessity be greater than BC, the Distance of the Power at B; so also must the Weight be less than the Power,

Plate 3. *Fig.* 22. Hence it is easy to conclude, that what has been demonstrated concerning the Leaver of the First Kind parallel to the Horizon, is also true of AB the Inclin'd Leaver if the Weight D hangs freely, and BE the Line of Direction of the Power at B is perpendicular to the Horizon; because in such

a caſe it will be parallel to AD, the Line of Direction of the Weight D, which is alſo perpendicular to the Horizon, by *Supp.* 2. whence the two Rectangular Triangles ACF, BCG, are ſimilar, ſuppoſing the Line FG parallel to the Horizon; and that the Weight D apply'd at A, and the Power apply'd at B, act the ſame way upon the Inclin'd Leaver AB, as upon the Horizontal One FG, *&c.* *Plate 3. Fig. 22.*

PROPOSITION II.

PROBLEM.

To raiſe a Body of known Weight with a ſmall Power, by means of a Leaver.

TO raiſe the Weight D, that weighs, for Example, 100 Pounds, apply'd at A, the end of the Leaver AB at the diſtance of an Inch from the *Fulcrum* C, with a given Force of 10 Pounds; conſider that Force as a Weight of 10 Pounds, as E; and to the Three Numbers 10: 100:: 1: which are the Weight E or given Force, the given Weight D, and its diſtance AC, find a fourth proportional, which will be 10 Inches for the diſtance CB. If then you take CB of 10 Inches, you will have the point B, where if the Weight E be apply'd, it will keep the given Weight D in *Æquilibrio* about the fix'd Point C, by *Prop.* 1. Wherefore if this Weight E be apply'd ever ſo little beyond B, that is, farther from the *Fulcrum* C, it will raiſe the Weight D, becauſe its Force will become greater, as we have obſerv'd in the foregoing Propoſition. *Plate 2. Fig. 18.*

If the length AB of the Leaver be determin'd, you muſt ſo divide the Leaver at the point C, that the Weight E, or the Power: may be to the given Weight D:: as AC: is to BC, as has been taught concerning the Balance, and then will C be the common Center of Gravity of the Quantity compounded of the Two Weights E, D; wherefore if the *Fulcrum* be any where betwixt A and C, it is plain that the Power at B will raiſe the Weight propos'd D.

PRO-

PROPOSITION III.

THEOREM.

As much as the Power gains in Force, as it moves a Body with a Leaver, so much it loseth in Time and Space.

Plate 3. *Fig.* 23. SUppose the Leaver AB, whose fix'd Point is at C, to have a Weight fix'd to it, whose Center of Gravity corresponds to the end A, and a Power at the other end B; this Power in moving the Weight will give the Position DE to the Leaver AB; in which case the Weight will describe the Arch AD, and the Power the Arch BE about the *Fulcrum* C. If the Power at B only sustain'd the Weight at A, it wou'd by *Prop.* 1. have the same *Ratio* to the Weight, as AC the distance of the Weight has to DC the distance of the Power; but as we suppose that it is able to Move it, it must of consequence have a greater *Ratio* to the Weight, than the Space AD has to the Space BE; so that if the Power be a great deal less than the Weight, so must AC the distance of the Weight be reciprocally a great deal less than BC that of the Power; and consequently the Space AD, which the Weight goes thro', must be a great deal less than the Space BE, which the Power goes thro'; because the Arches AD, BE, which measure the equal Angles ACD, BCE, are similar, and consequently as their *Radii* AC, BC.

Whence we may easily conclude, that the Power runs thro' a greater Space than the Weight, proportionably as it is less than the Weight; because the less the Power is, the greater must its Distance BC be, to enable it to move the Weight, which must after the same manner encrease the Space BE, which it runs thro'. So that if the distance BC is, for Example, Ten Times greater than the distance AC, likewise the Space BE of the Power will be Ten Times greater than the Space AD of the Weight; because, as we have already said, the Two Arches AD, BE, being Similar, are to one another as their *Radii* AC, BC. Whence it follows, that the Power will be Ten Times as long in moving the Weight, by means of the Leaver AB, as it wou'd be in moving it without a Leaver.

Thus you see, that if on the one Hand, your Force be encreas'd by applying the Power farther from the fix'd Point, on the other Hand you lose something of the Space, or of the Time. So that if you can raise a Body of 100 Pound Weight with the Leaver AB, the Power being at B, and the Weight

at A; you may likewiſe raiſe 200 Pound Weight applied at A, if you double BC the diſtance of the Power; but if you take off thus half the Gravity of the Weight, you muſt employ as much more Time in the Operation, becauſe in this Suppoſition the Power muſt run thro' a greater Space. *Plate* 3. *Fig.* 23.

Hence it is plain, that † the Farther the Power moves, the Greater is its Force, which happens not only in the Leaver, but alſo in all the other Engines, as you will ſee in the Sequel: And it is by this Principle of Velocity and Space that *Galilæus* and *Carteſius* have explain'd the Effect of Mechanical Engines; and tho' this Principle does not ſatisfy the Mind enough to ſerve for a Demonſtration, yet we have no reaſon to doubt of it after what we have ſaid hitherto, and what we ſhall ſay concerning the other Engines.

SCHOLIUM.

Becauſe the Leaver paſſes thro' the Center of Gravity of the Weight, it is evident that the Force of the Power will be alike in all Poſitions; that is, that the Power will weigh no more upon the Horizontal Leaver AB, than upon the Inclin'd one DE: But this will not hold when the Leaver does not paſs thro' the Center of Gravity of the Weight, as you will ſee in the Two following Propoſitions.

PROPOSITION IV.

THEOREM.

If a Power whoſe Line of Direction is perpendicular to a Leaver, bears up by means of that Leaver a Weight, whoſe Center of Gravity is above *the Leaver, it muſt be greater to bear it up, when the Leaver is Horizontal, than when it is Inclin'd, and the Weight rais'd; and greater yet when the Weight is lower.*

I Say that a Power (which has its Line of Direction perpendicular to a Leaver, and which by the help of that Leaver, whoſe fix'd Point is C, ſuſtains a Weight apply'd to the Leaver in ſuch manner that its Center of Gravity O be above the Leaver) ſuſtains that Weight with more difficulty when the Leaver is Horizontal, as at AB, than when it is Inclin'd, *Fig.* 24.

† *That is, the greater the Velocity of the Power is, in reſpect of that of the Weight.*

and

Plate 3. Fig. 24. and the Weight rais'd up, as DE, and with most difficulty when the Weight is let down as at FG: That is, supposing every thing else equal, a greater Force is requir'd for the Power to sustain the Weight O, when the Leaver is in the Position AB, then when it is in the Position DE, and a greater Force still when the Leaver is in the Position FG.

DEMONSTRATION.

Because OI the Line of Direction of the Weight cuts the Leaver at I, which is farther from the *Fulcrum* C in the Leaver FG than in the Leaver AB, and farther yet in this Leaver than in the Leaver DE; considering the Weight as if it was apply'd at I, it has less Force downwards in the Leaver DE than in the Leaver AB, and less still in the Leaver AB than in the Leaver FG, and consequently may be kept in *Æquilibrio* with a less Weight applied to the Leaver DE, than to the Leaver AB; and a less still when applied to the Leaver AB, than to the Leaver FG, if so be that the Line of Direction of that Weight is perpendicular to the Leaver; because the distance of that Weight from the fix'd Point C remains always the same.

SCHOLIUM.

This Theorem holds also when the Line of Direction of the Power is perpendicular to the Horizon, as if a Weight was to hang freely in the place of the Power; because tho' by reason of the different position of the Leaver, the distance of the Power alters as well as that of the Weight; yet it does not alter in the same proportion: As for Example, if CK the distance of the Weight becomes less in the Leaver DE, the distance CL of the Power will not grow less proportionably; so that if in the Leaver AB, the distance BC of the Power be, for Example, twice the distance CI of the Weight; in the Leaver DE, the distance CL of the Power will be more than twice CK the distance of the Weight; because CI, in this Leaver, is less than CI in the Leaver AB; and in the similar Triangles CLE, CIK, the Hypotenuse CE contains as often the Hypotenuse CI, as the side CL contains the side CK: And as CE contains CI more times in the Leaver DE, than BC equal to CE contains CI in the Leaver AB; so also CL contains CK more times than BC contains CI; whence it happens that CL must be more than twice CK, and consequently the Power has more Force when apply'd at E, than when apply'd at B, *&c.*

PRO-

PROPOSITION. V.
THEOREM.

If a Power whose Line of Direction is perpendicular to a Leaver, by means of that Leaver bears up a Weight, whose Center of Gravity is below the Leaver, it must be less to Æquiponderate when the Leaver is Horizontal than when it is Inclin'd and the Weight rais'd, and yet less when the Weight is lower.

I Say, That the Power, (whose Line of Direction is perpendicular to a Leaver, and which by means of that Leaver, whose *Fulcrum* is C, sustains a Weight apply'd to the Leaver in such manner, that the Center of Gravity O lies below it) sustains the Weight with less difficulty when the Leaver is Horizontal, as AB, than when it is Inclin'd and the Weight rais'd up, as at DE; and still with less difficulty when the Weight is let down, as at FG: that is, supposing every thing else equal, a less Force is requir'd for the Power to sustain the Weight O, when the Leaver is in the position AB, than when it is in the position DE; and a less still when it is in the position FG. *Plate 3. Fig. 25.*

DEMONSTRATION.

Because the Line of Direction OI cuts the Leaver at I, which is nearer to the *Fulcrum* C in the Leaver FG than in the Leaver AB, and nearer yet in this Leaver than in the Leaver DE; therefore if you consider the Weight as apply'd at I, it will have more Force downwards in the Leaver DE than in the Leaver AB; and more still in this Leaver than in the Leaver FG, and consequently it can keep an *Æquilibrium* with a greater Weight apply'd to the Leaver DE, than to the Leaver AB; and a greater yet apply'd to this Leaver AB, than to the Leaver FG, if so be that the Line of Direction of that Weight, which is instead of a Power, is perpendicular to the Leaver, because the Distance of that Power from the fix'd Point C, will always be the same.

SCHOLIUM.

This Theorem holds also, when the Line of Direction of the Power is perpendicular to the Horizon, as if in the

Plate 3. Fig. 25. place of the Power a Weight shou'd hang freely from the end of the Leaver; because, tho' by reason of the different position of the Leaver, the distance of the Power alters, as well as that of the Weight, yet it is not alter'd in the same proportion: as for Example, if CK the distance of the Weight becomes less in the Leaver DE, CL the distance of the Power will not become proportionably less; so that if in the Leaver AB, the Distance BC of the Power be, (for Examp.) twice CI the Distance of the Weight, in the Leaver DE, the Distance CL of the Power will not be quite double the Distance CK of the Weight; because CI in this Leaver is greater than CI in the Leaver AB, and in the Similar Triangles CLE, CKI, the Hypotenuse CE contains the Hypotenuse CI as many times as the side CL contains the side CK: and as CE does not contain CI of the Leaver DE so many times as BC, which is equal to CE, contains CI in the Leaver AB; so also CL does not contain CK so many times as BC contains CI; whence it is plain that CL is less than twice CK, and consequently the Power has less Force when at E, than when at B, *&c.*

PROPOSITION VI.

THEOREM.

If two Powers bear up a Weight with a Leaver parallel to the Horizon; That which is nearest the Weight, will bear a greater part of it than That which is farther from it.

Fig. 26. I Say that if the two Powers apply'd at the ends A,B, of the Leaver AB parallel to the Horizon, sustain the Weight EF, whose Line of Direction is CD, which goes thro' its Center of Gravity D, the Power at A which is nearest to the Weight, sustains a greater part of that Weight, than the Power at B, which is farther from it.

DEMONSTRATION.

The Power being at A, the point B may be look'd upon as the *Fulcrum*, and likewise the Power being at B, the point A is to be look'd upon as the *Fulcrum*, and in that Leaver of the Second Kind, you will find, by *Prop.* 1. that the Power at A: is to the Weight EF:: as BC the Distance of the Weight: is to AB the Distance of the Power; and likewise that the Power at B: is to the Weight EF:: as AC the Distance

Diſtance of the Weight: to AB the Diſtance of the Power. *Plate* 3. *Fig.* 26.
Whence it is eaſy to conclude, by changing theſe Two Analogies, that the Power at A: is to the Power at B:: as BC: is to AC: and becauſe BC is greater than AC, ſo alſo will the Power at A be greater than the Power at D. *Q.E.D.*

COROLLARY.

The Method of finding out what part of the Weight each Power ſuſtains is a conſequence of the foregoing Propoſition, if the Weight be known, as well as the length of the Leaver, and AC, BC, the Diſtances of the Powers: *as for Example*, If the Diſtance AC be Two Foot, and the Diſtance BC Three Foot, ſo that the Leaver AB be Five Foot long, and the Weight EF of 60 Pounds; you muſt conſider that ſince the Power at A: is to the Power at B:: as BC: is to AC: *by Compoſition*, the Sum of the two Powers, or the Weight EF, which is of 60 Pounds: will be to the Power at B:: as AB the Length of the Leaver, ſuppos'd of Five Foot: to the Diſtance AC, which has been ſuppos'd of Two Foot: wherefore if to theſe Three Numbers 5: 2:: 60: a Fourth proportional be found, you will have 24 Pounds for the Power at B, and if you ſubſtract theſe 24 Pounds from the whole Weight EF, or from 60 Pounds, you will have 36 Pounds left for the Power at A.

CHAPTER. III.

Of the Pulley.

A Pulley is a Wheel of Wood or Metal, as AB, moveable about a ſmall Axel which goes thro' the middle of it, call'd the *Center* or *Center-Pin* (in French *Goujon*) by Handy-Crafts Men, to which we allow no Thickneſs in the Theory. It is fix'd in a piece of Wood or Iron, as CD, call'd the Box, (in French *Echarpe*, *Chape*, *Mouſle* and *Palan*; and in Latin *Trochlea*,) tho' the French uſually call *Mouſle* ſeveral Sheevers or Wheels in the ſame Box upon the ſame Axel, which we call in Engliſh a *Pulley*; or when Two of 'em are us'd, *a Pair of Blocks*; and a ſingle Wheel or Pulley, as Fig. 27. a *Snatch-Block*. This Engine is of great Force by means of a Rope (which is kept about each Wheel by a Groove which confines it to the Sheevers,) and faſten'd at one end to the Weight that is to be rais'd or ſuſtain'd, and drawn at the other end by the Power. *Fig.* 27. *Fig.* 28.

PROPOSITION I.

THEOREM.

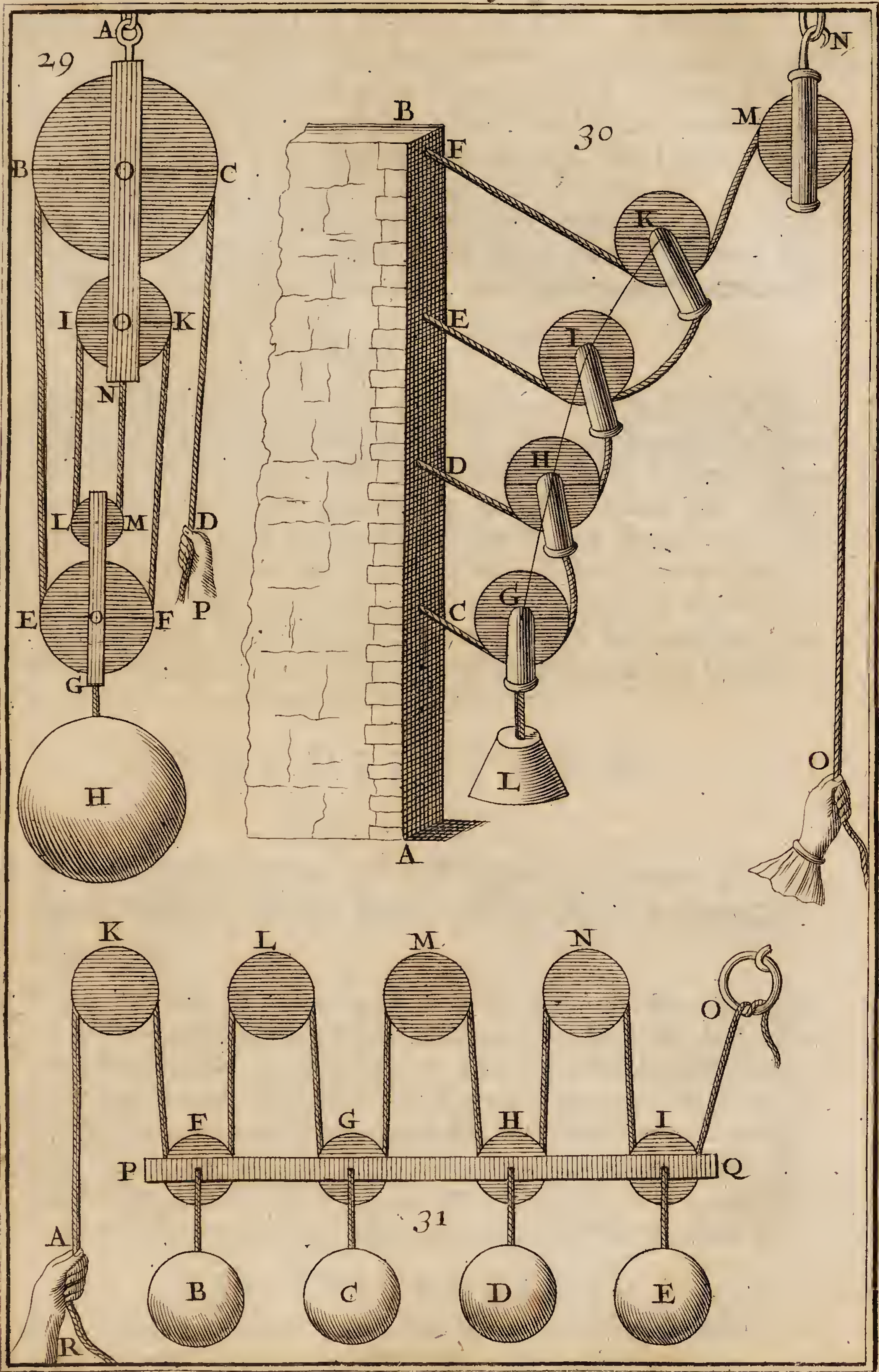
When a Power sustains or draws a Weight by means of several Pulleys, Each Pulley over which the Rope goes, is equivalent to a Leaver of the First Kind; and Each Pulley under which the Rope goes, is equivalent to a Leaver of the Second Kind.

Plate 4. *Fig.* 29. LET the Pulley BC be fasten'd by its Box to the fix'd Point A, and let CD be a Rope, which going over this Pulley, comes back under the Pulley EF, which sustains the Weight H, fasten'd to its Box G. Let the same Rope again go over the Pulley IK, to whose Box it is fix'd at N, after it has gone under the Fourth Pulley LM. Now in this case I say, that each of the Pulleys BC, IK, over which the Rope goes, is equivalent to a Leaver of the First Kind; and that each of the Pulleys EF, LM, under which the Rope goes, represents a Leaver of the Second Kind.

DEMONSTRATION.

Because each of the Pulleys BC, IK, LM, EF, is Moveable about its Center, and that part of the Rope, which is of the same side as the Power draws downwards, by causing the Wheel to turn about its Center O; That Center O may be consider'd as the Center of Motion of a Leaver of the First Kind, which wou'd be the Line BC or IK, which passes thro' the Center O, and also thro' the points where the Rope touches the Circumference of the Pulley on each side; for it is plain that such a Leaver wou'd have just the same effect as the Pulley; that part of the Rope which is towards the Power and draws downwards, as CD, and KF, being taken for the Power; and the other part BE, and IL which opposes that Motion, by reason of the Gravity of the Weight H, being taken for a Weight; which in such a case is equal to the Power: because the Weight and the Power are equally distant from the Center of Motion O.

Besides; The Union of all these Pulleys, by means of the Rope which comes round them all, causes the part of the Rope BE to draw Upwards, whilst the Power at D draws the part of the Rope DC Downwards, to sustain or raise the Weight H, which makes the Pulley EF equivalent to a Leaver of the Second Kind, such as wou'd be the right Line EF, whose fix'd Point wou'd be at F, the Weight at O, and the Power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Power at E, its Line of Direction being BE. Likewise the part IL of the said Rope drawing Upwards, causes the Pulley LM to be also equivalent to a Leaver of the Second Kind, such as wou'd be the Line LM, whose fix'd Point wou'd be at M, the Weight at O, and the Power at L, the Line of Direction being the Line IL. Thus you see that each of the Upper Pulleys BC, IK, over which the Rope goes, is a Leaver of the First Kind; and each of the Lower Pulleys LM, EF, under which the Rope goes, is a Leaver of the Second Kind. *Q.E.D.* *Plate* 4. *Fig.* 29.

SCHOLIUM.

Since the Upper Pulleys are Leavers of the First Kind, whose fix'd Point is in the middle, it is plain that the Power is equal to the Weight, and therefore that such Pulleys contribute nothing to the encreasing of the Force, but only facilitate the Motion by hindering the Ropes from Sticking. But as the Diameter of each Lower Pulley is as a Leaver bearing upon one end and rais'd at the other, it is easy to understand, that by such a Pulley the Force is doubled, because the Distance of the Power is double that of the Weight, as we shall say more particularly hereafter.

PROPOSITION II.

THEOREM.

When a Power bears up a Weight by means of several Pulleys, all the Parts of the Rope are equally Stretch'd.

SUppose that a Power apply'd at D, sustains the Weight H, by means of the Four Pulleys BC, IK, LM, EF, the two First of which as BC, IK, are Leavers of the First Kind, and the two others LM, EF, are Two Leavers of the Second Kind, *by Prop.* 1. whose Uppermost BC, which is joyn'd to the others, by means of the Rope which comes round them all, is fasten'd to some fix'd thing by its Hook A. I say, that in such a case all the Parts of the Rope are Equally stretch'd.

DEMONSTRATION.

It is already evident, that the Two Parts CD, BE, are Equally Stretch'd, because the Pulley BC being a Leaver of the First Kind, whose fix'd Point is in the middle O, the Power at C and the Weight at D are equal, which makes the part CD of the Rope to be drawn by the Power with the same force that the part BE is drawn by the Weight, and consequently those two parts are Equally Stretch'd. It is so

Plate 4. *Fig.* 29. likewise of the two parts IL, KF, which are fix'd to the ends of the Leaver IK, which is likewise of the First Kind.

It is also plain, that the two parts BE, KF, which are apply'd to the ends of the Leaver EF, by means of which they sustain the Weight H hanging from G, which answers to the middle of that Leaver, are Equally Stretch'd, because if one of 'em, (BE for Examp.) which draws Upwards, was more Stretch'd than the other KF which draws Downwards, it wou'd draw up the Weight H and put it into motion, which is contrary to the Supposition, because we have suppos'd that the Power cou'd sustain the Weight; that is, that the Power and the Weight are in *Æquilibrio*, by the same Reason it appears that the Two Parts IL, MN, are Equally Stretch'd; whence it is easy to conclude, that All the Parts of the Rope are Equally Stretch'd. *Q.E.D.*

COROLLARY.

From this Proposition it is easy to conclude, that since All the Parts of the Rope are Equally Stretch'd; Those that are apply'd to the Lower Pulleys, which are Leavers of the Second Kind, *viz.* the Four BE, KF, IL, MN, carry Equal Parts of the Weight H, which they sustain.

PROPOSITION III.

THEOREM.

When a Power bears up a Weight by means of several Pulleys, the Power is such a Part of the Weight, as One *is of the Number of the Parts of the Rope applied to the Lower Pulleys.*

Fig. 29. I Say, That if a Power apply'd at D, sustains the Weight H, by means of the Four Pulleys BC, IK, LM, EF, the First of which is hook'd to the Point A; such a Power is the Fourth Part of the Weight H; because there are Four Parts of the Rope, *viz.* BE, KF, IL, MN, apply'd to the Lower Pulleys, EF, LM, which are Leavers of the Second Kind, as has been shewn.

DEMONSTRATION.

Plate 4. *Fig.* 29. Since, by the Corollary of the foregoing Proposition, all the Parts of the Rope, apply'd to the Lower Pulleys sustain equal parts of the Weight, it follows by reason of the Four Ropes, that Each of them sustains the Fourth Part of the Weight, and that consequently the Rope CD, whose Force is equal to the Resistance at B from the Gravity of the Weight, sustains the Fourth Part of the said Weight exactly; that is, the Power at D is the Fourth Part of the Weight H. *Q. E. D.*

COROL-

COROLLARY.

From this Propoſition it is evident, that if the Number of the parts of the Ropes apply'd to the Lower Pulleys, and alſo the Weight H be Given or Known, the Power, which ſuſtains it by means of theſe Pulleys is alſo Known; as here it is equal to the Fourth part of the Weight; ſo that if this Weight be, for Example, of 200 Pounds, the Power will be of about 50 Pounds: I have ſaid *about*, becauſe it ought here to be ſomething more, by reaſon of the Stickage of the Rope, and the Gudgeons or Center-Pins, of their Weight, and the Weight of the Pulleys; for it is certain that all theſe things encreaſe the Weight of H, and leſſen the Force of the Power; but this is nothing in compariſon of the Force, which is gain'd by the lower Pulleys. *Plate* 4. *Fig.* 29.

SCHOLIUM.

As the Upper Pulleys, (which are unmoveable, being in the ſame Box, faſten'd up to the point A, and which are, as has been ſhewn, Leavers of the Firſt Kind,) ſerve only to hinder the fretting of the Rope, and to facilitate its motion, without multiplying the force; you may only make uſe of the Lower Pulleys, (which are moveable, and Leavers of the Second Kind, as has been alſo ſhewn) after the manner repreſented in this Figure, which is no ſooner ſeen but underſtood; for one may eaſily ſee that all the Ropes which are faſten'd to the points C,D,E,F, of the Wall AB, and which ſuſtain the Pulleys G, H, I, K, and by their help the Weight L tied to the middle of G the loweſt of them, communicate with the unmoveable Pulley M fix'd to N with its Hook, over which the Rope goes, which is drawn downwards by the Power at O, which in this caſe is the Sixteenth part of the Weight, becauſe there are Four moveable Pulleys, and by each of 'em the Weight loſes half of its reſiſtance, becauſe they are Leavers of the Second Kind. *Fig.* 30.

After the ſame manner one may know, that the Power at A is but the Eight part of the Quantity made up of the Four equal Weights B,C,D,E, which it ſupports by means of the Four Moveable Pulleys, F,G,H,I, and the Four Unmoveable ones K, L, M, N, which are united with the Four Firſt F,G,H,I, by means of the Rope faſten'd to the fix'd Point O. *Fig.* 31.

PROPOSITION IV.

THEOREM.

As much as the Power gains in Force, when it moves a Weight by means of several Pulleys; so much it loseth in Time and Space.

Plate 4. Fig. 31. SUppose a Power apply'd at A, which draws the Rope downwards towards R, to raise up the Weights B,C,D,E, or the Box PQ to which they are tied, I say that in such a case, the Power will run thro' a Great Space, whilst the Weight goes thro' a small Space; that is, the Power will draw a great deal of Rope, to raise the Weight but a little Way; so that to raise the Weight One Foot, for Example, the Power in this Engine must go down Eight Foot, because Eight parts of the Rope are apply'd to the lower Pulleys.

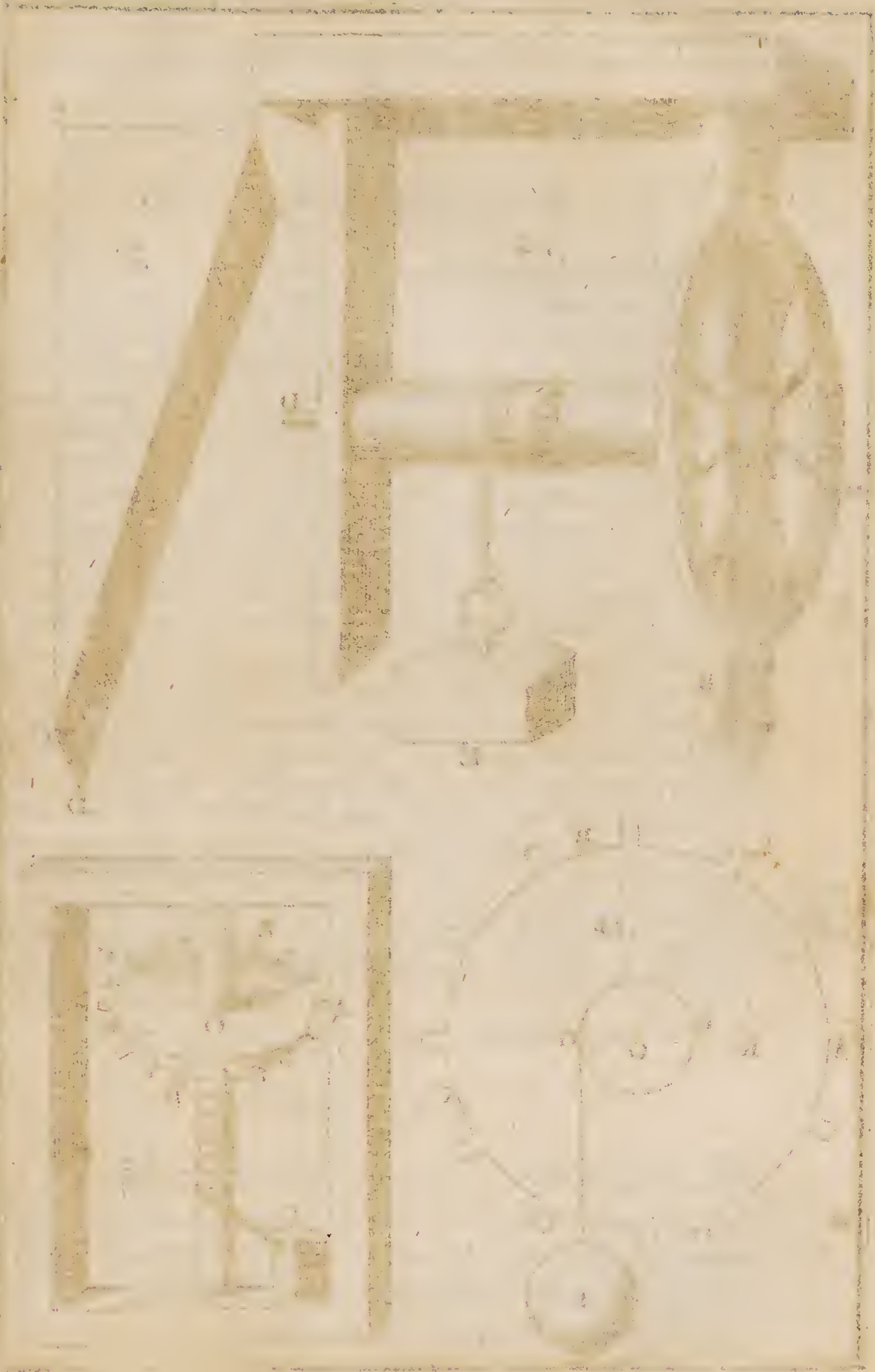
DEMONSTRATION.

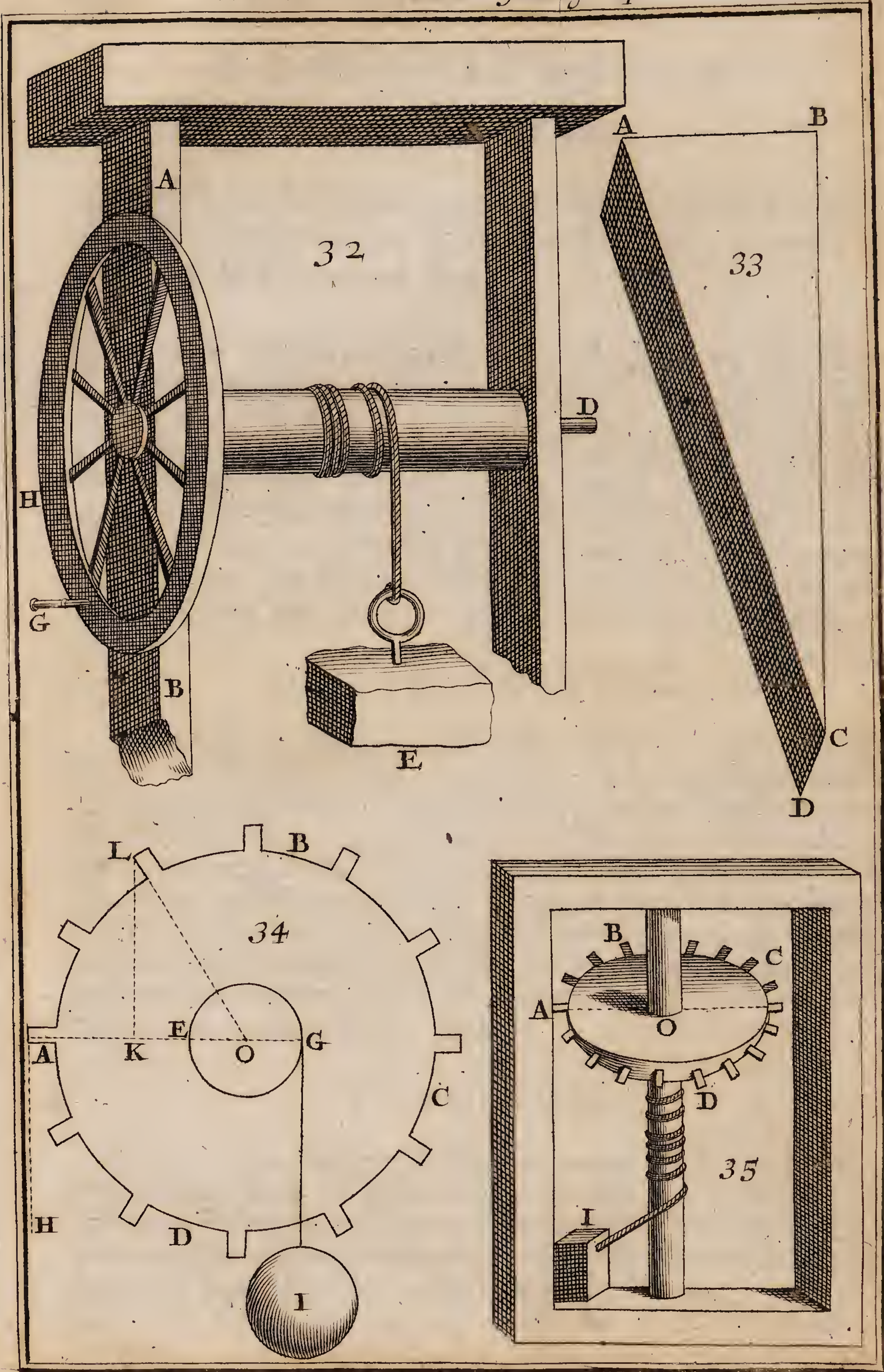
In the Use of Pulleys, as in the Use of the Leaver, the Space which the Weight runs thro': is to the Space which the Power runs thro':: as the Power: to the Weight; or as the Number One: is to twice the Number of the lower Pulleys; because the Weight cannot be rais'd, for Example, one Foot, unless each Rope, which is apply'd to the lower Pulleys, be shorten'd One Foot, and consequently all of them together Eight Foot, because they are Eight in Number; and this cannot happen unless the Power draws Eight Foot of Rope from A towards R. *Q. E. D.*

SCHOLIUM.

Thus you see, that in this Engine as well as in the Leaver, this general Law of Mechanicks is observ'd, namely, that the more Velocity the Power has the greater is its Force proportionably, which is observable in the Wheel that draws by its Axel, as will appear in the following Chapter.

CHAP.

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CHAPTER IV.

Of the Wheel *by its Axel.*

THE *Axel in the Wheel*, (in Latin *Axis in Peritrochio*) and the common *Turn-Barrel*, is a Wheel, as AB with a Cylindrical Axel as CD, call'd the *Tympanum*, *Tumbrel*, *Barrel*, or *Wallow*, &c. that passes thro' it at right Angles, which together with its Axel CD is moveable about it Center C. About this Axis is a Rope wound, being first fastned to it, which carries the Weight E, and draws it when the Axel is made to turn by means of the Wheel, which has small Teeth that it may be mov'd the easier, or a Handle as G, when the Power acts on the Circumference of the Wheel. *Plate 4. Fig. 31.*

It is plain that this Engine is nothing but a Leaver perpetually turn'd round, which is represented by any one of the Spokes of the Wheel as CH. It is call'd a *Turn* (in French *Tour*) only when its Axel CD being supported by two pieces of Timber turn Horizontally, and the Wheel AB Vertically, as is usual to draw Stone out of Quarries, Water from deep Wells, or to let down Men into Mines, and sometimes to draw up Coal or Oar: for the Axis is sometimes Vertical and the Wheel turns Horizontally, as when it is us'd to draw Water out of Places that you wou'd Build in, or out of * Mines, *&c.* All the Engines which are contriv'd to raise Burthens from the Ground with the Axel in the Wheel, are call'd by the common name of Rolls or Winds; in French *Guindas.*

* *Then it is call'd a Gin, or Barrel-Gin.*

PROPOSITION I.

THEOREM.

If a Weight is rais'd by means of a Wheel, (which with its Axel moves round its Center) by a Power whose Line of Direction touches the Circumference of the said Wheel; the Power : will be to the Weight :: as the Radius *of the Axel: is to the* Radius *of the Wheel.*

LET ABCD be a Wheel strongly fix'd to its Axel, so that together with it, it may turn about the Center O, as you may see in the Circle EFG, its Profil. Let the Power be apply'd *Plate 5. Fig 34.*

Plate 5. Fig. 34. apply'd at any place of the Circumference of this Wheel, as at A, in such manner that its Line of Direction AH may touch the Circumference, and consequently the Angle HAO may be Right; and that drawing downwards it may sustain the Weight I, which hangs from the end of a Rope fasten'd at the other end to EFG, the Circumference of the Axel. I say, that in such a case, the Power at A, or at H: is to the Weight I:: as OG the *Radius* of the Axel to AO the *Radius* of the Wheel.

DEMONSTRATION.

It is plain, that if by Thought you take away all the parts of the Wheel, except the *Radius* AO, this *Radius* AO will act after the same manner as the Wheel wou'd, if the Line of Direction AH is always perpendicular to it, and then that Line or Inflexible *Radius* AO, or AOG, will in no wise differ from a Leaver of the First Kind; whose fix'd Point is at the Center O, with the Power at A one of its ends, and the Weight at G the other end; and it has been demonstrated, *by Prop. 1. of the Leaver*, that the Power at A: is to the Weight I, apply'd at G:: as OG, the Distance of the Weight: is to AO, the Distance of the Power; that is, as the *Radius* of the Axel: is to the *Radius* of the Wheel. *Q.E.D.*

SCHOLIUM.

From this Proposition it is plain, that as Much as the *Radius* of the Wheel is greater than the *Radius* of the Axel; so Much is the Force of the Power encreas'd; always supposing the Line of Direction of the Power to touch the Circumference of the Wheel; because in such a case the Distance of the Power from the fix'd Point O will always be the Same, whatever point of the Circumference it is apply'd at: for were it otherwise this wou'd not hold; as *for Examp.* if the Power were apply'd at L, and its Line of Direction LK shou'd be perpendicular to the Horizon, then wou'd its Distance from the fix'd Point O be the right Line KO, perpendicular to the Line of Direction, and being less than the *Radius* AO, or LO, it wou'd cause the Force of the Power to be lessen'd, and the Resistance of the Weight to be encreas'd.

Fig. 35. After the same Manner it may be demonstrated; That when the Wheel is Horizontal, as ABCD, so that its Axel be perpendicular to the Horizon, as it often happens when the Weight I is to be drawn along an Horizontal, or an Inclin'd Plain; the Power: is to the Weight:: as the *Radius* of the Axel; is to the *Radius* of the Wheel; always supposing the Line of Direction of the Power to touch the Circumference of the Wheel, *&c.*

PRO-

PROPOSITION II.

THEOREM.

As much as the Power gains in Force, when it moves a Weight by the means of a Wheel and Axel, ſo much it loſes in Time and Space.

THIS Engine, as well as the Two foregoing, ſhews us that Nature can neither be Deceiv'd nor Conquer'd; that is, Nature loſes nothing on the one Hand, but what is regain'd on the other; ſo that whatever is got by the Wheel ABCD, is loſt in Time and Space; becauſe, by the foregoing Propoſition, if the Weight I has, for Example, Ten Times more reſiſtance than the Power; ſo alſo muſt AO the diſtance of the Power be Ten Times greater than OG the diſtance of the Weight, if you wou'd have that Power ſuſtain it; which makes ABCD, the Circumference of the Wheel, to be Ten Times greater than EFG the Circumference of the Axel; and conſequently the Power has Ten Times the Velocity of the Weight, when it is able to Move it: For when it has made one entire Revolution about the Wheel, the Weight alſo will have made one entire Revolution about the Axel, which is but the Tenth part of the Space which the Power has run thro'. *Fig.* 34.

SCHOLIUM.

Thus you ſee, that in this Engine the Law common to the Two foregoing is plainly obſerv'd, *viz.* that the Force of the Power is encreas'd in proportion to its Velocity; ſo that we may ſafely depend upon this Mechanical Principle as infallible, when we explain the Effect of the Screw and Wedge; which without it cannot, in my Opinion, be ſo clearly explain'd.

Hence is the Reaſon known, why in Pocket-Watches, which inſtead of a Counterpoiſe have a Spring, as AB, the Fuzè CD is rather a Cone than a Cylinder; becauſe when you have wound up the Spring AB, in which caſe the Chain or String CE is at the Top or Point of the Fuzè, it has more Force, which it loſes by opening it ſelf; and reciprocally the String CE has the leaſt Force at C, but more and more as it goes down towards D: And that the Strength may all along be Equal, the great Force, which the Spring AB has at the beginning, muſt be diminiſh'd by the Reſiſtance of the *Plate* 6. *Fig.* 38.

String

String at firſt, or at the Top of the Fuzè, and the Weakneſs of the Spring towards the laſt, muſt be recompenc'd by the greater Facility, that the String has to draw at the Bottom of the Fuzè CD, where the Fuzè being Broadeſt, its Axis is drawn Round with moſt Eaſe.

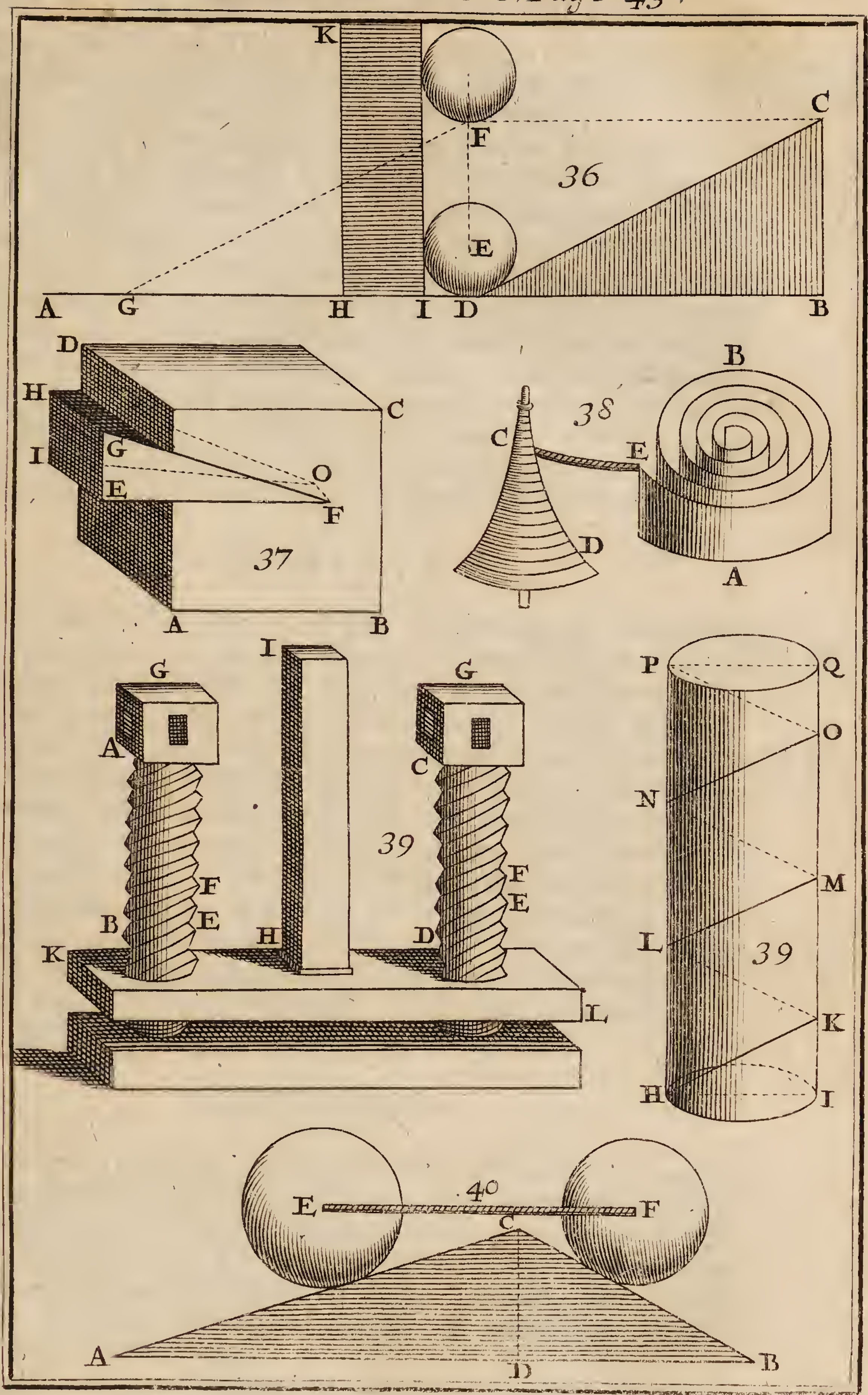
CHAPTER V.

Of the Wedge.

Plate 5. Fig. 33. THE *Wedge* is the moſt Simple or Plain Engine that is, being in the ſhape of a Solid Triangle, as ABCD, which is ſometimes of Wood; but uſually of Iron, that by reaſon of its Smoothneſs it may be more Uſeful in cleaving Bodies, becauſe it acts by ſliding againſt the Parts of the Body, which it divides.

To underſtand the power of the Wedge; One of the Two flat Sides, which incline to one another, is to be conſider'd as an Inclin'd Plain, and the other as a Horizontal Plain; and we muſt conceive, that by the help of this Inclin'd Plain, a Power ſhall raiſe a Weight, which without this Engine it cou'd not ſo much as bear up.

Plate 6. Fig. 36. Let the Triangle DBC Rectangular at B, repreſent a Wedge, D the Point or Edge, BC the Head, (and to be plainer underſtood, let BD the length of the Wedge be twice its Height BC) and the Baſis BD perfectly Smooth, ſo that being apply'd to the Horizontal Superficies AB, which alſo I ſuppoſe perfectly Smooth, the Wedge DBC may ſlide upon that Horizontal Plain AB without any difficulty. Then again, let us ſuppoſe that the Weight E, be hinder'd from going to A, by the Plain HIK perpendicular to the Horizon, which yet does not hinder the Wedge from ſliding along the Horizontal Plain AB, when it ſhall be drawn or puſh'd from B towards A by a Power, whoſe Line of Direction is parallel to the Horizon. If then the Power puſhes the Wedge DBC regularly from B towards A, in cauſing it to ſlide upon the Horizontal Plain AB, it will cauſe the Weight E to Riſe up, by ſo regular a Motion, that its Center of Gravity E will never go out of the line EF perpendicular to the Horizon; ſo that when the point B ſhall come to D, the point C to F, and the point D to G, (that is, when the Wedge DBC ſhall be in the poſition GDF) the Weight E, by the reſiſtance of the Plain HIK, ſhall have been forc'd to Riſe by the Inclin'd Plain CD, or FG, which will have puſh'd it upwards to F, ſo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ſo that it will have Riſen the whole Length of the line DF, when the Power ſhall have mov'd the Length of the line BD, or DG, which is twice DF by the Suppoſition. *Plate 6. Fig. 36.*

Since then in this Suppoſition, the Power has double the Velocity of the Weight, it ought to have double the Force of the Weight; therefore it needs not be more than half the Relative Weight of that *Pondus* upon the Inclin'd Plain CD, to be able to bear it there, according to that general Law of Mechanicks, which we have taken notice of in the foregoing Engines; (*viz.*) that the Power encreaſeth proportionably as its Velocity does encreaſe. Whence we may eaſily conclude, that "*When a Power, whoſe Line of Direction is parallel to* "*the Horizon, ſuſtains a Weight by the means of a Wedge, whoſe* "*Baſe is alſo parallel to the Horizon; that Power: is to the Weight* "*that it bears up:: as the Height of the Wedge to its Baſe.*

COROLLARY.

From what has been ſaid and demonſtrated, it follows, That the more Acute the Wedge is, the Greater will its Effect be; becauſe GD the Velocity of the Power will be great in Compariſon of DF the Velocity of the Weight: And that when this Wedge is applied to cleave a Body, as ABCD, the Plains EFOI, GFOH, which make up the Wedge, being more Inclin'd one to another, the parts E,G, may ſlide more Eaſily; where you will obſerve that the Plain EFOI, being conſider'd as a Horizontal Plain, and the other Plain GFOH as an Inclin'd Plain, as it really is in reſpect to the firſt Plain; the Reſiſtance of the Upper part of the Body ABCD, when you endeavour to diſunite it from the Lower, may be look'd upon as a Weight, whoſe Line of Direction is perpendicular to the Lower or Horizontal Part. *Fig. 36. Fig. 37.*

SCHOLIUM.

From what we have ſaid in the Theory of the Weight, muſt be ſubſtracted a Force able to overcome the Roughneſs and Irregularity of the Horizontal Plain on which the whole Wedge muſt ſlide, and likewiſe a Force able to overcome the Roughneſs of the Inclin'd Plain, along which the Heavy Body is made to Riſe, and the Roughneſs of the Weight it ſelf, when it is not Spherical.

This Friction or Stickage is not great in other Engines, tho' very conſiderable in the Wedge; Experience ſhewing that a Wedge laden with a vaſt Weight has hardly any Effect; becauſe, as well the Surfaces of the Wedge, as of the parts of the Body which you cleave, are always Rough, and ſo Cloſe, that

that their Friction very much hinders the Motion, which Obstacle we endeavour to remove by Percussion, which here is of wonderful Use; for Experience shews, that a Blow upon the Head of a Wedge makes it Enter easily into a Hard Body; the Reason of which, in my Opinion, is, That a Blow by putting all the parts of the Wedge in Motion makes them Tremble and be Disunited, so as to lessen the Stickage, and facilitate the Motion of the Wedge. And here it is observable, That the effect of Percussion will be Greater in proportion as the Percutient Body is Heavier, and moves Swifter.

CHAPTER VI.

Of the Screw.

Plate 6. Fig. 39. THE *Screw*, which the Greeks and Latins call *Cochlea*, is a Cylinder cut into several Concave Surfaces, continually Inclin'd in a Spherical Form, or a Spiral Plain inclin'd and wound about an *Arbor*, or *Barrel*, or *Axis*, as AB, or CD, whose each Circumvolution is call'd a *Helix* or *Thread of the Screw*, half of which is seen in the Figure, as BC, or DE. This Engine is very useful to Stop or Move any Thing, and Press with a great Force.

Fig. 36. To make an Estimate of this Force, we must consider, That if a Power shou'd push up the Weight E along the Inclin'd Plain CD, (whose Base DB is parallel to the Horizon) from D to C in a Line of Direction parallel to the Length BC, the Space which the Power goes thro', or its Velocity wou'd be represented by the line DC, and the Space or Velocity of the Weight by the line BC perpendicular to the Horizon; because the Weight wou'd have risen above the Base DB the whole Height BC of the Inclin'd Plain CD: Now in this case, by the General Law of Mechanicks, one may easily know that the Power wou'd have a Force proportionable to its Velocity, and wou'd be to the Weight which it shou'd sustain upon the Inclin'd Plain CD, pushing or drawing it in a Line of Direction parallel to its Length CD, (or what is all one, the Relative Gravity of the Weight wou'd be to its Absolute Gravity) as the Height BC; is to the Length CD:

Plate 6. Fig. 36. Instead of supposing the Power to draw the Weight E Upwards, you may imagine it to push the Solid Triangle BCD all its Length CD, which is the same thing; for thus will

will the Weight E, being Stopp'd by the perpendicular Plain HIK, as we have ſuppos'd concerning the Wedge, Force the Solid Triangle to Deſcend by preſſing it with its Gravity, or (which is equivalent) it ſelf will Riſe along the Inclin'd Plain CD above the Baſe BD, which repreſents the Horizon, running thro' the whole Length CD, without moving out of its perpendicular DF, which is its Line of Direction, this Length CD being the Space which the Power runs thro', when it puſhes the Solid Triangle BCD under the Weight E, till the point B comes on to D. *Plate 6. Fig. 36.*

Hence the nature of the Screw is very eaſily Explain'd by the Triangle BCD, which being puſh'd lengthways, the whole diſtance CD, ſlides upon the Horizontal Plain AB, and raiſes the Weight, ſo that BE or DE half the Length of One Helix is repreſented by the Length of the Inclin'd Plain, whoſe Height EF repreſents half the Height of the ſame Helix; the Baſe of the Inclin'd Plain repreſents the Horizontal Plain, or Baſe of the Arbor of the Screw, and the Weight is inſtead of the *Female* or *Inſide Screw*, which is a Spiral Hole made in the *Collar* G, with a *Top*, for the Male Screw, which we have deſcrib'd, to turn in. The French call alſo *Ecrou* (Female Screw) the Collar G, when it is moveable upon the Male Screw, and the Burthen to be rais'd is laid upon it; which is lifted with the greateſt Eaſe imaginable by turning G, which riſes together with the Weight. *Fig. 39.*

The Triangle or Inclin'd Plain gave the firſt hint to the Inventors of the Screw, which was made by winding the ſaid Triangle round an *Arbor* or Cylinder HIPQ, to make an Engine of more Uſe and leſs Bulk. For which end the Height of the Triangle has been allow'd for IK the Height of the Cylinder, and the Inclination of the Hypotenuſe of the ſaid Triangle has been given to the *Helix* HK, and to all the other *Helices* that go upwards round about the *Arbor* of the Screw, which make the Spiral Plain HKLMNOP, uſually call'd the *Thread of the Screw*, (in French *Trait de la Vis.*)

Thus one may ſee, that *if a Power ſuſtains a Weight by means of a Screw, that Power: will be to that Weight:: as the Height of the Screw: is to the Thread of the Screw*; that is, the Line which the whole Thread or all the *Helices* wou'd make if unwound and laid at Length. Whence it is eaſy to conclude, that in a Screw the Force of the Power is the Greater, the Cloſer the *Helices* are, and the more they are Inclin'd to the Horizon, *cæteris paribus*, or every thing elſe being equal; becauſe the Length of the Hypotenuſes of the Triangles upon which they have been made, have a greater *Ratio* to their Height.

Never-

Plate 6. Fig. 39. Neverthelefs to make an Eftimate of the Force of any Screw, it is not neceffary to meafure the Length of the whole Thread, nor the Height of the whole Cylinder; for it is fufficient to know, how often a line equal to one *Helix*, contains the Height of that *Helix*; for Example, how often the Height HL is contain'd in the Circuit of the *Helix* HKL, becaufe HP, the whole Height of the Cylinder, is contain'd juft as many Times in the whole Thread of the Screw *HKLMNOP*, as it is eafy to Demonftrate.

SCHOLIUM.

Without having recourfe to the Inclin'd Plain, one may know the Force of the Screw, by confidering, that when the Power has made one entire Revolution to raife a Weight, (for Example from E to F, the height of One Helix) it will have run thro' a great Space in Comparifon to the Space EF, which the Weight has gone thro'; whence may be known by the general Principle of Mechanicks, that the Power ought to gain a great Force by means of this Engine; fo that if the Space which it has gone thro' to raife the Weight up to the Height EF, be, for Example, Ten Times Greater than That Space, a Heavy Body will, by means of this Engine, be fuftain'd by One of about a Tenth part of its Weight.

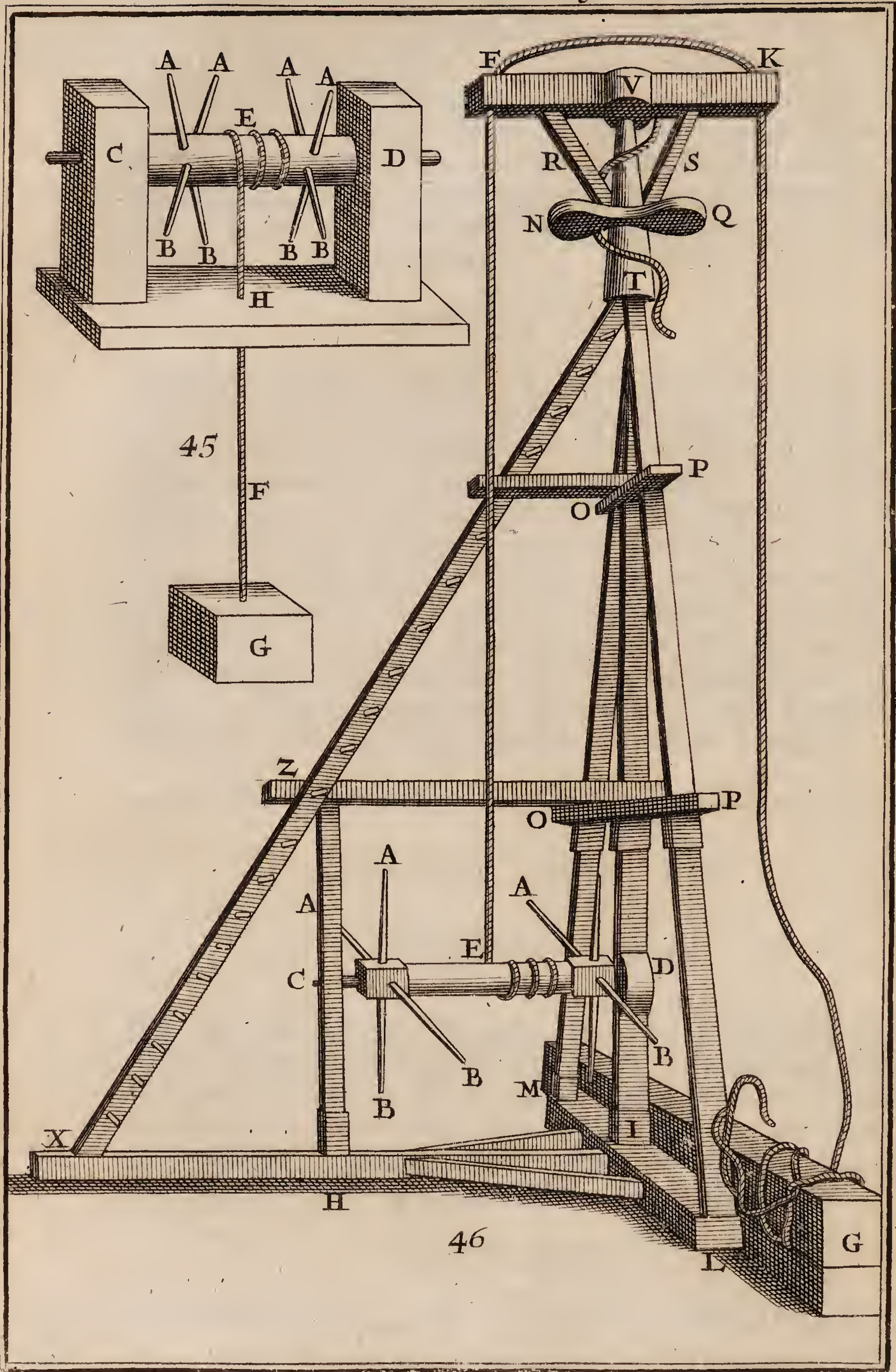
CHAPTER VII.

Of Compound Engines.

WE call *Compound Engines*, One that is compounded of feveral Simple Engines, which may be made ufe of a Thoufand different Ways, as Occafion ferves or Need requires; for which reafon we cannot give any Account of the Number of Compound Engines: Wherefore we fhall only fpeak of fuch as are moft Obvious and moft Us'd.

Of the Balance.

Plate 7. Fig. 41. THO' the Balance be a very Plain or Simple Engine, yet may be reckon'd amongft the Compound Engines, when it is made ufe of to Blow; which is done with fo much more Eafe as the Power at A, which draws downwards, in the Line of Direction AB, is more diftant from the fix'd Point C, and as the faid Line of Direction AB comes nearer to a perpen-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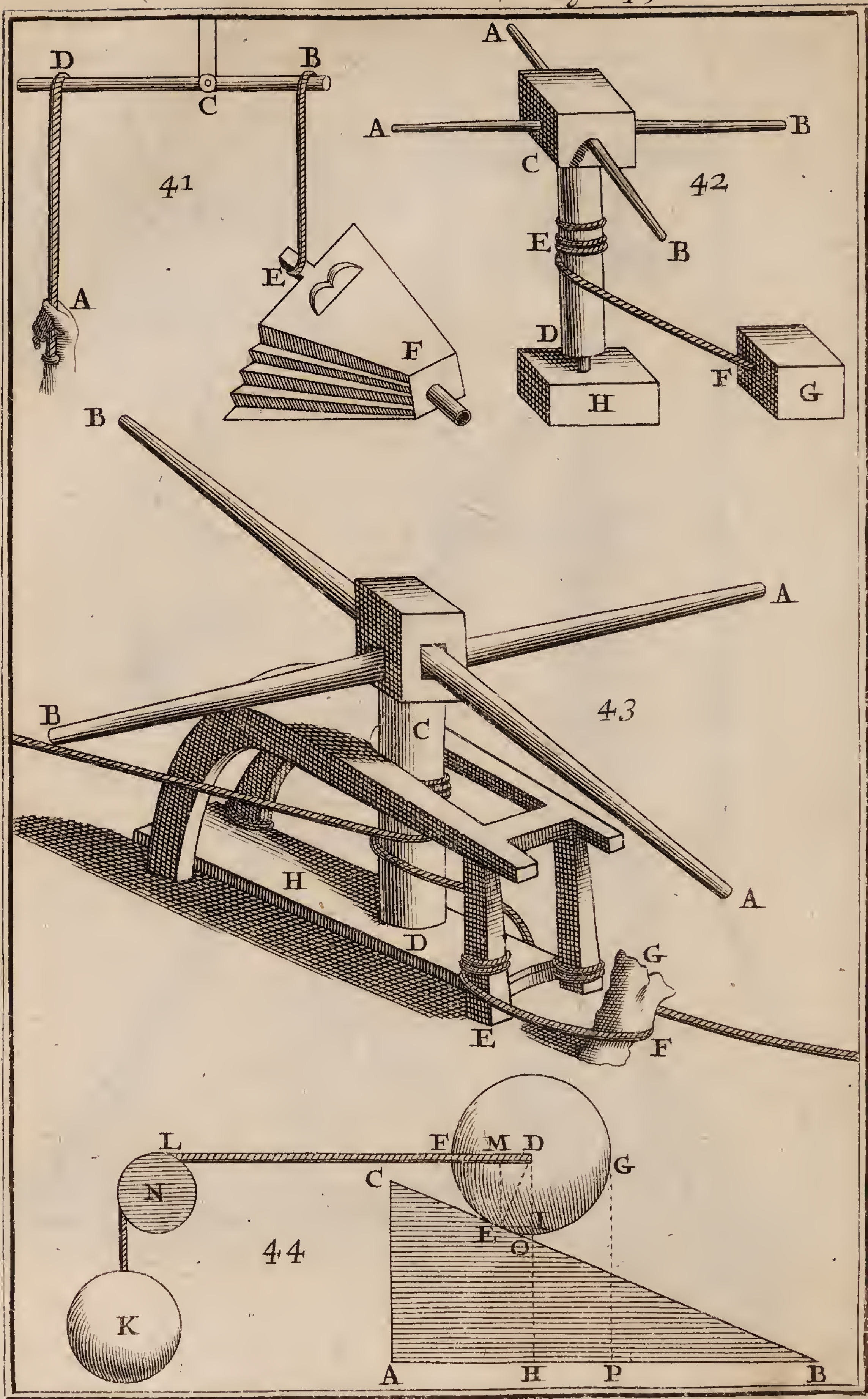
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pendicular to the Rod or Beam BD, whose end D being rais'd, opens the Bellows EF, whose point F is as the Center of Motion of a Leaver, which will move with so much more Ease as its Length EF is Greater: For this reason may this Engine be reckon'd amongst the Compound Engines, because it consists of Two Leavers; or of a Balance and a Leaver.

Of the Leaver.

THE Leaver AB apply'd Horizontally to the *Roll* or Axel CD, which is perpendicular to the Horizon, about which is wound the Rope EF, that has one end fasten'd to the heavy Body G, which is laid upon the Ground, is of great use in turning horizontally the Axel CD by the Strength of Men, when several Powers are apply'd to the ends of the Leavers AB, in order to draw the Body G towards H. *Plate 7. Fig. 42.*

This Engine usually call'd a *Winch* or *Windlass*, in Latin *Ergata*, and by Mariners a *Capstane*, is very useful to draw Stones out of Boats, or out of the Banks of Rivers, or to draw along the Boats themselves. It is usually made as represented by the 43d *Fig.* and is very serviceable in Ships to heave up Anchors, which require a great Force to draw them from the Ground. *Fig. 43.*

The Capstane is also us'd in Ships to draw up the Top-Masts, or Sail-Yards; and when it may be taken from one Place to another, it is call'd a *Flying-Capstane*, or *Crab*: But it is call'd *Jeer-Capstane*, or *Little-Capstane*, when it is upon the Second Deck, and is only us'd to draw the Top-Masts, Yards, and such Things as do not require so great a Force as what is requir'd to heave the Anchors; for the Capstane, which is for that Use, is call'd *Main-Capstane*, or *Double-Capstane*; it is fix'd upon the Lower Deck, and is useful for both Decks; because it may be rais'd Four Foot above the Upper Deck.

Sometimes the Leavers AB are applied Vertically, to turn Horizontally the Axel CD, and raise the Weight G fasten'd to the end of the Rope EF, which is twisted about the Cylinder CD, whilst several Powers applied at the ends A, B, of the Leavers, cause it to turn round the Two immoveable Points CD. *Plate 8. Fig. 45.*

Such an Axel, without a Wheel, is call'd a *Roll* (in French *Moulinet*, and in Latin *Succula*,) and the Mariners which use it to raise their Anchors, call it a *Windlass*: And when this Axel is made use of to raise a Weight very high, by means of a Rope going over Two Pullies very high above it, to facilitate its Motion, such a Machine is call a *Gin*, which is of great use to raise Stones in a Building.

Plate 8. Fig. 45. The Piece of Timber FK, which has in it Two Pullies which are usually of Brass, is call'd the *Head:* (in French *Etourneau*) You go up to it by means of the *Brace* and *Ladder* XT, full of little cross Pieces call'd *Pins* or *Rounds.* This Brace supports the Engine, and is pinn'd into a Mortise made at X upon the Sill XI, as also into another at T in the Spindle TV, below the Piece NQ, to which are fasten'd the little Braces R,S, which support the Head FK.

The *Perpendicular Shaft* ITV is not only supported by the Brace XT, but also by the Two *Arms* or *Braces* TL, TM, which rest upon the *Sole* LM, that is, perpendicular to the *Sill* XI, and are fix'd at top into a *Shoulder* T: These Braces, together with the Perpendicular Shaft IV, are kept tight together by the *Binding Pieces* OP, which have Tenons and Mortises, and support the *Transsoms* which are parallel to the Sill XI, that is fasten'd to the Sole by help of Braces. These *Transoms* strengthen and support the Brace and Ladder, and keep tight the upright *Puncheon* ZH, which being perpendicular to 'em sustains the Roll CD.

A *Tenon* is the End of a piece of Wood, which goes into a *Mortise*: and a *Mortise* is a Hole, usually Square, cut into another Piece of Wood, to take in the Tenons, which also are usually Square. The *Main Brace* XT, and the Braces TL, TM, are fix'd to the Perpendicular-shaft with Binding Pieces that have Mortises and Tenons fasten'd with Iron or Wooden Pins made to put in or take out at pleasure; to set up, or take down the Engine when you wou'd carry it from one place to another. The Lowermost of the Binding pieces OP is call'd, The *Main Binding Piece.*

When this Axel is laid on two pieces of Timber, so plac'd as to represent St. *Andrew*'s Cross, to draw Water or raise and let down Stones, or unload Barges, such an Engine is call'd a *Roll.* An Engine call'd a *Crane*, whose Head is made of a long Beam tending upwards, is also made use of to raise Timber or Stone in Building.

Of the Pulley.

Plate 9. Fig. 47. AN Engine made use of either to encrease the Force of the Power or to facilitate Motion, is call'd *Monospastum*, when it has but One Pulley; *Dispastum*, when it has Two; *Trispastum*, when it has Three; *Tetraspastum*, when it has Four; *Pentaspastum*, when Five; and usually *Polyspastum* when it has many.

Almost all Engines encrease the Force of the Power, except that which the Latins call *Fistuca*, and the French *Sonnette*, which is made use of to drive Piles, by means of a

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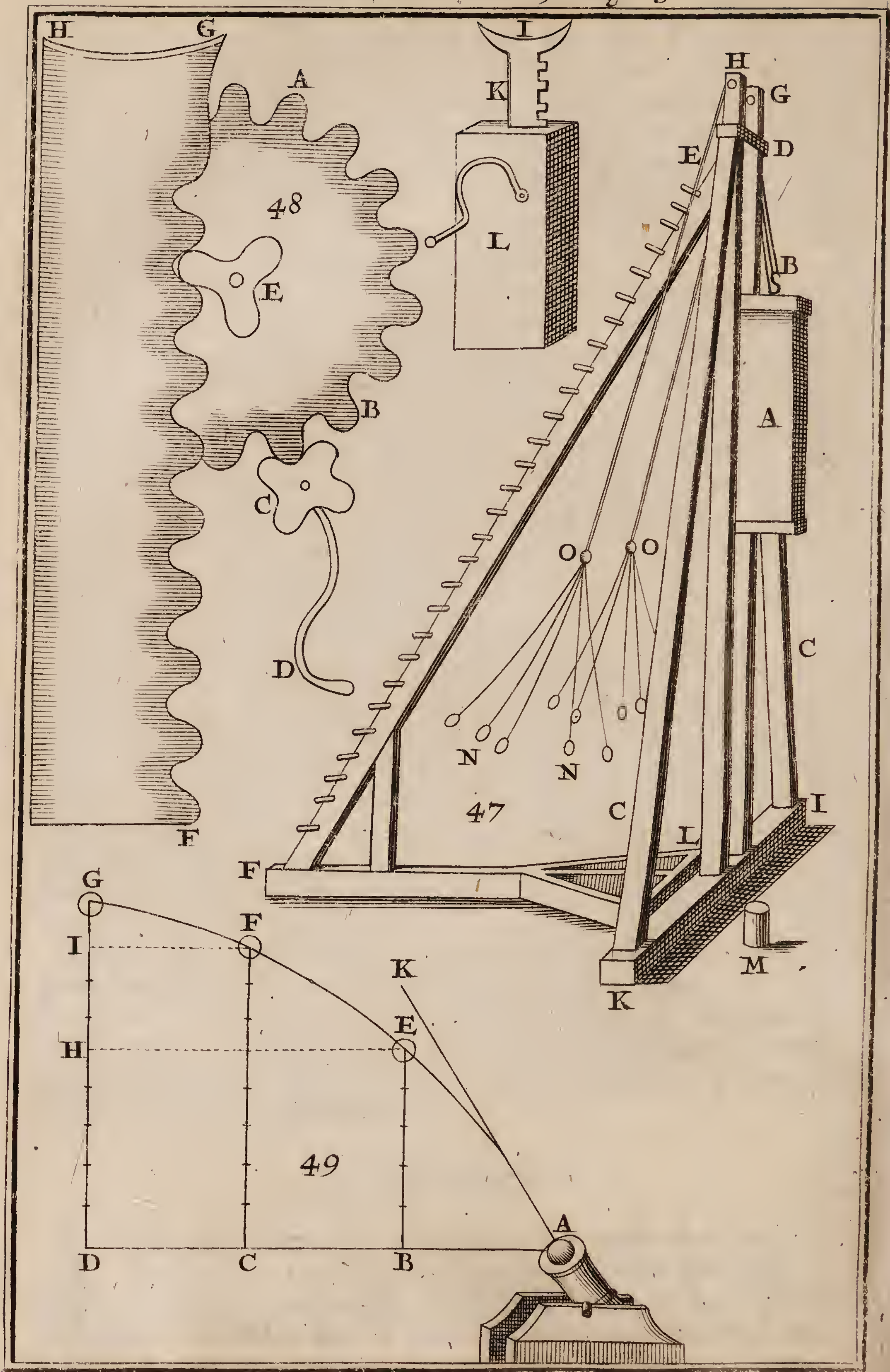
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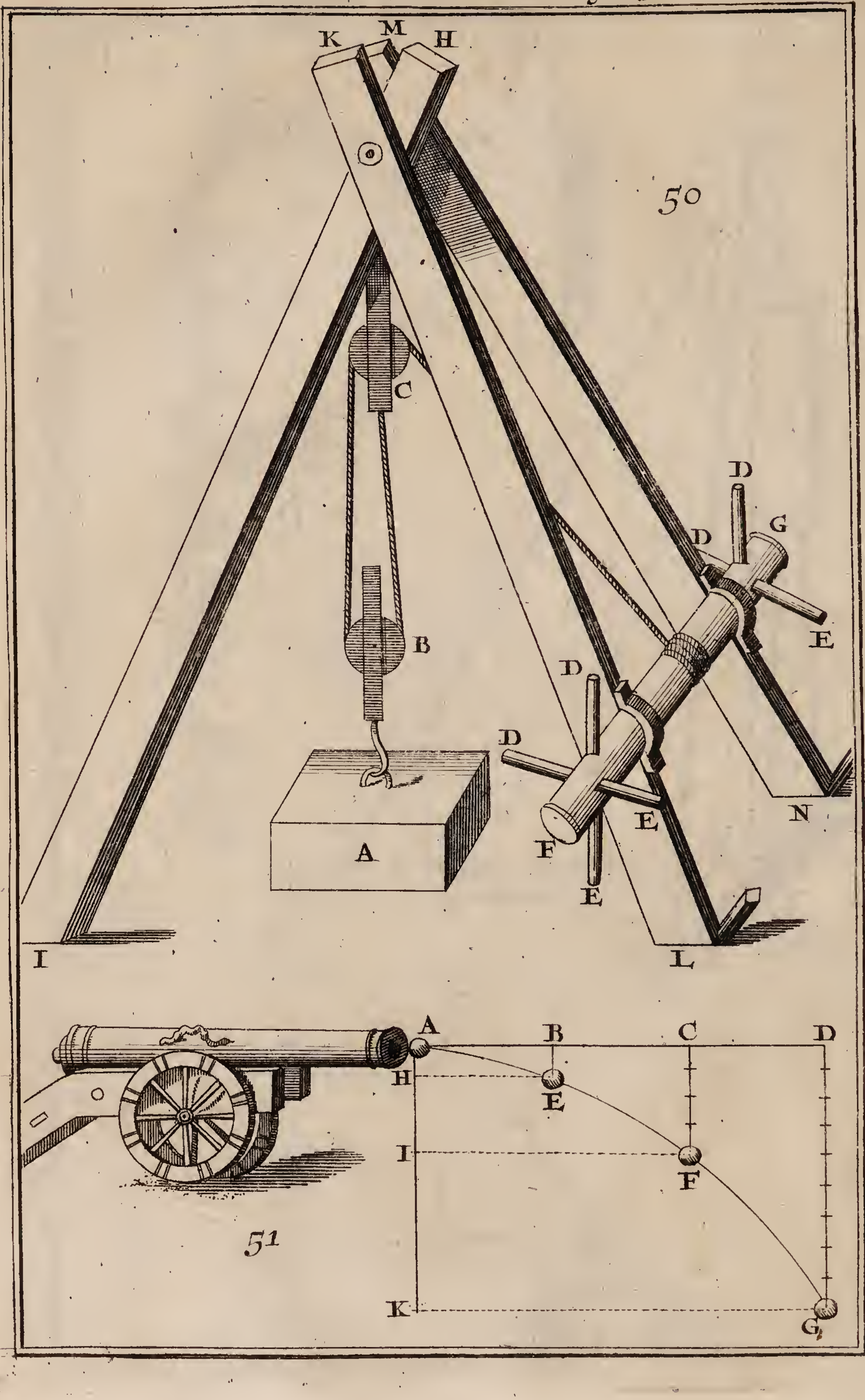
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Heavy Piece of Wood, or *Upright Rammer* A, so call'd in a Large Engine; but *Beetle* in a small one, which is us'd to draw out old and useless Piles, by striking gently upon their Heads, whilst they are pull'd by a Strong and Stretch'd Rope. *Plate 9. Fig. 47.*

This * *Rammer* is by Two Strong Iron Hooks fasten'd to Two Ropes, which go over the Pullies GH, and have usually at their Ends O, Sixteen smaller Ropes N tied fast, for as many Men to draw at once, when they raise the Rammer towards D, and let it fall at once upon the Head of the Pile M. *To Drive Piles*, by means of this Instrument, is call'd in French *Battre le Mouton.*

* *In French Mouton.*

The said Rammer has Two *Tenons* fasten'd behind with *Keys*, which are Square Wooden Pins to be taken off at Pleasure. These Tenons keep the Rammer in the Grooves which are made in the Two *Upright Puncheons* G, H, which are perpendicular to the *Sole* IK; and supported by Two *Braces* C, and a *Brace and Ladder* EF, which at F rests on the *Sill* LF. It is usually bound at bottom with a Strong Iron Hoop, lest it shou'd be cloven in striking on the Piles, &c.

The *Wind* may be reckon'd one of the Compound Engines; it is made use of in Building to draw Heavy Stones straight up as A, by means of the Two Blocks or Pulleys B, C, of which the lowest B doubles the Force of the Power, which is still encreas'd by help of the Leavers or Arms DE, applied to the *Tumbrel* FG instead of a Wheel. This *Tumbrel* rests on the Two long Pieces KL, MN, which at Bottom are at some distance from one another; but are Pinn'd together at Top. These Beams bear against a Wall, or if there is no Wall to sustain them, the *Stay* HI is added, and then the Engine is call'd a *Triangle.* *Plate 10. Fig. 50.*

Of the Axel *in the Wheel.*

THO' the *Crane* be very plain or simple, as having but one Wheel, which is call'd a *Counter-Wheel* as A, and is made to Turn together with its Axel, by being drawn on the outside, or when Men walk in it, to Move by that means the Rope DE which is fasten'd to the Axel, and going over the Pulleys or *Sheevers* F,M,N, raises the Weight H; yet, as it is a very Considerable Engine, and very Useful in Building to raise great Stones and take them to any design'd Place, and seeing it is made of several great Pieces of Timber, it may very justly be look'd upon as a *Compound Engine.* *Plate 11. Fig. 52.*

This Machine is so common, that almost every Body knows its Use; a sight of the Figure being enough to make you understand it. Wherefore we shall only say that the

Plate 11. Fig. 52. * French call the End C, of the Axel BC, *Lumiere*, and the other End B, *Mammelon.* The Piece K which supports the Brace and Ladder FG, that together with the Wheel A Turns upon I the End of the Spindle, which is Headed with Iron, is call'd the *Gudgeon of the Crane*; its Upper part being the *Spindle*, and its Lower being fix'd at Right Angles to Eight Cross-Pieces of Timber, call'd *Cross-Sills* or *Ground-Sills*: in French, *Embrassures*, *Empatements* and *Racineaux.* The Piece O which sustains the Brace and Ladder FG and the Counter-Wheel A, is call'd the *Stay*; in French *Soupente.*

Plate 13. Fig. 54. By means of a Wheel with Teeth, the Power may be encreas'd as much as you please; for if you have but One Wheel, whose *Radius* is, for Example, Ten Times greater than That of its Axel, the Power applied at the Circumference of this Wheel, will have Ten Times more Force than otherwise: and if you add a Second Wheel as A, which is call'd a *Pinion* when its Circumference is Little, whose Teeth take those of the Greater Wheel; the Force will still be encreas'd as much as the *Radius* of this *Pinion* is greater than That of its Axis: as for Example, if the *Radius* of this *Pinion* be Six Times longer than That of its Axis; the Force of the Power will be encreas'd Sixty Times, and more still, if you add the Handle DBC, which will encrease the Force in proportion to the Length of the Line BD: Thus if E weighs Sixty Pounds, One Pound apply'd at C will raise it.

Plate 12. Fig. 53. If you multiply the Number of Wheels and Pinions, as in *Fig.* 53. the Force of the Power is prodigiously encreas'd; whence the Greeks and Latins have call'd this Engine *Pancratium*; because there is no Burthen so Heavy but what it may Raise; and it was not without reason that *Archimedes* wou'd undertake to raise the Whole Earth, if he had but a fix'd Point to set his Engine on. *Da mihi Punctum, & Terram movebo.* But as by the Multiplication of Wheels and Pinions you encrease the Force of the Power: So likewise you Lose Time in raising the Weight A, fasten'd to the Wheel BC, which will turn very Slowly.

Plate 9. Fig. 48. The Force of the Power is likewise very much encreas'd by means of the *Jack*, which is made use of to lift up Coaches and overthrown Waggons, by means of AB a Wheel with Teeth, and the *Handle* or *Windlass* CD, which causes the *Pinion* C to turn, whose Teeth taking those of the Wheel AB, causes it also to turn together with its *Pinion* that has Three *Leaves* or Teeth, which taking the Teeth of the *Rack* FG, raise it up together with the Weight that bears on the Forked Top of it GH. The Figure ILK shews the Out-

* *There are no peculiar Words in English to express those Ends of the Axel.*

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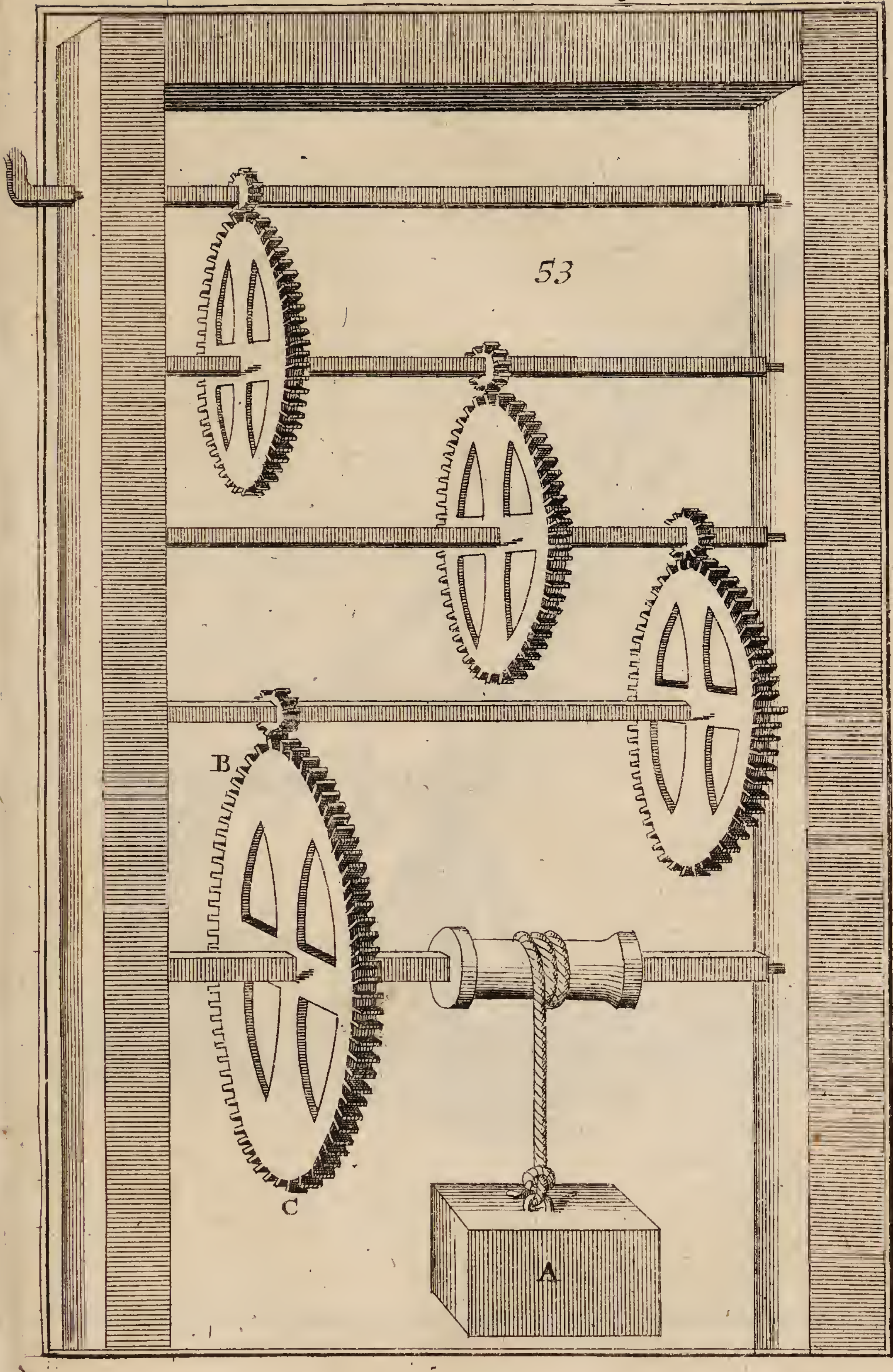
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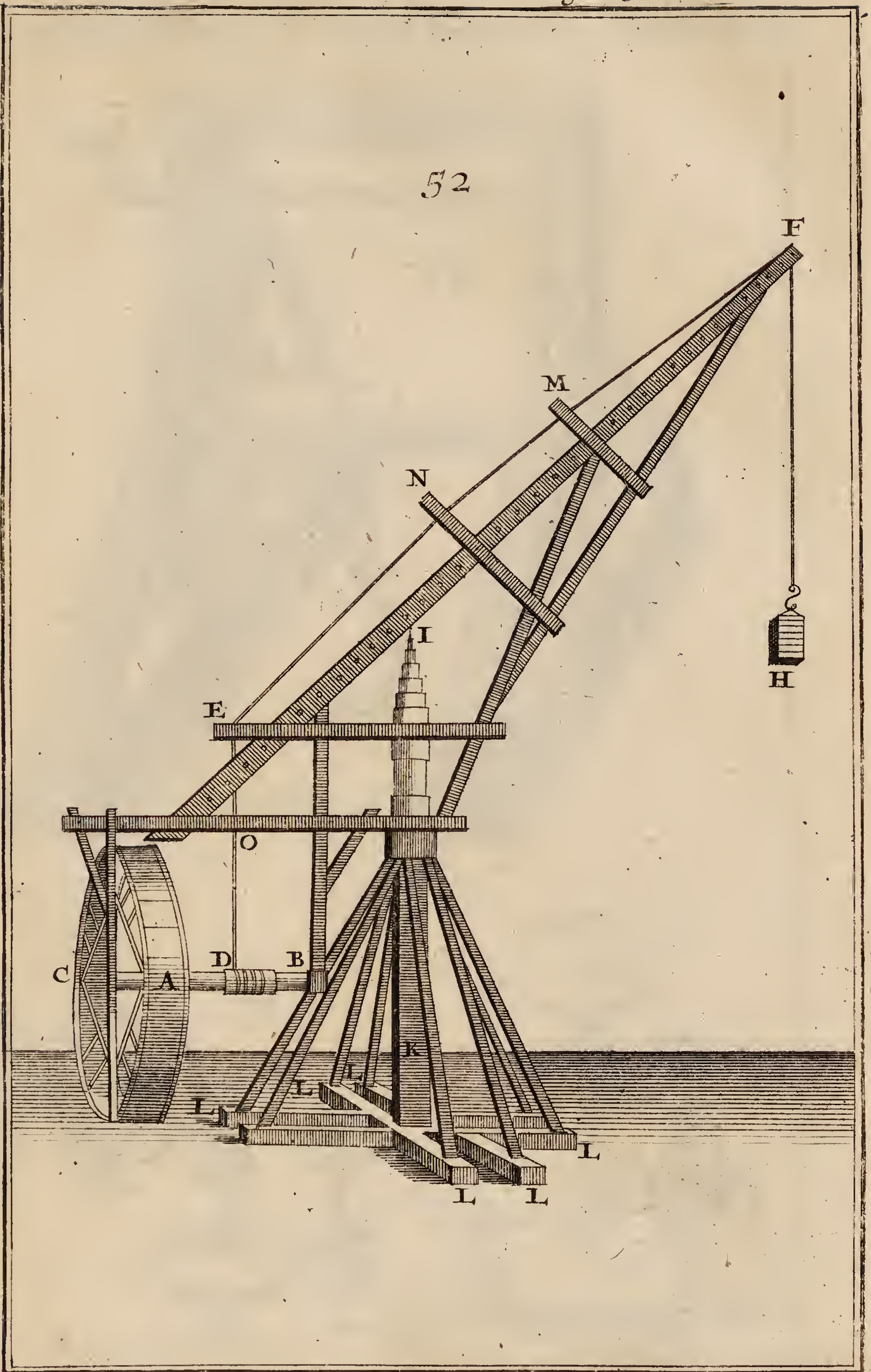
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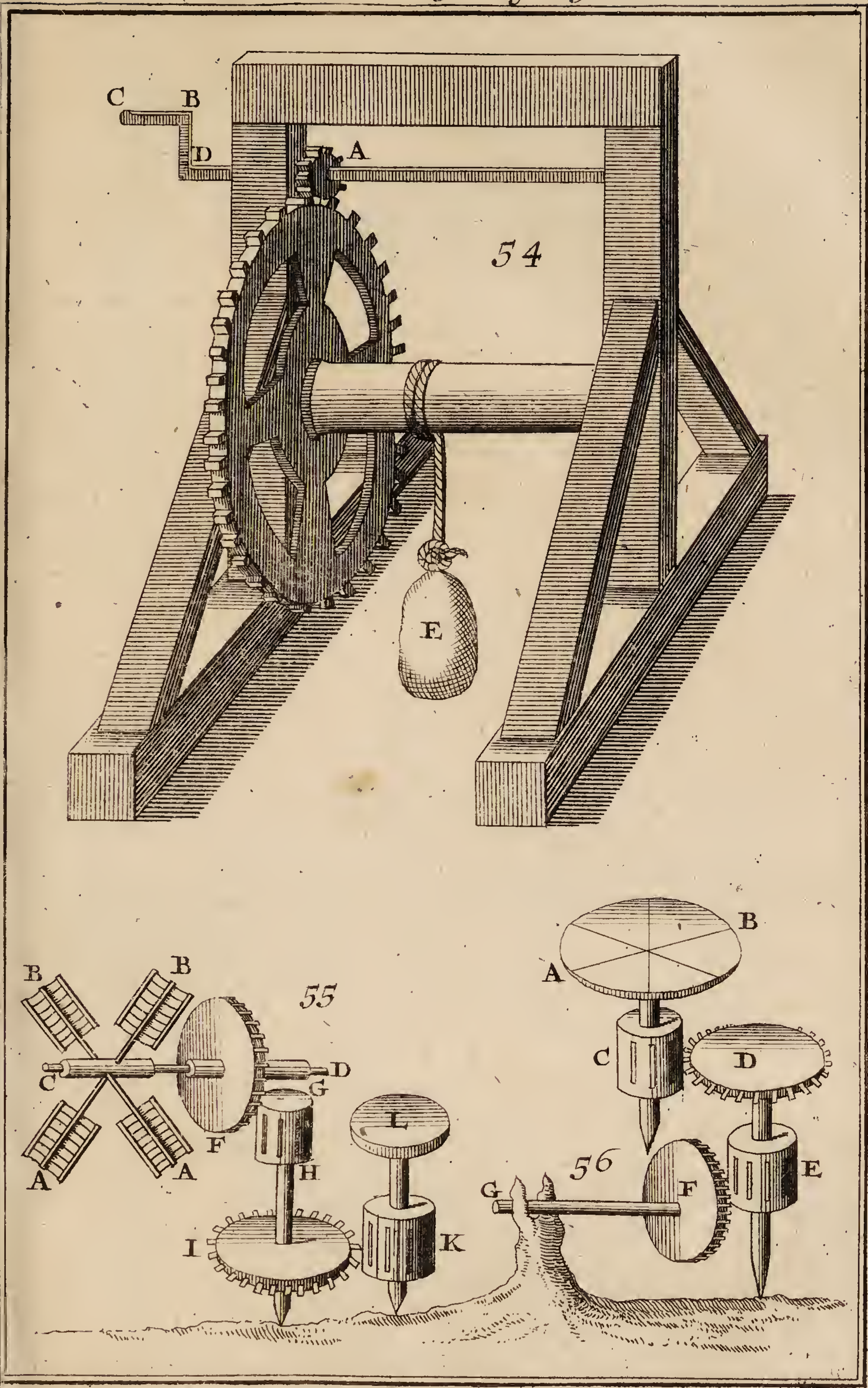
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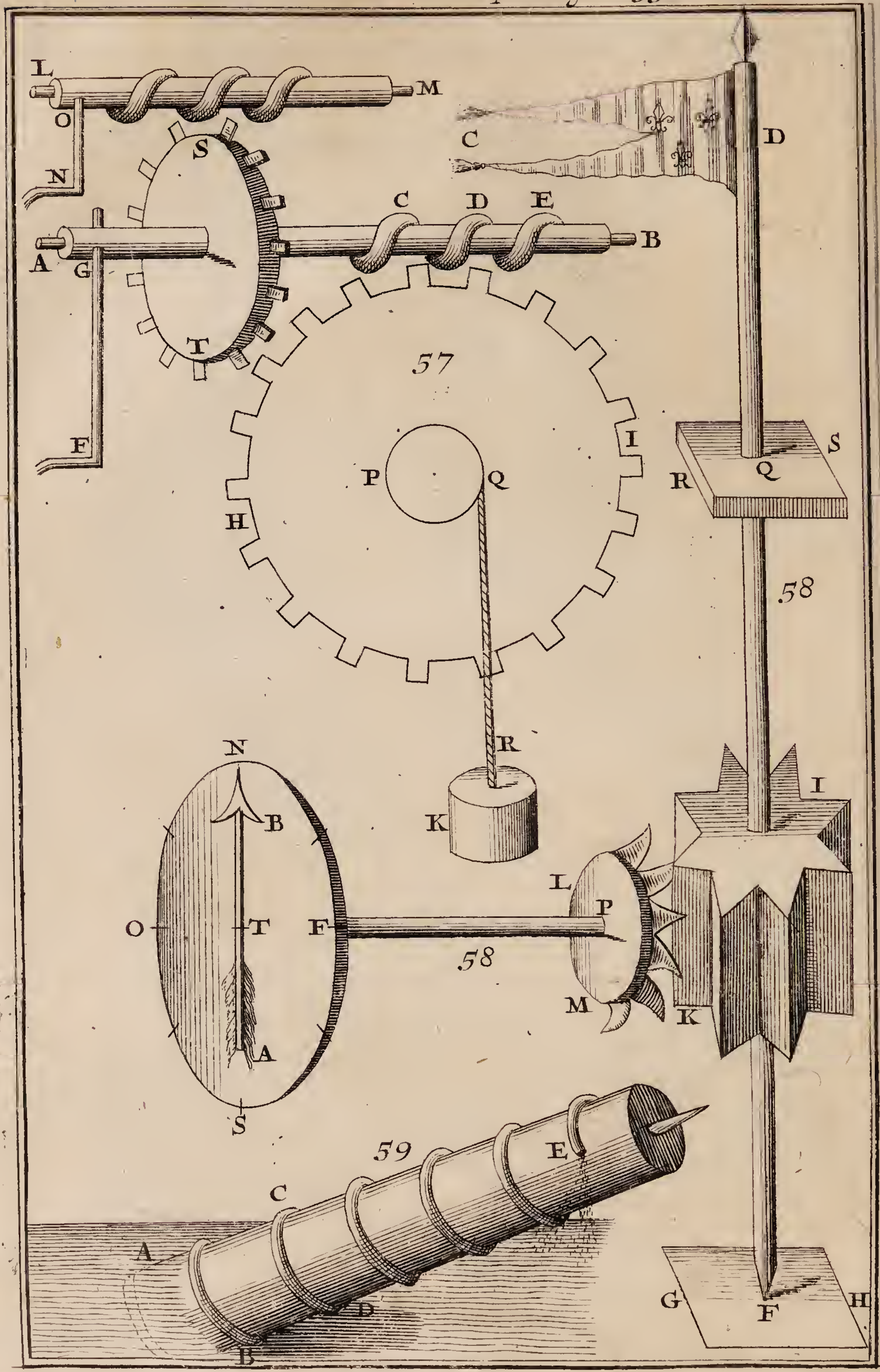
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A
D
B
G
F
H

ſide of the *Jack*, which by Multiplication of Wheels may be made Strong enough to raiſe up a Houſe; but then it will work more Slowly.

There are ſome Compound Engines wherein the Wind is made uſe of for a Power, as in the *Wind-Mill*, where the Wind blowing round the *Sails* AB, (when their Canvas is ſtretch'd) which in ſuch a caſe are Leavers, cauſes the Axel CD to turn Horizontally, and the Wheel EF Vertically, whoſe *Cogs* taking the *Rounds* of the *Trundle* GH, cauſe it to turn, and at the ſame time to turn the *Spur-Wheel* I, which taking the *Spindles* or *Rounds* of the *Trundle* K, turns it about together with the *Mill-Stone* L, which Grinds the Corn. *Plate* 13. *Fig.* 55.

A *Trundle* has 9 Rounds, and a *Wallower* 14; The Firſt has its Axis uſually Vertical, and the Laſt its Axis Horizontal. The Whole Body of the Mill, call'd in French *Cage*, may be turn'd round upon its *Poſt*, to put the Sails in the Wind. The Mill-Poſt is arm'd with Iron. CD is the *Shaft* or *Roll* of the Mill, AB the *Whips* which have *Barrs* to ſtretch the Sails upon.

The Wind is alſo applied to an Engine call'd an *Anemoſcope*, where the *Hand* AB ſhews what Point of the Compaſs the Wind blows from, the Names of the Winds being mark'd upon the Circumference NOSE, as it is in the Mariner's Compaſs: The Weather-Cock CD, whoſe End D is fix'd to the Top of the long Axis DF, perpendicular to the Horizon, turns with the leaſt Breath of Wind; becauſe F the end of the Axis ends in a Point where it is ſuſtain'd by the plain GH; this Axis runs thro' the Pinion IK that it may turn it: This Pinion has Eight Wings or Leaves at equal diſtances for the Eight Chief Winds. *Plate* 14. *Fig.* 58.

Theſe Leaves take the Eight equal Teeth of the *Crown-Wheel* LM, which is perpendicular to the Horizon, and cauſe it to turn together with its Axis PE, and the Hand AB, which is fix'd to the end of the ſaid Axis, and with its end B points to the Wind that Blows, and is mark'd upon the *Dial-Plate*. The Long Axis DF goes thro' a Hole made at Q thro' the Horizontal Plain RS, that it may always remain Upright or Perpendicular to the Horizon: and the Axis PE of the Wheel LM goes thro' a Wall, and alſo thro' the Center T of the *Dial-Plate*, as may be ſeen at *Paris* in the French King's Library, and upon the *Pont-neuf* (New-Bridge) on that Clock which is call'd *l' Horloge de la Samaritaine*.

Inſtead of Wind, *Smoke* is ſometimes made uſe of to turn a Spit, and the Meat to be roaſted with wonderful Eaſe; for tho' the Smoke be of it ſelf very Weak, yet its Force is very much encreas'd by means of the Wheels C, D, E, F, whoſe Teeth take one another: As the Smoke riſes it turns the *Plate* 13. *Fig.* 56.

 Wheel

Plate 13. Fig. 56. Wheel AB with its Trundle C, whose Rounds taking the Teeth or Spurs of the Wheel D, cause it to turn together with its Trundle E, whose Rounds likewise taking the Cogs of the Wheel F, cause it to turn with its Axis or Spit FG. Running Water is also very useful to turn Water-Mills, and other Engines to raise Water, or draw away the Water from a Place where you wou'd build: for Running Water by its Swiftness drives before it the *Floats* of a great Wheel, which have one Part in the Water, and thus has Force enough to turn round a Wheel with its Axis, and so the whole Engine.

As these Machines are very common, we shall say no more of them. One may see at *Paris* upon the *Pont-neuf* a very pretty Engine, call'd the *Samaritan*'s Engine; but the Finest in *Europe* is that which is at *Marly* near *Paris*, which is admir'd by all Travellers, and will be a Monument to Posterity of the Greatness and Magnificence of King *Lewis the Fourteenth*.

Of the Wedge.

THE Wedge has no Force of it self as you have seen, because it must be driven by some Power; and of all Force Percussion, and especially that which is the Fall of some Weight, is the most effectual.

To this Engine, which is the Plainest of any, may be reduc'd all Edge-Tools, and Tools that have a sharp Point; to Cut, Cleave, Slit, Chop, Pierce, Bore, *&c.* as Knives, Hatchets, Scissors, Swords, Bodkins, *&c.*

Of the Screw.

Plate 6. Fig. 39. THE Force of the Screw is encreas'd according to the Length of the Leaver which is applied to its Collar G to turn it about. It is Useful to make Jacks to lift great Stones into Carts, or to raise up a House with the *Point* or Bearing-piece HI, which is a Piece of Timber perpendicular to the lower Piece KL; which has Two Female Screws to receive the Male Screws, which are turn'd by the means of Leavers applied to their Collars G, which mightily encreases the Force of the Screw, especially if its Threads are very close.

A *Printing-Press* is made of a Leaver and a single Screw, which is also us'd in the *Mint* to Stamp the King's Image on the Coin. Much such an Engine as is represented by Fig. 39. is made use of to press Linnen or Books; and it is sometimes made very Large for a *Wine-Press*.

Plate 14. Fig. 57. A Screw is sometimes made to take the Teeth of a Wheel and turn it round; and such Screw is call'd a *Perpetual Screw*;

Screw; becauſe being turn'd by means of a Handle, it cauſes the Wheel and its Axel to turn continually, together with the Weight faſten'd to it by means of a Rope, which winds it ſelf about the Axel as the Weight is drawn up: This Engine has a vaſt Strength, which may be encreas'd by multiplying the Wheels and Screws. *Plate* 14. *Fig.* 57.

As in this Figure if by means of the Handle or Leaver FG the Axel AB which has upon it the Screw C, D, E, be made to turn; the Screw taking the Teeth of the Great Wheel HI, will cauſe it to turn together with its Axel PQ, which will raiſe the Weight K faſten'd to the Rope QR. But if you add a ſecond Axel LM, whoſe Screws take the Teeth of the ſecond Wheel ST; by turning this Axel LM by means of the Leaver NO, the Wheel ST will turn with its Axel; and thus the Force of the Power will be prodigiouſly encreas'd, *&c.*

It is not improper to ſpeak here of *Archimedes*'s Screw, *Fig.* 59. which the French call *Limace*, whoſe Effect is ſo much the more Wonderful as it ſeems Contrary to Reaſon; becauſe this Engine makes Water Riſe by Deſcending. It is a Pipe wound Screw-wiſe about a Cylinder, which the French call *Noyau*; when it is made uſe of, One of its Ends as A is put into the Water that you wou'd raiſe; for as the Water goes into the Pipe at A it muſt deſcend to B which is lower; and as the Engine, which ought to be Inclin'd, turns round, the Part B will come up to C, and the Part C will go down to D, which will cauſe the Water to run from B to C, and from C to D, and ſo on as far as E the Upper Hole of the Pipe, which it will run out at. It is ſaid, that *Archimedes* contriv'd this Engine for the Uſe of the *Egyptians*, to drain their Marſhes caus'd by the Inundation of the *Nile*.

Tho' this Engine will draw up a great deal of Water, it will not draw it very High, becauſe it ought to be very much inclin'd to the Horizon: But Water may be rais'd as high as you will with an Engine which is very Common, call'd a *Jack Pump* or *Chain Pump*, which has ſeveral little Buckets wider at top than at bottom fix'd to an Iron Chain, which is mov'd round upon an Axel by means of a Wheel which is turn'd by Men, Horſes, or Water: The end of the Chain hangs in the Water, and the Buckets bring up the Water as they come up and pour it out into any place deſign'd.

Sometimes ſmaller Engines of this kind are us'd, which Two or Three Men turn by means of a Handle as AB, which is faſten'd to the Cylinder CD, call'd the *Rag-Wheel*, and thus the Chain DG which is upon the ſaid Cylinder is roll'd round: Such a Chain is call'd an *Endleſs Chain*, be- *Plate* 15. *Fig.* 60.

Plate 15. Fig. 60. cauſe it moves continually in the Pipe EF which is in the Water; as it moves round it brings up the Water with Pieces of Leather cut in the Form of Half-Globes, which are fix'd to the Chain inſtead of Buckets. This Engine is moſt properly call'd the *Jack-Pump.*

This, and all other Engines, which raiſe Water by Water, or any other Moving Force, are call'd *Hydraulick* Engines: And we call *Pneumatick* Engines ſuch as by the Impulſe of the Air imitate the ſound of Muſical Inſtruments with Strings, as the Organ; or the Voice of Man, or other Animals, as the Clock at St. *John*'s Church at *Lions*, where the Crowing of a Cock is heard before the Clock ſtrikes.

The

The Second Book.

Of STATICKS.

AS *Mechanicks*, in a ſtrict Senſe, only conſider the Moving Powers as they are apply'd to Engines; We thought it not improper to Treat of them ſeparately from *Staticks*, which conſider the different Weight, and Centers of Gravity of Heavy Bodies. This Part takes in a great many Phyſical Queſtions, which are out of the Province of a Mathematician, and which conſequently we ſhall paſs over, becauſe we wou'd not go from our firſt Deſign, which is to ſpeak to Military Men, who love the Practical, but hate the Diſputing part.

CHAPTER I.

Of the Free Deſcent of Heavy Bodies.

BY the Free Deſcent of a Heavy Body, we underſtand here the Fall of that Body in the Air when it meets with no other Body to oppoſe its Motion. We have obſerv'd in the beginning of the foregoing Book, that a Body which falls freely in the Air, acquires in equal Moments or Times of its Fall, equal degrees of Velocity, and that the Spaces, which the Body goes thro' encreaſe each Moment or Inſtant of Time, according to the Series of the firſt uneven Numbers 1, 3, 5, 7, 9, *&c.* which are the differences of the Squares 1, 4, 9, 16, 25, *&c.* of the Arithmetically proportionable Numbers 1, 2, 3, 4, 5, *&c.* and that conſequently the Spaces which Bodies go thro' from the beginning of their Fall, are in a Duplicate *Ratio*, or as the Squares of the Times or Moments: And it is only by Experience that we can account for this Proportion, which will be as a Foundation for the greateſt Part of what we are about to ſay.

PRO-

PROPOSITION I.

PROBLEM.

The Space which a Heavy Body goes thro' in a Determinate Time being known; to find out what Space it will go thro' in a Given Time.

SUppose a Body to have descended 24 Foot in One Minute of Time, to find how far the same Body will fall in the same Medium in Three Minutes, for Example; to the Three Numbers 1 : 9 : : 24: (the Two First of which (*viz.*) 1, 9, are the Squares of the Given Times 1, 3, and the third (*viz.*) 24, is the Space gone thro' in the first Time) find a fourth Proportional; which will be 216 Foot, for the Space which the Body will go thro' in the second Time, that is, in Three Minutes.

DEMONSTRATION.

Because the Spaces gone thro' are as the Squares of the Times, and that in this Example the Times are 1, 3, and their Squares 1, 9; it is evident, That since 1, the Square of the first : has the same *Ratio* to 9, the Square of the second : : as 24, the Space gone thro' in the first Time : has to the Space which the Body will go thro' in the second Time; this Space must be known by finding a fourth Proportional to these Three Numbers 1 : 9 : : 24: as has been done.

PROPOSITION. II.

PROBLEM.

The Time being known in which a Heavy Body descends thro' a Determinate Space, to find in how long time it will descend thro' a Given Space.

SUppose a Body has spent One Minute in falling 24 Foot; to find what Time it will spend in falling, for Example, 216 Foot in the same Medium; to these Three Numbers 24: 216:: 1: (the Two first of which (*viz.*) 24, 216, are the first and second Space given, and the third (*viz.*) 1, is the Square of the given Time) find a fourth Proportional, which will be 9 Square Minutes, for the Square of the Time requir'd, which consequently will be 3 Minutes, as is known by

by extracting the Square Root of 9, the fourth Number found.

DEMONSTRATION.

Since the Spaces gone thro' are as the Squares of the Times, and that in this Example the Spaces are 24, 216, it is evident, that since 24 the first Space: has the same *Ratio* to 216, the second Space:: as 1, the Square of the first Time, which answers to 24, the first Space: has to the Square of the second Time, which answers to 216, the second Space; the Square of the second Time must be known by finding a fourth Proportional to the Three Numbers 24: 216:: 1: as has been done.

PROPOSITION III.

THEOREM.

The Force which carries up a Body Perpendicularly, grows less Equally.

I Say, That if a Heavy Body be thrown upwards perpendicularly, by giving it a Perpendicular Force which continues, that Motion will gradually Decrease.

DEMONSTRATION.

Because the Gravity of the Body which is thrown up, constantly, pushes it downwards; its Motion upwards must continually decrease, and be wholly destroy'd, when the *Impetus* upwards, which it receiv'd from the Power that threw it up, becomes equal to that *Impetus* which its own Gravity gives it downwards; that is, the Body thrown upwards must cease to rise as soon as the Two Impulses are become Equal; and then immediately begin to descend, because then the Impule of Gravity is greater than that of the Projection. Since then Gravity lessens the Velocity of the Impulse upwards, and, by its contrary Action, destroys the Motion upwards with the same Force that it wou'd produce a Motion downwards, which is Uniformly accelerated, the Force which pushes upwards must also decrease Uniformly. *Q. E. D.*

SCHOLIUM.

It is plain, That since a Body in falling acquires equal degrees of Velocity in equal Times, and that on the contrary, in its rise it loses equal degrees of Velocity in equal Times; that

that is, that since the Velocities decrease in Rising in the same Inverse Proportion that they encrease in Descending: Such a Body goes thro' the same Spaces in equal Times in Rising, as it does in Falling. Whence it follows, That the Spaces which the Body goes thro' when thrown upwards, are in an Inverse Order, the same which the Body goes thro' in its Descent: So that if the said Body is Five seconds in Rising to the height of 25 Foot, and that the Space which it goes thro' the first Second of Time be 9 Foot, for Example, the second *Second* of Time, it will go thro' a Space of 7 Foot, the third a Space of 5, the fourth a Space of 3, and the fifth and last, a Space of One Foot; to the very Instant when it is in *Æquilibrio,* without either rising or falling: then it will first begin to descend, in the same Inverse Proportion running thro' the same Spaces in the same Times; so that the first Second it will descend a Foot, the second 3 Foot, the third 5, the fourth 7, and the fifth and last the Space of 9 Foot, thus taking up five Seconds of Time to Descend 25 Foot; as it took up the same Time to Ascend 25 Foot.

PROPOSITION IV.

PROBLEM.

The Time being known in which a Heavy Body falls from a known Height, to find out how far it will fall in each Part of that Time.

*Fathoms. SUppose a Heavy Body has taken up 5 Seconds in Descending 125 * Toises; to find out how many Toises it will Descend each Second; let x be put for the Number of the Toises that it must run thro' the first Second; and then because the Spaces which the Body runs thro' in equal Times, encrease according to the Progression of the odd Numbers 1, 3, 5, 7, 9, *&c.* the Space which the Body has faln thro' the second *Second* will be $3x$, the Space gone thro' the third Second will be $5x$, the Space gone thro' the fourth Second will be $7x$, and the Space gone thro' the fifth and last Second will be $9x$: And as the Sum of all those Spaces is $25x$, which are suppos'd equal to 125, you will have this Equation $25x = 125$, which being divided by 25, you will have $x = 5$, which shews that the first Second the Body has Descended 5 Toises, and consequently it will have Descended 15 Toises during the second *Second*, because of $3x$, and 25 Toises the third Second, because of $5x$, and 35 Toises the fourth Second, because of $7x$, and lastly 45 Toises the fifth and last Second, because of $9x$. *Q. E. I.*

SCHO-

SCHOLIUM.

This Problem is ſo eaſy, that it may be ſolv'd without the help of *Algebra*: For it is eaſily ſolv'd, if you divide the given Number 125 into Five other Numbers proportional to to theſe Five 1,3,5,7,9, which may be eaſily done by the *Rule of Fellowſhip*. But to come to the Practice; Multiply each of theſe Numbers 1,3,5,7,9, by the given Number 125, and divide each of the Products 125, 375, 625, 875, 1125, by 25 the Sum of the ſaid Numbers 1,3,5,7,9, and the Quotients 5, 15, 25, 35, 45, will be the Spaces gone thro' during the firſt, the ſecond, the third, the fourth, and the fifth and laſt Second of Time.

LEMMA.

In an Arithmetical Progreſſion, every one of the Sums of Two Terms equally diſtant from the Two Extremes, are Equal to the Sum of the Two Extremes.

LET the following Progreſſion be an Arithmetical One, of Seven Terms, a, $a+b$, $a+2b$, $a+3b$, $a+4b$, $a+5b$, $a+6b$. 'Tis plain that $2a+6b$ the Sum of the Two Terms $a+b$, $a+5b$, or of the Two $a+2b$, $a+4b$, equally diſtant from the Two Extremes a, $a+6b$, is equal to the Sum of theſe Two Extremes, and that conſequently all thoſe Sums are equal to one another. *Q. E. D.*

COROLLARY.

It follows from this Propoſition, that when the Number of the Terms is odd, as here, the ſaid Sum $2a+6b$ is double the Middle Term $a+3b$.

PRO-

PROPOSITION V.

THEOREM.

The Force which pushes a Heavy Body upwards to a certain Height, if it were not diminish'd, wou'd in the same Time carry the same Body twice as High.

SUppose that with a determinate Force a Heavy Body be push'd upwards, Seven Toises, for Example, in One Minute; I say that if that Force had not been diminish'd, but continued the same as at first, it would have carried the Body double that height; that is Fourteen Toises in the same Minute of Time.

DEMONSTRATION.

Divide the Time into Seven Moments, for Example, and likewise the Force into Seven Degrees, and it will easily be known; that since by *Prop.* 3. this Force decreases Uniformly, and that it was suppos'd at the beginning of the first Moment to have Seven Degrees of Velocity, at the end of that Moment it will have but Six Degrees of Velocity, at the end of the second but 5, at the end of the third but 4, at the end of the fourth but 3, at the end of the fifth but 2, at the end of the sixth but One; and at the end of the seventh and last Moment, it will have no Degree of Velocity left.

Thus we have an Arithmetical Progression made up of these Eight Terms 7, 6, 5, 4, 3, 2, 1, 0, where the Sum of the Two Extremes and of any Two Terms equally distant from those Extremes is, *by the foregoing Lemma*, every where the same (*viz.*) 7; which makes in all 4 times 7, or 28. Now if the Force had not been diminish'd, it wou'd have had each Moment Seven Degrees of Velocity, and as it makes the Body run thro' a Space proportionable to its Strength, and as it wou'd have had in all 8 times 7 Degrees, or 56 Degrees of Velocity, which is twice 28, so it wou'd have carried the Body to a double Height. *Q. E. D.*

PROPOSITION VI.

THEOREM.

Two Powers force a Heavy Body upwards to Heights, that are to one another, as the Square of Two Numbers which expreſs the Ratio *of the Two Powers.*

I Say, that if a Power be, for Example, Three Times greater than another Power, ſo that the Two Powers be in the ſame *Ratio* as theſe Two Numbers 3, 1, it will by its Triple Force raiſe a Body to a Height Nine Times greater than that to which the little Power will by its Force raiſe the ſame Body, ſo that the Two Heights will be as theſe Two Numbers 1, 9, which are the Squares of the Two Numbers 1, 3, which expreſs the *Ratio* of the Two Powers.

DEMONSTRATION.

Becauſe the Body, or the Gravity which diminiſhes the Force of both theſe Powers is the ſame, the Force ought to be diminiſh'd by Equal Degrees, and that which is Three Times as much as the other, ought to be Three Times as long in decreaſing: If then the Greateſt is Three Minutes in decreaſing, and the Leaſt One Minute, the Greateſt ought to make the Body in its third and laſt Minute to run thro' a Space equal to that which the Little one made the ſame Body run thro' the firſt Minute; and the ſecond Minute it ought to make it run thro' a Space Three Times greater, and the firſt a Space Five Times greater, becauſe the Spaces decreaſe in the riſe, or encreaſe in the fall, according to the proportion of the odd Numbers, as we have obſerv'd in *Prop.* 3. So that if the firſt Minute the little Power puſhes up the Body the height of One Toiſe, for Example, the other Power which is Three Times Greater, will likewiſe puſh up the ſame Body the third Minute, the Space of One Toiſe, and of Three Toiſes in the ſecond; and laſtly, of Five Toiſes in the firſt Minute, which makes in all Nine Toiſes. Thus you ſee that when the Leaſt Power has rais'd its Body the height of One Toiſe, the triple Power has rais'd its Body Nine Toiſes; and conſequently thoſe Heights are to one another, as 1, to 9, which are the Squares of thoſe Two Numbers 1, 3, which expreſs the *Ratio* of thoſe Two Powers. *Q. E. D.*

COR-

COROLLARY.

From this Propoſition one may eaſily conclude, That if the Force which acts directly Upwards, be Double, the Space will be Quadruple; and conſequently that a Bow, as Strong again as another, will ſhoot an Arrow Four times as high.

PROPOSITION VII.

THEOREM.

The Force which a Body acquires in Falling, makes it Riſe up again to the ſame Height which it fell from.

THIS Propoſition is evident, firſt by Experience, then because a Body in deſcending acquires a Velocity, which makes it riſe again by puſhing it upwards as ſoon as it is got to the loweſt place that it can go, if nothing has hindred its fall: and if the acquir'd Force or Velocity ſhou'd continue the ſame, it wou'd by *Prop.5.* make the Body riſe to a Double height; but as the Gravity of the Body makes its Velocity continually Decreaſe after the ſame manner that it had Encreas'd, it can only carry up the Body to the ſame Height from which it fell. *Q.E.D.*

SCHOLIUM.

In all that we have ſaid, you muſt *abſtract* from the Gravity and Reſiſtance of the Air, which is the cauſe why all the foregoing Propoſitions do not exactly agree with Experiments, eſpecially this, for we ſee that Pendulums do not exactly return to the ſame Height from which they fell, or elſe they wou'd have a Perpetual Motion; but we ſee that they Stand ſtill in a little time.

Several Conſequences are drawn from the Gravity and Reſiſtance of the Air, which are confirm'd by Experiments. The firſt is, that the *Motion of a heavy Body is not always Accelerated, but at a certain Height it becomes Equal and Uniform in the Air*; becauſe the Reſiſtance of the Air encreaſing in the ſame proportion as the Spaces encreaſe; and conſequently in a Duplicate *Ratio* of the Times, or of the Velocities, this Reſiſtance may become ſo great as to deſtroy as much of the Velocity as ſhou'd be produc'd, and by that means hinder the Velocity of the Moving Body from being encreas'd any more.

The

The Second is, That *Different Bodies moving in the same Medium have not their Motions Accelerated after the same Manner*, by reason of the difference of their Bulk, which meets with more or less Resistance; because Those of a greater Bulk drive more Air before them than Those of a less.

The Third is, That the *Motion of Heavy Bodies is differently Accelerated in different Mediums, and in the most Dense Medium it becomes Equal soonest*; because the more Dense the Medium is, the more difficulty it has to make its Circulations, and consequently it resists Motion the more easily.

The Fourth is, That the *Least Bodies of the same Homogeneous Matter fall with less Velocity, and come soonest to an Equality*; because that Body which has a Greater Surface is more resisted than that which has a Less, and the Less Bodies have a greater Surface than the Great Ones in respect of their Weight or Solidity; for we are taught by Geometry that if a Cube has its Surface, *for Example*, of One Foot, another Cube Eight times as heavy will have its Surface but of Four Foot. According to this Principle, the Dust falls very slowly when it is rais'd, Birds sustain themselves in the Air by spreading their Wings, and a Pike thrown in the Air, or in the Water, falls upon its Point, *&c.*

The Fifth is, That *there is a Determinate Height which produces in a Heavy Body the greatest Velocity that it can acquire in Falling*; so that if it shou'd fall from an Higher place it wou'd have no more Velocity, which is evident from the first Consequence, where we have said that the Motion of a Heavy Body is not continually Accelerated; but that at a determinate Height it becomes Equal.

The Sixth is, That *there is a Determinate Height, the greatest of all those to which the Velocity which a Body has acquir'd in Falling, can make the same Body Rise up again*; because by the foregoing Consequence, there is a determinate Height, which produces the greatest Velocity that a falling Body can acquire, and that Velocity can make it rise up again but about to the same Height.

The Seventh is, That *a Body thrown Upwards by a Force Greater than the Greatest that it can acquire in Falling, ought to be longer in Falling than in Rising*; because the Velocity of the Body thrown up to any Height whatever, is continually diminish'd; whereas the Velocity of the same Body in its fall encreases but till it comes to such a Height, it being certain that if it shou'd encrease continually, the Body wou'd be just as long in falling as it was in rising.

The Eighth is, *That if a Body be thrown Downwards by a Force which exceeds the Greatest Force that it can acquire in falling*, it has a Retarded *Motion*; because by the first Consequence the

Body which falls with the Greateſt Velocity that its fall wou'd give it, meets with a Reſiſtance in the Air, equal to its Gravity; and when it goes with a Greater Force, the Reſiſtance of the Air becomes Greater than its Gravity, and muſt deſtroy part of the Motion, which thus will be Slackened and Retarded.

This laſt Conſequence ſhews why a Cannon-Ball ſhot Downwards Retards its Moton; becauſe ſuch a Ball is put in Motion by the Force of the Powder which gives it a greater Velocity, than that which its Abſolute Gravity wou'd have given it in falling: And the Seventh Conſequence ſhews likewiſe the reaſon of this Experiment, which Father *Merſennus* takes notice of in his *Baliſtica*, or Art of throwing Heavy Bodies, *Prop.* 13.

This Author ſays, that he has found by ſeveral Experiments, that an Arrow which has been Three Seconds in Riſing has been Five in Deſcending: And tho' he adds that an Iron Bullet of Three Pound-weight having been ſhot Upwards perpendicularly by a Mortar-piece a Foot long, has ſpent as much time in Riſing as in Deſcending, *viz.* Six Seconds; yet it does not follow that it muſt always happen ſo, the Difference not being ſo conſiderable in a Bullet as in an Arrow, whoſe Motion comes ſooneſt to an Equality, by reaſon of its Lightneſs.

PROPOSITION VIII.

THEOREM.

If a Power puſhes horizontally a Heavy Body upwards, it will cauſe it to move in a Parabolick Line, both in its Aſcent and Deſcent.

PErpendicular Projections either Upwards or Downwards, which we have ſpoken of in the Foregoing Propoſitions, in reſpect to us are always in a Right-line perpendicular to the Horizon, whoſe Direction is not alter'd by the Gravity of the Body, which only Shortens the Right-line of a Body tending Upwards, or Lengthens that of a falling Body.

It is not ſo of the Motion of Bodies thrown Horizontally, or Obliquely, whoſe Line of Direction is alter'd by Gravity, which hinders that Line from continuing Streight, becauſe of the Horizontal or Oblique Motion which mixes with the perpendicular Force of Gravity; and this cauſes the Body which is thrown horizontally, or ſideways, to move in

in a Curve line, which is the Circumference of a Parabola, as we ſhall firſt demonſtrate in the horizontal Projection, ſuppoſing the Air not to reſiſt the motion at all, and the Lines of Direction of Heavy Bodies to be parallel to one another.

DEMONSTRATION.

Suppoſe then, *for Example*, that the Bullet A, which we will ſuppoſe perfectly Round, of a very Hard and Uniform Matter, to be driven by ſome external cauſe, ſuch as might be the force of Powder, with a determinate degree of Velocity, which directs it towards D, along the horizontal Line AD; the equal Spaces of which *(viz.)* AB, BC, CD, it wou'd go thro' in equal Times, if it had no Gravity, or if it ſhou'd run along the horizontal Plain AD; but if that horizontal Plain be taken away, and the Bullet A be left to it ſelf with a liberty to move according to the Force impreſs'd by the Powder it wou'd go on towards D, if it were not for a new Impreſſion which it receives from its own Gravity, which obliges it to turn out of its Streight direction AD and move in the Curve AEFG, which is a compound of the Two Motions; One of which *(viz.)* that which is impreſs'd by the Force of the Powder, is Equal and Uniform; and the Other, which it receives from its own Gravity is Uniformly Accelerated. So that if the firſt Moment, the Bullet A, has according to its Line of Direction, run thro' the Space AB, by the Equal Motion which it had from the Impulſe of the Powder, and the Space BE by the Accelerated Motion of its Gravity; the ſecond Moment whilſt it goes thro' the Space BC equal to the firſt AB by its equal Motion, it will go thro' the Space CF which is Four times as great as the firſt BE, or AH, by the Accelerated Motion; and the third Moment whilſt it goes thro' CD, equal to the firſt Space AB by its equal Motion, it will by the Accelerated Motion go thro' DG, which is Nine times BE, and ſo on, according to the Squares of the Times; which Times are repreſented by the Lines AB,AC,AD, or their equals HE,IF,KG, which are parallel to the Horizon, and terminated by the Line AK perpendicular to the Horizon; as the Lines BE,CF,DG, or their equals AH,AI,AK, repreſent the Spaces which the Bullet A has faln thro' in Each Time: And ſince theſe Lines are as the Squares 1, 4, 9, of the Lines HE,IF,KG, it is eaſy to conclude from the Definition of a Parabola, that the Curve AEFG, is a Parabolick line, whoſe *Axis* is AK, and whoſe *Ordinates* are HE,IF,KG. *Q.E.D.*

Plate 10. *Fig.* 51.

PROPOSITION IX.

THEOREM.

The Lines of Oblique Projections are also Parabolick.

Plate 9. Fig. 49. I Say, that the Curve line AEFG, which the Bullet A has describ'd being driven Obliquely; that is, in a Direction between a Parallel and a Horizontal one, is also the Circumference of a Parabola.

DEMONSTRATION.

Since, as we have observ'd in *Prop.* 3. when a Body is driven Upwards, the Velocities decrease in Rising after the same proportion as they encrease in Falling; the equal Times being represented as before, by the Three equal Parts AB,BC,CD, of the horizontal Line AD, which represents the Time that the Bullet A has spent to come to G the highest Place; if in the First Moment AB, this Bullet has risen to E, *for Example*, Five Foot, the Second Moment BC, it will have risen to F Three Foot more than the first; and the Third and Last Moment CD, it will have risen to G One Foot higher then the Second, so that in all it will have risen 9 Foot. Thus will the Perpendicular DG be 9, when the Line AD is 3, the square Root of 9; the Line GH will be 4, when the Line EH equal to BD is 2, the square Root of 4; and the Line GI will be 1, when the Line FI equal to CD is 1, the square Root of 1; where you see that the Squares of the Ordinates AD, EHFI, are to the Axis GD, as the Correspondent parts GD, GH, GI, and consequently that the Curve AEFG, is a Parabola. *Q.E.D.*

SCHOLIUM.

It is plain that the Line of Direction AK by which the Bullet A driven up by the Powder, touches the Parabola at the point A, because at the very instant that the Force drives the Bullet A according to the Line of Direction AK, its Gravity causes it to descend a little, by turning it out of the Right line AK, and making it describe the Parabola AEFG. The Angle DAK, made by the Line of Direction AK, and the horizontal Line AD is call'd the *Angle of Inclination*, and the Breadth of the Parabola, which is terminated on the horizontal Line AD produc'd, is call'd the *Amplitude of the Parabola*, whose half here is AD.

CHAP-

CHAPTER. II.

Of the Descent of Heavy Bodies upon Inclin'd Plains.

WHEN a Heavy Body Rolls upon an inclin'd Plain, to go to the lowest place that it can, it goes with less Velocity than if it shou'd fall freely in the Air, its Relative being less than its Absolute Gravity; because its perpendicular Descent is hinder'd by the inclin'd Plain which in part sustains it. Whence it is easy to conclude, that the Less the Plain is Inclin'd, the Less is this Relatve Gravity; so that it becomes equal to nothing when the Plain is not Inclin'd, but Horizontal. There are several Curious and Useful things to be observ'd concerning Relative Gravity, which we shall take notice of in the following Propositions.

PROPOSITION I.

THEOREM.

If a Power sustains a Spherical Weight, that endeavours to roll along an Inclin'd plain whose Base is parallel to the Horizon, by a Line of Direction, which passing thro' the Center of Gravity, of the Weight, is parallel to the Hypotenuse of the Rectangular Triangle, which determines the Inclination of the Plain; That Power : will be to the Weight which presses the Plain : : as the Height of the Rectangular Triangle : is to the Hypotenuse.

I Say, that if a Power, whose Line of Direction ED passes thro' the Center of Gravity D of the Spherical Weight FGH, (which endeavours to roll along the inclin'd Plain BC,) and is parallel to BC the Hypotenuse of the Rectangular Triangle ABC, whose Base AB is parallel to the Horizon, sustains the Weight FGH; the Power : will have the same *Ratio* to the Weight : : or what the Power sustains will have the same *Ratio* to what the Plain sustains, as the Height AC : has to the Hypotenuse BC. *Plate* 15. *Fig.* 61.

PREPARATION.

Draw from the point F, where the Spherical Body FGH touches the Plain BC, thro' the Center D, the Diameter FH, which *by the 18th of the 3d Element*, will be perpendicular to the Hypotenuse BC, and consequently to its parallel ED. Draw again from the Center D, the line DK perpendicular to the Horizon, and consequently to the Base AB also, which will be the Line of Direction of the Weight EFG. Lastly, Draw from the point F the line FI parallel to the Horizon, or to the Base AB, which will be perpendicular to DK, so that the Rectangular Triangle DFO will *by* 8.6. be divided into Two Rectangular Triangles FID, FIO, Similar to one another and also to the Triangle OKB, or to ABC Similar to it.

Plate 15. *Fig.* 61.

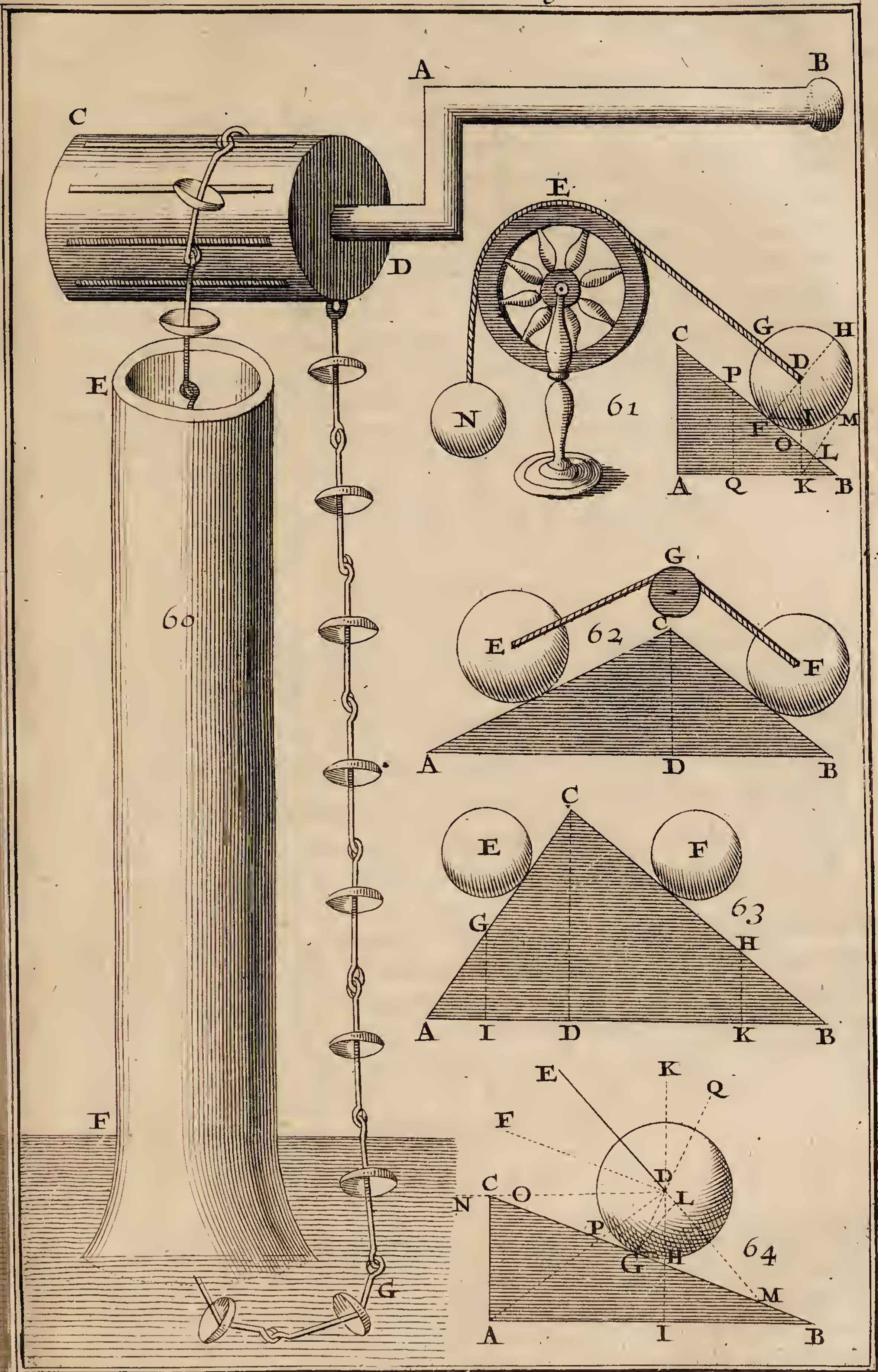
DEMONSTRATION.

Having made this Preparation, it will easily be known, that ED being the Line of Direction of the Power, and DK the Line of Direction of the Weight, it is as if the Power were apply'd at D, and the Weight suspended at I, and therefore DFI may be consider'd as a Bended Leaver, whose fix'd Point is F, the distance of the Power FD and the distance of the Weight FI; and in such a case, as has been elsewhere Demonstrated, the Power : is to the Weight : : as FI, the distance of the Weight : is to FD the distance of the Power : Wherefore if instead of the Two last Terms FI, FD, the Two AC, BC, which are in the same *Ratio* (by reason of the Similar Triangles FDI, ABC,) be us'd, it will be found that the Power : is to that part of the Weight which bears upon the Plain : : as the Height AC : is to the Hypotenuse BC. *Q. E. D.*

SCHOLIUM.

It is plain from this Proposition, that if instead of supposing the Power to sustain the Weight FGH, by means of the Rope ED tied to its Center D, and parallel to the Plain BC, you imagine the Weight to be stopp'd by the plain LM perpendicular to the Plain BC, the Relative Gravity whereby the Weight will bear against the Plain LM : will be to that whereby it bears against the Plain BC : : as the Height AC : to the Hypotenuse BC.

It is also plain, that if instead of a Power apply'd at E, there was a Weight N tied to a Rope passing over a Pulley so plac'd that the part ED of the Rope shou'd be parallel to the Hypotenuse BC, and this Weight N shou'd keep the Weight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Weight D in *Æquilibrio*; the Weight N: wou'd be to the Weight D:: as the Height AC: to the Hypotenuſe BC; and reciprocally, if the Height AC: was to the Hypotenuſe BC:: as the Weight N: is to the Weight D; thoſe Two Weights N,D, wou'd be in *Æquilibrio*. *Plate 15. Fig. 61.*

Laſtly, 'tis evident, that the Power thus apply'd is always leſs than that part of the Weight which bears upon the Plain; becauſe the Line AC muſt needs be leſs than the Hypotenuſe BC. By the Weight D is underſtood here, not its Abſolute Gravity, but as much of it as the Plain BC bears up, and by the Power, which is equal to the Weight N, the remaining part of the Weight D which is ſuſtain'd in the Air and not ſupported by the Plain BC. If you conſider the Abſolute Gravity of the Weight D, we ſhall Demonſtrate in the *Fifth Prop.* that it is to the Weight N, as AC is to BC.

PROPOSITION II.

THEOREM.

If a Power ſuſtains a Spherical Weight, that endeavours to roll along an inclin'd Plain, whoſe Baſe is parallel to the Horizon, by a Line of Direction, which being parallel to that Baſe, goes thro' the Center of Gravity of the ſaid Weight; the Power: will be to the Weight:: as the Height of the inclin'd Plain: to the Length of its Baſe.

I Say, That if a Power, (whoſe Line of Direction DL goes thro' the Center of gravity D, of the Spherical Weight EFG, which endeavours to roll along the inclin'd Plain BC, and is parallel to the Baſe AB, which I ſuppoſe parallel to the Horizon) ſuſtains that Weight EFG, the Power: will have the ſame *Ratio* to the Weight:: as the Height AC: has to to the Lengh AB. *Plate 7. Fig. 44.*

PREPARATION.

Draw from the point E, where the Spherical Body EFG, touches the Plain BC, thro' its Center D, the *Radius* DE, which *by* 18.3 will be perpendicular to BC the Hypotenuſe of the Rectangular Triangle ABC. Draw again from the Center D, the Line DH perpendicular to the Horizon, and conſequently to the Baſe AB, which will be the Line of Direction of the Weight EFG. Laſtly, Draw from the point E, where the Body touches the plain, the Line EI perpendicular to DH

Plate 7. *Fig.* 44. the Line of Direction of the Weight; to which Line of Direction the Power is alſo perpendicular: Thus will the Rectangular Triangle DEO, be *by* 8.6. divided into Two Rectangular Triangles EDI, EIO, Similar to one another, and to OHB, or ABC Similar to it.

DEMONSTRATION.

Having made this Preparation, it will be known *by Axiom* 9. that if the Power be apply'd at L, it is the ſame as if it was apply'd at M, where DE its Line of Direction is cut at Right-angles by EM parallel to DH the Line of Direction of the Weight; and as if the Weight ſhou'd hang from I, where its Line of Direction DH is cut at Right-angles by EI parallel to DL the Line of Direction of the Power; and therefore that MEI may be conſider'd as a Bended Leaver, whoſe fix'd Point is at E, the diſtance of the Power EM, and the diſtance of the Weight EI; and in ſuch a caſe, as has been elſewhere Demonſtrated, the Power: is to the Weight:: as EI: is to EM, or DI its equal: Wherefore if inſtead of EI, DI, the Two laſt Terms, you ſhou'd put AC, AB, which are in the ſame *Ratio* by reaſon of the Similiar Triangles ABC, EDI, it will be known that the Power: is to the Weight:: as AC: is to AB, *Q.E.D.*

SCHOLIUM.

It is plain by what has been Demonſtrated, that when the Angle B of Inclination is half a Right one, in which caſe the Angle C muſt alſo be half a Right, the † Power will be Equal to the Weight, becauſe in ſuch a Suppoſition, the Lines AB, and AC are equal: And when the Angle of Inclination B is leſs than half a Right, in which caſe the Angle C will be greater than half a Right, the Power will be leſs than the Weight, becauſe then AB will be greater than AC: And laſtly, when the Angle of Inclination B is greater than half a Right, in which caſe the Angle C muſt be leſs than half a Right, the Power will be greater than the Weight; becauſe then AB will be leſs than AC.

† *See* Page 71. *the laſt Paragraph of the* Scholium.

It is alſo plain, that if inſtead of a Power apply'd at L, to ſuſtain the Weight EFG, by means of the Rope DL tied to its Center D, and parallel to the Horizon, or to the Baſe AB, this Rope was made to paſs over the Pulley N, to hang the Weight K at it, which ſhou'd keep the Weight D in *Æquilibrio*; this Weight K: which wou'd be here inſtead of a Power, wou'd be to the Weight D:: as AC: is to AB; and reciprocally if K: was to D:: as AC: is to AB, theſe Two

Two Weights KD, wou'd be in *Æquilibrio*, because the Weight K acts just as a Power wou'd do. *Plate 7. Fig. 44.*

Lastly, it is evident, That if, instead of the Power at L, or of the Weight at K, the Surface PG perpendicular to the Horizon, or to the Base AB, shou'd be so apply'd as to touch the Weight EFG at G, and keep it from falling, the Relative Weight of EFG, by which it shou'd press against the Surface PG : wou'd be to that Gravity by which it shou'd bear on the Plain BC :: as AC : to AB.

PROPOSITION III.

THEOREM.

If Two Spherical Weights fasten'd to one another with a Rope parallel to the Horizon by their Centers of Gravity, keep one another in Æquilibrio *upon Two inclin'd Plains of the same Height, whose Bases are upon the same plain parallel to the Horizon; they will be to one another, as the Lengths of those Bases.*

THE Triangle ABC, whose perpendicular is CD, is the Profil of the Two inclin'd Plains AC,BC, whose common Height is CD, and whose Bases AD, BD, are upon the same Plain AB parallel to the Horizon: And there are upon these Two inclin'd Plains AC, BC, the Two Weights E,F, which keep one another in *Æquilibrio* by means of the Rope EF, which passing thro' their Centers of Gravity is parallel to the Horizon, or to the Plain AB. Now in this case, I say, that the Weight E: is to the Weight F: : as AD : is to BD. *Plate 6. Fig. 40.*

DEMONSTRATION.

Since the Two Weights EF keep one another in *Æquilibrio*, in such manner that the One does not draw the Rope more than the Other, the same Power that wou'd sustain the Weight E upon the Inclin'd Plain AC, by the Line of Direction EF, wou'd also sustain the Weight F upon the Inclin'd Plain BC, by the same Line of Direction EF: And as *by Prop.* 2. the Power which shou'd sustain the Weight E upon the Inclin'd Plain AC: wou'd be to that Weight E: : as CD: is to AC; and that the same Power which shou'd sustain the Weight F upon the Inclin'd Plain BC: wou'd be

to

Plate 6. *Fig.* 40. to that Weight F : : as CD : is to BD ; it will be concluded by *Equality*, that the Weight E : is to the Weight F ; : as AD : is to BD. *Q. E. D.*

SCHOLIUM.

Here you muſt note, that when we ſpeak of a Weight ſet on an Inclin'd Plain, as of the Weight E, to compare it with another Weight, or a Power that might ſuſtain it, we do not ſpeak of its Abſolute Gravity, with which it tends toward the Center of the Earth, but of that Gravity wherewith it preſſes on the Plain ABC, which muſt neceſſarily be leſs than the Abſolute, becauſe there is but part of the Heavy Body that preſſes on the Plain; ſince it endeavours to roll along the Plain.

PROPOSITION IV.

THEOREM.

If Two Sperical Weights (join'd by their Centers of Gravity with a Rope, which paſſing over a Pulley, folds in ſuch a manner that its Two Parts are parallel to Two Inclin'd Plains, of the ſame Height, whoſe Baſes are upon the ſame horizontal Plain,) keep one another in Æquilibrio *upon the Two Inclin'd Plains, they will be to one another as the Length of one of thoſe Inclin'd Plains is to the Length of the other.*

Plate 15. *Fig.* 62. THE Triangle ABC, whoſe Perpendicular CD, is the Profil of the Two Inclin'd Plains AC, BC, whoſe common Height is CD, and whoſe Baſes AD, BD, are upon the ſame Plain AB, parallel to the Horizon ; and there are upon theſe Two Inclin'd Plains AC, BC, the Two Weights E,F, which keep one another in *Æquilibrio* by means of the Rope EGF, which going over the Pulley G, and thro' their Centers of Gravity, hangs on each ſide, in ſuch manner, that the part EG is parallel to the Inclin'd Plain AC, and the part FG parallel to the Inclin'd Plain BC. I ſay, that in ſuch a caſe, the Weight E : is to the Weight F : : as AC : to BC.

DEMONSTRATION.

Since the Two Weights E,F, keep one another in *Æquilibrio*, in ſuch manner that each of 'em endeavours with equal Force

Force to deſcend along its Inclin'd Plain, drawing the Rope Equally; the ſame Power which cou'd ſuſtain the Weight E upon the Inclin'd Plain AC, by the Line of Direction EG, cou'd alſo ſuſtain the Weight F upon the Inclin'd Plain BC, by the Line of Direction FG: And as by *Prop.* 1. the Power which ſhou'd ſuſtain the Weight E upon the Inclin'd Plain AC: wou'd be to that Weight E:: as CD: is to AC; and that the ſame Power which ſhou'd ſuſtain the Weight F upon the Inclin'd Plain BC: wou'd be to that Weight F:: as CD: is to BC; it will be concluded by *Equality*, that the Weight E: is to the Weight F:: as AC: is to BC. *Q. E. D.* *Plate* 15. *Fig.* 62.

SCHOLIUM.

The Inverſe Propoſition is alſo true, *viz.* If the Two Weights E,F, are to one another as the Lengths AC, BC, they will keep one another in *Æquilibrio*; which holds alſo in Priſms plac'd perpendicularly upon Inclin'd Plains, and tied by their Centers of Gravity.

PROPOSITION V.

THEOREM.

If the Abſolute Weight of a Body as it lies upon an Inclin'd Plain, is to that of another Body that falls Perpendicularly, as the Height of the Inclin'd Plain to its Length; thoſe Two Weights will Equiponderate.

I Say, that if the Abſolute Gravity of the Weight D ſet upon an Inclin'd Plain BC, is to that of the Weight N, which falls perpendicularly, as the Height AC is to the Length BC; theſe Two Weights N,D, will be in *Æquilibrio*; that is, each of them will endeavour to deſcend with an equal Force, ſo that if they are join'd by a Rope, as in *Prop.* 1. in ſuch manner that the One has as much Velocity as the Other, each will draw its part of the Rope, which goes over the Pulley E. *Fig.* 61.

DEMONSTRATION.

If on the Length BC, you take the part BP equal to the Height AC, and from P draw PQ perpendicular to AB, and then ſet the Weight N at C, and the Weight D at B; you will find that if you cauſe the Weight N to deſcend from C to A, the Weight D will Riſe upon the Inclin'd Plain BC, from

Plate 15. Fig. 61. from B to P, because BP has been made equal to AC; so that it will have risen the Height PQ, when the Weight N has faln the Height AC.

Thus the line AC which is the Velocity of the Weight N: will be to the Line PQ which is the Velocity of the Weight D: : reciprocally as the Weight D: is to the Weight N, by this Rule of Mechanicks, which we have observ'd in the Leaver, and the other Engines, *viz.* that the Weights are reciprocally proportional to their Velocities. Wherefore, if instead of the Two first Terms AC, PQ, you shou'd put the Two BC, AC, which are in the same *Ratio*, by 4.6. because of the Similar Triangles ABC, QBP, it will be true to say that BC : is to AC: : as the Weight N: is to the Weight D, when these Two Weights are in *Æquilibrio*; and consequently, if the Weight D : is to the Weight N: : as the Height AC : to the length BC, these Two Weights N, D, will be in *Æquilibrio.* *Q. E. D.*

PROPOSITION VI.

THEOREM.

If of Two Equal Weights the One descends Perpendicularly, and the Other along an inclin'd Plain; their Relative Gravity will be Reciprocally Proportional to the Length, and the Height of the Plain.

I Say, That if the Weight D, which is upon the inclin'd Plain BC, is Equal to the Weight N, which descends perpendicularly; the Force with which the Weight D endeavours to descend upon the inclin'd Plain BC, is to the Force with which the Weight N endeavours to descend perpendicularly, as the Height AC, is to the Length BC.

DEMONSTRATION.

By making the Foregoing Construction, and causing the Weight N to descend from C to A, the Weight D will go from B to P, and be only risen to the Height PQ less than the Height AC, whence the Weight N having more Velocity than the Weight D, will proportionably have more Force to descend than the Weight D, by that general Rule of Mechanicks, *viz.* that the Relative Gravity of the Weight D : must be to that of the Weight N: : as the Height PQ : is to the Height AC; or as AC : to BC. *Q. E. D.*

CORO-

COROLLARY. I.

It is evident from this Propoſition, that the Force which a Heavy Body has to fall down by its own Weight, that is, its Abſolute Gravity, becomes leſs upon an inclin'd Plain, in proportion as the Length of that Plain is to its Height, or as the Whole Sine is to the Sine of the Angle of Inclination: So that if the Length BC ſhou'd be, *for Example*, twice the Height AC, which will happen when the Angle of Inclination is exactly of 30 Degrees, the Abſolute Weight of N, will be twice its Relative Weight upon the inclin'd Plain BC. *Plate* 15. *Fig.* 61.

Whence it follows, that if a Horſe draws a Loaded Cart up an inclin'd Plain, as a Mountain; beſides the ſame trouble which he wou'd have to draw that Cart upon a Flat, he muſt feel the Relative Weight of the Load which he draws upon the inclin'd Plain, which is the ſame part of the Abſolute Gravity of the Load, as the Height of the inclin'd Plain is of its Length. As, *for Example*, if the Length of the Mountain be twice its Height, and the Load be 2000 Weight, the Horſe will carry 1000 pounds. So that if the Horſe cou'd draw but 1000 pounds up ſuch a Mountain, there wou'd be occaſion for Two Horſes to draw 2000 pounds up the ſame Mountain.

COROLLARY. II.

It follows alſo that the Velocity of the Body D moving upon the inclin'd Plain BC, grows leſs according to the Proportion in which BC the Length of the Plain exceeds its Height AC; that is, the Velocity of the Body D upon the inclin'd Plain BC: is to the Velocity of the ſame Body when it moves perpendicularly:: as AC the Height of the Plain: is to its Length BC, becauſe the Velocities of the ſame Bodies ought to be in the ſame *Ratio* as the Relative Weights, it being certain that a Body which has a double Force will have a double Velocity, *&c.*

PRO-

PROPOSITION VII.

THEOREM.

If of Two Equal Weights, the One descend along an Inclin'd Plain, and the Other along another Inclin'd Plain of the same Height; their Relative Gravities will be reciprocally proportional to the Lengths of those Two Plains.

Plate 15. *Fig.* 63. I Say, that if the Weight E, which lies on the Inclin'd Plain AC, is equal to the Weight F, which is upon BC, an Inclin'd Plain of the same Height, the Force that the Weight E has to descend upon its Inclin'd Plain AC : is to the Force that the Weight F has to descend upon its inclin'd Plain BC ; : reciprocally as the Length BC of that Plain : is to AC, the Length of the other Plain.

DEMONSTRATION.

Suppose a Third Weight equal to the Weight E, or to the Weight F, to fall perpendicularly along the common Height CD, the Force of that Weight : will be to that of the Weight E : : as AC : to CD, and to that of the Weight F : : as BC : is to CD, by *Prop. 6.* Therefore *by Equality*, the Force of E : will be to that of F : : as BC : to AC. *Q.E.D.*

PROPOSITION VIII.

THEOREM.

The Relative Gravities of Two Equal Weights set upon Two inclin'd Plains of the same Height, are to one another as the Heights which answer to Equal Parts of their inclin'd Plains.

Fig. 63. I Say, that if the Weight E, being set upon the Inclin'd Plain AC, be equal to the Weight F upon the Inclin'd Plain BC ; having taken at pleasure upon the Lengths AC, BC, the Two equal parts AG,BH, and drawn from the Two Points G,H, the Lines GI, HK, perpendicular to the Bases AD, BD, or parallel to the common Height CD ; the Force which the Weight E has to descend along its Inclin'd Plain AC :

AC; is to the Force which the Weight F has to descend upon its Inclin'd Plain BC;; as the Height GI; to the Height HK. *Plate* 15. *Fig.* 63.

DEMONSTRATION.

If you imagine a Third Weight equal to E, or to F, to fall perpendicularly from the Heights GI, HK; the Relative Gravity of that Weight: will be to the Relative Gravity of the Weight E:: as AG: is to GI, and to the Relative Gravity of the Weight F:: as BH, or AG: is to HK, by *Prop.* 6. Wherefore *by Equality*, the Relative Weight of the Weight E: will be to that of the Weight F:: as the Height GI: to the Height HK. *Q. E. D.*

COROLLARY.

It is evident from this Proposition, that the Relative Gravities of Two equal Weights laid upon Two inclin'd Plains of the same Height, are in proportion as the Sines of the Angles of Inclination of those Two Plains, because the perpendicular GI is the Sine of the Angle A, in respect of the Whole Sine AG, or BH; and the perpendicular HK is the Sine of the Angle B, in respect of the same Whole Sine BH.

PROPOSITION. IX.

THEOREM.

If a Power sustains a Spherical Weight which endeavours to roll along an inclin'd Plain, whose Base is parallel to the Horizon, by a Line of Direction, which passing thro' the Center of Gravity of the Weight, falls upon the Hypotenuse of the Rectangular Triangle, which determines the Inclination of the Plain; that Power: will be to the Weight:: as the Sine of the Angle of Inclination: to the Sine of the Complement of the Angle of Traction.

I Say, That if a Power (whose Line of Direction DE goes thro' the Center of Gravity of the Spherical Weight D, *Fig.* 64. which endeavours to roll along the inclin'd Plain BC, and being produc'd to M, cuts the Hypotenuse BC of the Rectangular Triangle ABC, whose Base AB is parallel to the Horizon) sustains that Weight D; the Power: will have the same *Ratio* to the Weight;: as the Sine of the Angle GDH equal

equal to the *Angle of Inclination* B: has to the Sine of the Complement of the Angle CME, call'd the *Angle of Traction.*

PREPARATION.

Plate 15. *Fig.* 64. Draw from the point G, where the Weight D touches the Plain BC, to the Center D, the *Radius* DG, which *by* 18.3. will be perpendicular to the Hypotenuse BC: Then draw GL perpendicular to the Line of Direction EM, and the Angle DGL, whose Sine will be DL, will *by* 8. 6. be equal to the Angle of Traction CME, and the Sine of the Complement GL, in respect of the Whole Sine DG; as in respect of the same Whole Sine DG, the line GH is the Sine of the Angle GDH, which is equal to the Angle of Inclination B. Draw from the Center D, the line DI perpendicular to the Base AB, which will be the Line of Direction of the Weight D, and the line DF parallel to the Hypotenuse BC, which you must consider as the Line of Direction of another Power so apply'd at F or at D, as likewise to sustain the Weight D upon the inclin'd Plain BC. Draw also from the point G the line GH parallel to the Horizon, or perpendicular to DI, the Line of Direction of the Weight, and the Triangle DGH will be Similar to the Triangle ABC, as we have observ'd in *Prop.* 1.

DEMONSTRATION.

Having made this preparation, it will be known that, as in *Prop.* 1. DE,DF, being the Lines of Direction, of Two Powers which separately sustain the Weight D, whose Line of Direction is DI; it is as if those Two Powers were apply'd at the end D of the Bended Leaver DGH, whose fix'd Point is at G; and as if the Relative Weight of the Body D, wherewith it presses upon the Plain BC, was reduc'd to the point H, and that thus GH shou'd be the distance of the Weight, GL the distance of the Power at E, and GD the distance of the Power at F, because it is perpendicular to the Line of Direction DE, as DE is perpendicular to the Line of Direction DP and DH perpendicular to the Line of Direction DN.

This being suppos'd, you must consider, that since the Power E sustains the Weight D by the Line of Direction DE, by means of the Bended Leaver DGH, where G is the fix'd Point, GH the distance of the Weight, and GL the distance of the Power, this Power: will be to the Weight:: as the distance GH of the Weight: is to the distance GL of the Power: And likewise since the Power at F sustains the same Weight D in the Line of Direction DF, by means of the same Bended Leaver DGH, where G is the fix'd Point,

GH

GH the diſtance of the Weight, and GD the diſtance of the Power; that Power: will be to the Weight:: as GH, the diſtance of the Weight: is to GD, the diſtance of the Power. Wherefore *by Equality* the Power at E: will be to the Power at F:: as GD: is to GL; and becauſe GD: is to GH:: as BC: to AC, by reaſon of the Similar Triangles DGH, ABC, or *by Prop.* 1. as the Weight D: is to the Power F; therefore one may conclude *by Equality* that the Power E: is to the Weight D:: as GH is to GL, or as the Sine of the Angle of Inclination, is to the Sine of the Complement of the Angle of Traction. *Q. E. D.* *Plate* 15. *Fig.* 64.

COROLLARY I.

It is evident from this Propoſition, that the Power at F, whoſe Line of Direction is parallel to the inclin'd Plain BC, is the leaſt of all; that is, that there is leſs Force requir'd to ſuſtain the Weight D upon the inclin'd Plain BC, if you draw that Weight in a line parallel to the inclin'd Plain, ſuch as DF, than if you draw it in any other line, ſuch as DE might be; ſo that the Power at E is greater than the Power at F, and it will always be greater as the Angle of Traction encreaſes, becauſe it has been demonſtrated, that the Power at E: is to the Power at F:: as GD: is to GL, or as the Whole Sine is to the Sine of the Complement of the Angle of Traction, and this Sine of the Complement GL ſtill grows leſs as the Angle of Traction grows greater.

Whence it is eaſy to conclude, that the Power is the greateſt that it can be, and exactly equal to the Weight, when the Angle of Traction is equal to the Complement of the Angle of Inclination, which will happen when the Line of Direction DE is perpendicular to the Horizon, as DK, becauſe in ſuch a caſe the lines GH, GL will be equal, which makes the Power be equal to the Weight, ſince that Power: is to the Weight:: as GH: is to GL. Thus you may know that the Power is the leaſt that it can be, when it draws by a Line of Direction parallel to the inclin'd Plain, and that it is the greateſt that it can be, when it draws by a Line of Direction perpendicular to the Horizon; hence we ſee that if a Horſe draws a Burthen by means of a Cart or any other rolling Machine, he will draw it with ſo much more eaſe, as the Line of Direction by which he draws the Load will come nearer to a parallel to the Slope of the Mountain along which he draws.

COROLLARY II.

Plate 15. Fig. 64. It follows alſo, that if the Line of Direction, as DN, and DF parallel to BC, make the Angle FDN equal to the Angle EDF, the Power apply'd at N, will be equal to the Power apply'd at E; becauſe in ſuch a caſe the Angles of Traction DMO, DOM, will be equal, ſince *by* 29. 1. the Angle of Traction DMO is equal to its External Oppoſite EDF, which is ſuppos'd equal to the Angle FDN, and conſequently to its Alternate DOM, *&c.*

Whence it follows, that if the Line of Direction, as DP, and the Line DG perpendicular to BC, make the Angle GDP equal to the Angle of Inclination B, the Power apply'd at P will be equal to the Weight, *by Cor.* 1. becauſe then the Angle of Traction DPM is equal to the Complement of the Angle of Inclination B.

COROLLARY III.

Laſtly, it follows alſo, that if the Line of Direction, as DQ, be perpendicular to the Inclin'd Plain BC, in ſuch manner that the Angle of Traction QGB be a Right; the Power apply'd at Q, or in any point whatever of its Line of Direction DQ, to ſuſtain the Weight D, muſt be Infinite; that is as much as to ſay, That a Power which ſhou'd draw that Weight D, by the Line of Direction DQ, wou'd not be able to ſuſtain it with ever ſo great a Force, becauſe the Sine of the Complement of the Angle of Traction is reduc'd to nothing, being Infinitely Small, which makes the Power apply'd at Q to be Infinitely Great, ſince that Power : is to the Weight :: as the Sine of the Angle of Inclination, is to the Sine of the Complement of the Angle of Traction.

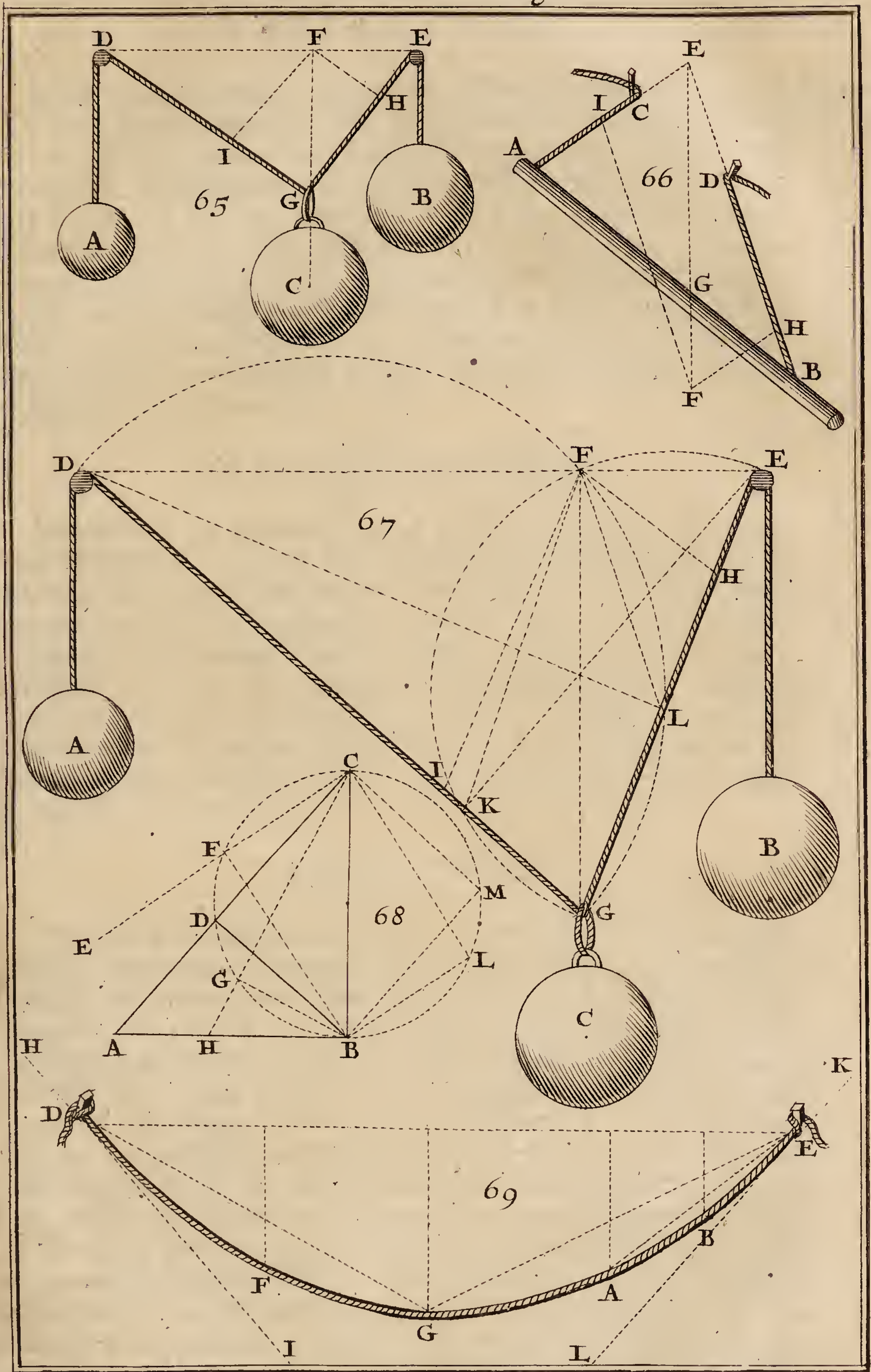
PROPOSITION X.

THEOREM.

If Two Powers ſuſtain a Weight by means of a Rope, which, folding by reaſon of the Weight plac'd between the Two Powers, makes a Right-angle; the ſaid Powers ſhall be Reciprocally Proportional to the Parts of the Rope.

Plate 16. Fig. 65. I Say, that if the Two Powers A, B, ſuſtain the Weight C, by means of the Rope DGE, which bending at the point G where the Weight C hangs, makes there a Right-angle, the Power A : is to the Power B :: as the Part EG, is to the Part DG.

PRE-



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PREPARATION.

Draw to the Line of Direction CG of the Weight C, the Two Perpendiculars DF, EF, and then the Weight C, which hangs from G, may be consider'd as hanging from F, the Power A, which draws by the Line of Direction DG, as apply'd at G, as also the Power B, which draws by the Line of Direction EG: So that GEF may be consider'd as a Bended Leaver, whose fix'd Point is E, the distance of the Weight C is EF, and the distance of the Power A is EG: And likewise GDF may be consider'd as a Bended Leaver, whose fix'd Point is D; the distance of the Weight C, is DF; and the distance of the Power B, is DG. *Plate* 16. *Fig.* 65.

DEMONSTRATION.

Because in the Bended Leaver GEF, the Power A: is to the Weight C:: as EF the distance of the Weight: is to EG the distance of the Power, or as EG: is to DE, because of the Similar Triangles DGE, FGE, *by* 8. 6. and that likewise in the Bended Leaver GDF, the Power B: is to the Weight C:: as the distance DF of the Weight: is to the distance DG of the Power, or as DG: is to DE, because of the Similar Triangles DGE, DGF, *by* 8. 6. it will be known by Equality, that the Power A: is to the Power B:: as EG: is to DG. *Q. E. D.*

SCHOLIUM.

From this Proposition 'tis plain, that if the Three Weights A,B,C, keep one another in *Æquilibrio* by means of the Rope DGE, so that as we have suppos'd in the foregoing Demonstration, the Line DE be parallel to the Horizon; these Weights are in proportion to the Sines of the Angles, from which they hang; that is, the Weight A: will be to the Sine of the Angle EDG:: as the Weight C: is to the Sine of the Angle DGE, and as the Weight B: is to the Sine of the Angle DEG; for it has been demonstrated that the Power A, which is instead of a Weight: is to the Weight C:: as EG: is to DE; or as the Sine of the Angle EDG: is to the Sine of the Angle DGE; and it has been also demonstrated, that the Power B, which is instead of a Weight; is to the Weight C:: as DG: to DE, or as the Sine of the Angle DEG: to the Sine of the Angle DGE; and last of all, that the Weight A: is to the Weight B:: as EG: to DG, or as the Sine of the Angle EDG: to the Sine of the Angle DEG.

If from the point F, FH and FI be drawn parallel to the Two sides DG, EG, it will be known that the Power A: is to the Power B:: as FH: to FI, because the Two lines FH, FI, or GH are proportional to the Two EG, DG, to which the Two Powers A and B are proportional, because of the Similar Triangles GFH, DGE; this is also true when the Angle G is oblique; but we must demonstrate it.

PREPARATION.

plate 16. *Fig.* 67. Draw from the point D, DL perpendicular to the Rope EG, which will be the distance of the Power B, in respect of the fix'd Point of the Bended Leaver GDF; and from the point E, draw EK perpendicular to the Rope DG, which will be the distance of the Power A, in respect of the fix'd Point E of the Bended Leaver GEF. Then draw the lines FK, FL, and the Triangle FEK will be Equiangular to the Triangle FGI, as may be known by describing on EG the Semi-Circle EFKG, which *by* 31. 3. will pass thro' the Two points F, K: For it will be known *by* 21. 3. that the Angle FGI which is in, or stands on the Arch FK is equal to the Angle FEK, which is in the same Arch FK; and likewise that the Angle FKE, which is in the Arch EF is equal to the Angle FGE, which is in the same Arch EF, and *by* 29. 1. to its alternate GFI; wherefore *by* 32. 1. the third Angle EFK is equal to the third FIG. The Triangle DLF is also Equiangular to the Triangle GFH, as may be known by describing on GD the Semi-Circle DFLG, which *by* 31. 3. will pass thro' the Two points FL, for it will be known *by* 21. 3. that the Angle FDL, which is in the Arch FL, is equal to the Angle FGH, which is in the same Arch FL; and likewise that the Angle DLF, which is in the Arch DF, is equal to the Angle DGF, which has the same Arch DF, for its Base or *by* 29. 1. to its alternate GFH; wherefore *by* 32. 1. the third Angle FHG is equal to the third DFL.

DEMONSTRATION.

Because in the Bended Leaver GEF, the Power A: is to the Weight C:: as EF the distance of the Weight: is to EK the distance of the Power; or as GI: is to GF, because of the Similar Triangles FEK, FGI; and that in the Bended Leaver GDF, the Power B: is to the Weight C:: as the distance DF of the Weight: is to DL the distance of the Power; or as GH: is to GF, because of the Similar Triangles DLF, GFH; one may conclude *by Equality*, that the

the Power A: is to the Power B:: as GI: is to GH; or as FH: is to FI. *Q. E. D.*

COROLLARY I.

It follows from the foregoing Demonstration that the Three Weights A, C, B, are proportional to the Three Lines GI, GF, GH; because it has been demonstrated that A: is to C:: as GH is to GF; and that C: is to B as GF is to GH. Whence it is easy to conclude, that the Three Weights A,C,B, are proportional to the Sines of the Three Angles of the Triangle FGI, or of the Triangle FGH, (*viz.* of the Three Angles GFI, GIF, FGI, because the line GI is the Sine of the Angle GFI, the line GF the Sine of the Angle I, and the line GH or FI the Sine of the Angle FGI:) or to the Sines of the Three Angles which are at the Point G, *viz.* That of the Angle FGE equal to the Angle GFI, That of the Angle DGE, whose Sine is the same as that of the Angle GIF; and the Sine of the Angle FGI.

COROLLARY II.

It follows also that the Weight A: is to the Weight C:: as EF: as is to the Sine of the Angle EDG, in respect to the Whole Sine ED; and that the Weight B: is to the Weight C:: as DF: is to the Sine of the Angle DEG, in respect to the same Whole Sine ED; because we have demonstrated that the Weight A; is to the Weight C:: as EF to EK, which is the Sine of its opposite Angle EDG, in respect of the Whole Sine ED; and that the Weight B: is to the Weight C:: as DF; is to DL, which is the Sine of the opposite Angle DEG, in respect of the same Whole Sine ED. Whence it follows, That knowing the Three Weights A,B,C, and the lines DF, EF, and consequently the Whole Line DE, you may likewise by Trigonometry know the Three Angles of the Triangle DGE. *Plate 16. Fig. 67.*

COROLLARY III.

Lastly, it follows, That since the Weight C, however little it be, causes the Rope whence its suspended to bend, how great soever the Powers AB which stretch it be suppos'd, a Rope cannot perfectly be Stretch'd, tho' it should be drawn by the greatest Force imaginable; because that Force, be it ever so great, may always be represented by the great Weights AB, which will not be able to hinder the Rope from bending, even tho' the Weight C should not hang at it; the

Plate 16. *Fig.* 67. Gravity of the Rope it ſelf being ſufficient to Bend it a little, and ſo raiſe the Weights A,B.

The Angle G of the Two Ropes EG,DG, is here an Acute Angle, and may be an Obtuſe Angle, in which Caſe the Perpendiculars DL, EK, will fall without the Triangle DGE, but ſtill the foregoing Demonſtration will be the ſame. It may alſo happen that the Two Points D,E, will not be of the ſame Height; that is, that the Two lines EF, DF, which have been drawn perpendicular to FG the Line of Direction of the Weight C, will not be One and the ſame Right-line; but this will not make the Theorem falſe, ſince you may draw the Two FH,FI, Parallel to the Ropes from which you will of the Two points, where FG the Line of Direction of the Weight C is cut by One of its Perpendiculars EF,DF,*&c.*

PROPOSITION XI.

THEOREM.

If a Slack Rope be ſuſpended by its Ends it will hang in a Curve Line.

Plate 16. *Fig.* 69. THE laſt Theorem ſhews that a Rope laden with a Weight makes an Angle where the Weight hangs; but it does not make an Angle merely by its own Weight and when it is a little ſlack; for in ſuch a caſe being tied up at each end, the weight of each of it Parts will cauſe it to deſcend, and bend in a Curve as all long flexible Bodies do: As ſuppoſing D,E, to be the Two Ends, the Weight of the Rope will make G its middle Point to ſink below the Right-line DE, and likewiſe the Point A will ſink below the Right-line GE, the Point B below the Right-line AE, the Point F below the Right-line GD, and ſo it will be of all the other Points, which will deſcend ſo low as to form the Curve DFGABE.

SCHOLIUM.

It is evident that this Rope thus bent will remain in the ſame Situation, if inſtead of being tied up by its Two Ends D, E, it be ſuſpended from the Points H, K, of the Two inflexible Lines HI, KL, which touch the Rope at the Points D, E, if ſo be that you attribute no Gravity to thoſe Two Tangents HI, KL. Now the Situation of a Rope thus ſuſpended will be ſuch, that its Center of Gravity G, will be ſomewhere in that line which being drawn from the Center of the Earth, will paſs thro' the Point where thoſe Tangents HI, KL

HI,KL produc'd, will meet; as it will appear from what we ſhall ſay in the

PROPOSITION XII.

THEOREM.

If a Heavy Body be ſuſpended by Two Ropes, which being lengthned wou'd meet, its Center of Gravity will be in a Right-line, drawn from the Center of the Earth thro' the Point where the Two ſaid Ropes wou'd meet.

I Say, That if the Body be ſuſpended by the Two Ropes CA, DB, which being produc'd wou'd meet at the Point E, thro' which the Plumb-line EF is to fall; That Body AB, will be in ſuch a Poſition, that its Center of Gravity G will be ſomewhere in the line EF: becauſe as we have elſewhere obſerv'd, the Center of Gravity deſcends as low as it can; and that it muſt riſe if it were ever ſo little out of the Line EF, which therefore muſt be the Line of Direction of the Body AB. *Plate 16. Fig. 66.*

SCHOLIUM.

If from the Point F, taken at pleaſure in the Line of Direction EF of the Body AB, FH be drawn parallel to the Rope AE, and FI parallel to the Rope BE, it will be known *by Prop.* 10. That the Force of the Weight AB being repreſented by the line EF, the line EH will expreſs the Force which draws the Rope BD, and the Line EI that of the Rope AC.

It is plain, that tho' the Body AB be ſuſpended by the Two Ropes faſten'd to the Points CD, it is as if it were ſuſpended by Two Ropes faſten'd to the ſingle Point E. And as theſe Two Ropes are inclin'd to one another in ſuch manner, that being continu'd they wou'd croſs one another in the Line of Direction EF, (or which is all one, the Center of Gravity gets into the Right-line EF, which is let fall perpendicular to the Earth, from the Point E where the Ropes wou'd cut one another being produc'd) ſo this will ſhew us an eaſy Method to find out the Center of Gravity of a Regular or Irregular Plain, *viz.* if we ſuſpend ſuch a Figure from Two different Points, that is, in Two different manners; for if from each of thoſe Points a Plumb-line be let fall along the Figure, the Point where theſe Two Right-lines cut one another will be the Center of Gravity requir'd.

PROPOSITION XIII.

PROBLEM.

Knowing the Absolute Weight of a Spherical Body set upon an inclin'd Plain of known Length and Height; how to know what Part of the Weight bears or gravitates on the Plain.

Plate 15. *Fig.* 61. LET us suppose the Absolute Weight of the Spherical Body D set upon the inclin'd Plain BC, to be 1000 lib. and the length of the inclin'd Plain BC to be 6 Foot, and its Height AC 4; to find what part of the Weight D the Plain BC bears, to these Three Numbers 10: 6:: 1000, (which are BC + AC; BC:: D,) find a Fourth Proportional, which will be 600 lib. for that part of the Weight which is supported by the Plain BC.

DEMONSTRATION.

Since by *Prop.* 1. AC: has the same *Ratio* to BC:: as that Part of the said Weight D, which is sustain'd in the Air: has to that part of it which is sustain'd by the Plain, you will find *by Composition*, that AC + BC; is to BC:: as the Whole Weight of D: is to that part of the Weight which presses on the Plain BC; and therefore, that to find this part, to these Three Quantities AC + BC: BC:: D: a Fourth proportional must be found, *as has been done.*

PROPOSITION XIV.

PROBLEM.

A Spherical Body of known Weight, being set upon an inclin'd Plain, whose Length and Height are also known; how to find what Power will sustain it drawing in a Line of Direction, which being parallel to the inclin'd Plain goes thro' the Center of that Sphere.

Fig. 61. TO know the Quantity or Degree of the Power N, (which must be able to sustain the Sphere D, drawing it in the Line of Direction ED, which passing thro' the Center D is parallel to the inclin'd Plain BC) we will suppose the

the Sphere D to weigh 1000 lib. the Length BC of the inclin'd Plain to be 6 Foot, and its Height AC 4; then it will only be requir'd to find a Fourth Number proportional to these Three 10 : 4 : : 1000, which are AC + BC : AC : : D : and this Fourth Number will be 400 lib. for the Quantity of the Power, or of the Weight N, which will be able to sustain the Sphere D on the inclin'd Plain BC. *Plate 15. Fig. 61.*

DEMONSTRATION.

Since *by Prop.* 1. BC : has the same *Ratio* to AC : : as that part of D, which is sustain'd by the inclin'd Plain BC : has to that part of D which is sustain'd in the Air, or by the Power; you will find *by Composition*, that AC + BC : is to AC : as the Whole Weight of D, is to that part of that Weight which is sustain'd in the Air; and therefore that to find out that Power, or the Power which may sustain D upon the inclin'd Plain B C, you must find a Fourth Quantity proportional to these Three AC + BC : AC : : D : as has been done.

PROPOSITION XV.

THEOREM.

The Velocities of the same Moving Body upon Two Plains differently inclin'd are to one another, as the Relative Weights upon the said Plains: and reciprocally as the Lengths of these Plains, if they are of the same Height.

THE First part of this Proposition is plain, *by Coroll.* 2. *Prop.* 6. *viz.* That the Velocity of the Body moving along the inclin'd Plain AC; is to the Velocity of the same Body moving along the other inclin'd Plain BC : : as the Force with which the Body endeavours to roll along the inclin'd Plain AC : is to the Force by which the same Body endeavours to roll along the Plain BC; because (the Force which the Body has to descend along the inclin'd Plain AC : being to that which it has to descend along the perpendicular Plain CD : : as the Velocity on the Inclin'd Plain AC : is to the Velocity on the perpendicular Plain CD; (and likewise the Force which the Body has to roll along the Inclin'd Plain BC : being to the Force which it has to descend along the perpendicular Plain CD : : as the Velocity of the Inclin'd Plain BC : to the Velocity on the perpendicular Plain CD;) it follows *by Equality*, That the Velocity of the Body on the inclin'd Plain AC : is to the Velocity of the same Body on BC the other inclin'd Plain; : as the Force wherewith it *Fig. 63.*

rolls

Plate 15. *Fig.* 63. rolls along the Inclin'd plain AC, is to that Force wherewith it rolls along BC the other Inclin'd Plain. *Q. E. D.*

The Second part is alſo evident, *viz.* That the Velocity of the Body on the inclin'd Plain AC : is to the Velocity of the ſame Body on the other inclin'd Plain BC : : reciprocally as the length of that plain BC : is to the length of the firſt plain AC of the ſame Height; becauſe *by Prop.* 7. thoſe lengths are reciprocally proportional to the Relative Gravities, or to the Force wherewith the Body endeavours to roll along each Plain; and by *Coroll.* 2. *Prop.* 6. thoſe Relative Gravities are proportional to the lengths of the inclin'd Plains.

PROPOSITION XVI.

PROBLEM.

How to find what Space a heavy Body muſt go thro' upon an Inclin'd Plain, in the ſame time that it wou'd go thro' a Determin'd Space along a Vertical Plain.

Plate 16. *Fig.* 68. TO determine what Space a heavy Body muſt run thro' along the Inclin'd Plain AC, whoſe Baſe AB is always ſuppos'd parallel to the Horizon, in the ſame time that it wou'd take up in falling from C to B thro' the determinate Space BC; draw from the Right-angle B, the Line BD perpendicular to the Hypotenuſe AC of the Rectangular Triangle ABC, and the Space CD will be the Space requir'd; that is, the Body being let fall at C will be as long in going thro' the Space CD along the Inclin'd Plain AC, as it wou'd be in falling perpendicularly thro' the Space BC.

DEMONSTRATION.

It is certain that the Space which the Body runs thro' along the Inclin'd Plain AC : is to that which it runs thro' in the ſame time along the perpendicular Plain BC : : as its Velocity along AC : is to its Velocity along BC; or *by Coroll.* 2. *Prop.* 6. as the Height BC of the Inclin'd Plain : is to its Length AC; wherefore if inſtead of the Two laſt Terms BC, AC, you put the Two lines CD, BC, which are in the ſame *Ratio by* 4.6. becauſe the Triangle ABC, BDC are Similar *by* 8.6. you will find that the Space which the Body has gone thro' along the Inclin'd plain AC : is to that which the ſame Body has gone thro' along the perpendicular Plain BC in the ſame time : : as CD : is to BC. Since then the Two Spaces AC, and BC, are proportional to the Two Lines CB, CD, it is eaſy to conclude; that

that if the Body goes thro' the Space BC falling perpendicularly in a determinate Time, it will be juſt as long in going thro' the Space CD along the Inclin'd Plain AC. *Q. E. D.* *Plate* 16. *Fig.* 68.

SCHOLIUM.

In the Demonſtration we have ſuppos'd that the Spaces, which the Body goes thro' upon Plains differently Inclin'd, are in equal Times proportional to the Velocities which it has on thoſe Plains, beginning from the Point of Reſt, becauſe the Motion of a Heavy Body is accelerated upon an Inclin'd Plain, not equally, but in the ſame proportion as when it falls perpendicularly, as Experience ſhews; and that according as its Velocity is greater or leſs, it ought to go thro' Spaces proportionably greater or leſs in equal Times, if the Velocities be conſider'd after the ſame manner; that is, as they are produc'd by the Gravity of the Body at the beginning of its fall.

One may in the ſame manner Demonſtrate, That if there be another Inclin'd Plain, as CE, and from the Right-angle B, BF be drawn perpendicular to it; the Body will run thro' the Space CF along the Inclin'd Plain CE, in the ſame time that it wou'd fall perpendicularly thro' the Space BC: And likewiſe, That if to a third Plain, as CH, the perpendicular BG, be drawn from the Right-angle B; the Body will run thro' the Space CG along this Plain CH, in the ſame time that it wou'd fall perpendicularly thro' the Space BC, *&c.*

COROLLARY I.

Hence it follows, That ſince by 31. 3. all the points F,D,G, are in the Circumference of a Circle, whoſe Diameter BC is perpendicular to the Horizon, all the Chords CF, CD, CG, which begin from the upper part C, are run thro' by the Body in equal Times; that is, if CF, CD, CG repreſent Plains differently Inclin'd; Three Bodies equally Heavy, beginning to move at C, will in the ſame Time run along the whole Lengths of thoſe Plains CF, CD, CG; becauſe it has been Demonſtrated, that they will run along their whole Length during the ſame time that they wou'd fall along the perpendicular Plain CB.

COROLLARY II.

It follows alſo that all the Chords of the ſame Circle, which meet at the loweſt Point of the Circle, are run thro' in the ſame time, as BL, BM: For if you join the Two Chords CL, CM,

Plate 16. *Fig.* 68. CM, and the Two Chords BF, BD, equal to them be drawn parallel to 'em, in which case the Two Chords CF, CD, will also be equal and parallel to the Two BL,BM; the Chord CD being parallel and equal to BM will be equally Inclin'd, and consequently those Two Chords CD, BM, will be run thro' in the same time: And likewise the Chord CF being equal and parallel to BL, is equally Inclin'd, and consequently run thro' in the same time; and as it has been Demonstrated that the Two CF, CD, are run thro' in the same time, BL and BM must also be run thro' in the same time.

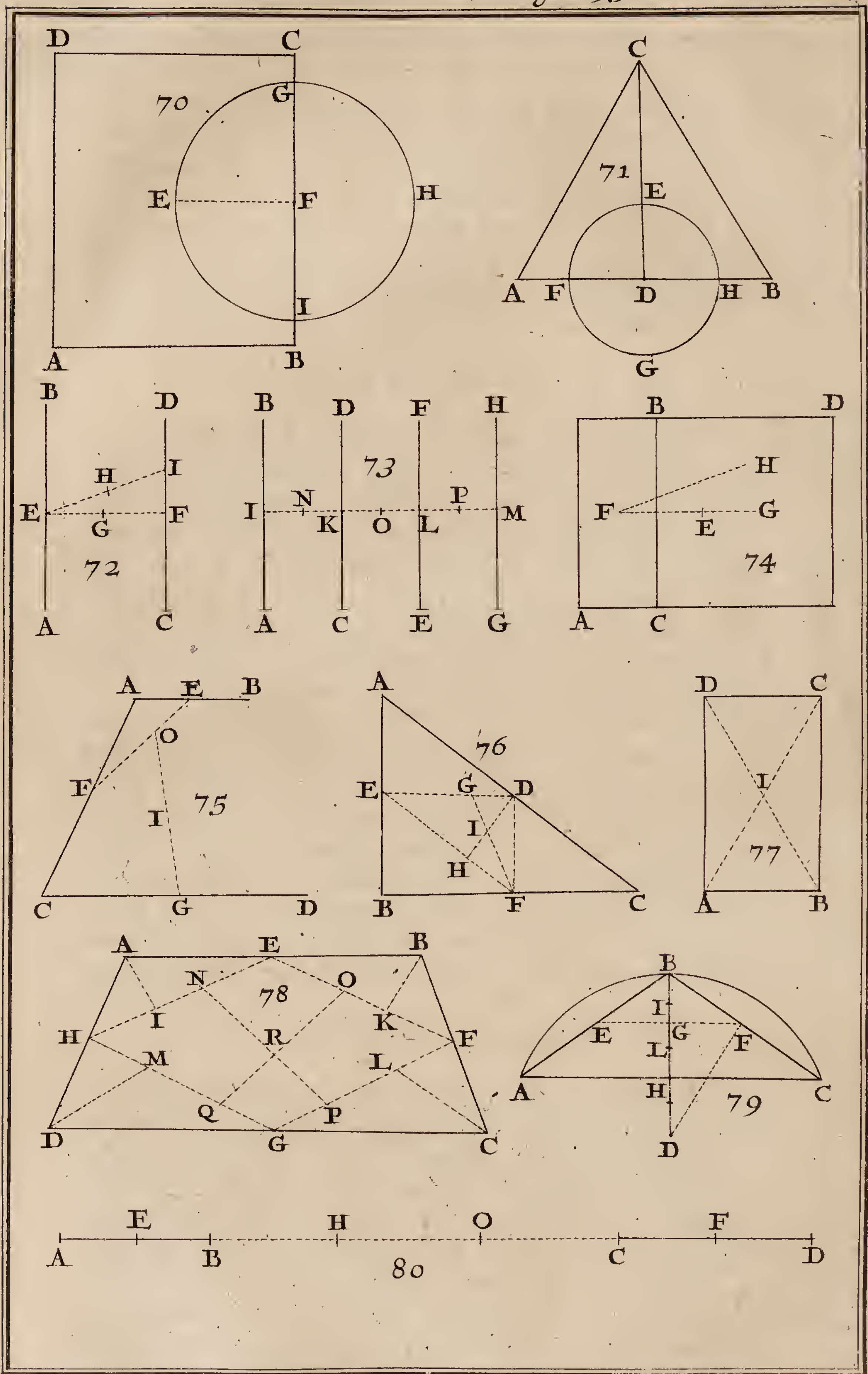
COROLLARY III.

This shews the Reason why the same Pendulum makes its Vibrations, whether great or small in the same time, as to Sense: For the Pendulum which runs thro' the Arch BD does not sensibly go from its Chord when the Arch is a small one, and it will go less out of it when the Arch is less, as BG; and as it wou'd in equal Times go thro' the Length of the Chords BG, BD, it must likewise go thro' the Arches BG, and BD, in about the same Time. I have said *about*, because the Pendulum will not be so long in going thro' the Arch as the Chord, tho' it be shorter, because the Arch is more Inclin'd at first, and the Chords do not encrease in the same proportion as the Arches do; whence there is some difference in the Duration of Two Vibrations which are considerably Unequal; and Father *De Chales* affirms, that in comparing Two Pendulums of equal Lengths, That which made short Vibrations, made 101, whilst That which made long Vibrations, made but 100. Hence it is easy to conclude, that the most just Pendulums are those whose Vibrations are the smallest

COROLLARY IV.

Lastly it follows from this Proposition, that a Heavy Body is longer in running along a Plain which is more, than a Plain which is less Inclin'd, if both be of the same Height, that is, it will be longer in running along the Plain CA, than the Plain CH which is less Inclin'd; because in equal Times it runs thro' CD which is a less part of the Inclin'd Plain CA, and CG which is a greater part of the Inclin'd Plain CH. Nevertheless it does not acquire more Velocity upon one Plain than upon another, because on each it acquires a Velocity equal to that which it wou'd acquire in falling thro' the Perpendicular CB, which we cou'd here demonstrate, as well as several other Propositions more curious than useful, but that we must end this Chapter, and proceed to the next.

CHAP-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CHAPTER III.

Of the Center of Gravity.

WE ſhall teach in this Chapter how to find the Center of Gravity of Lines, Plains, and Solids; but before we come to the Practice, we ſhall occaſionally ſpeak of a remarkable Property of the Center of Gravity, which may ſerve to find the Center of Gravity of a Figure, when you know its Square or Quadrature, or to find out its Quadrature or Arëa, when you know its Center of Gravity, as you will ſee in the Sequel. *Plate* 17. *Fig.* 70.

If by Thought you move the Rectangle ABCD, whoſe Center of Gravity is at E round about the ſide BC which muſt remain unmoveable; the Cylinder deſcrib'd by that motion, whoſe Baſe is the Circle, whoſe *Radius* is AB, and Height the unmoveable ſide BC, is equal to a Priſm whoſe Baſe is the propos'd Plain ABCD, and Height a Line equal to the Circumference of the Circle EGHI deſcrib'd by the Pircumvolution of the Center of Gravity E, whoſe *Radius* EF is equal to half the ſide AB, becauſe the Center of Gravity of a Parallelogram is the ſame as its Center of Magnitude, as we ſhall hereafter demonſtrate.

To demonſtrate what we have juſt ſaid, let a be put for AB, b for BC, c for the Circumference EGHI, whoſe *Radius* EF being but half of AB, the Circumference whoſe *Radius* is AB will be $2c$: and then the Arëa of the Plain ABCD will be ab, and that Solid whoſe Baſe is the ſaid Plain ABCD or ab, and whoſe Height is the Circumference EGHI, or c, will be abc; which is equal to the Cylinder deſcrib'd by the Circumvolution of the Plain ABCD round its unmoveable Side BC, becauſe its Baſe is ac, which is found if you multiply the *Radius* AB, or a, by half its Circumference, or c, and its Height is BC, or b.

Likewiſe if the Equilateral Triangle ABC, whoſe Center of Gravity is E, be made to revolve about the unmoveable Side AB; the Solid Rhomb produc'd by that motion, made up of Two equal Cones, whoſe equal Heights are AD, BD, and common Baſe a Circle whoſe *Radius* is CD, will be equal to a Priſm whoſe Baſe is the Plain ABC, and Height a Right-line equal to the Circumference EFGH, deſcrib'd by the Circumvolution of the Center of Gravity E, whoſe *Radius* is DE the third part of the Perpendicular CD, as we ſhall hereafter demonſtrate. *Fig.* 71.

For

Plate 17. *Fig.* 71. For the Demonstration, let a be put for AD, or for BD, b for CD, and c for the Circumference EFGH, whose *Radius* DE being but the third part of the *Radius* DC of the Circle, which is the common Base of the Two Cones, which make up the Solid Rhomb, the Circumference of this second Circle will be $3c$: And then the Arëa of the Plain ABC will be ab, and the Solid whose Base is this Plain ABC, or ab, and Height the Circumference EFGH, or c, will be abc, which is equal to the Solid Rhomb made up of the Two Cones, whose common Height is a, and common Base a Circle whose *Radius* is CD or b: For if the *Radius* CD, or b, be multiplied by $\frac{3c}{2}$ half of its Circumference, you will have $\frac{3bc}{2}$ for that common Base, which being multiplied by the third part of AB, or $\frac{2a}{3}$, you will have abc for the Solid Rhomb, *&c.*

SECTION I.

Of the Center of Gravity of Lines.

THO' there is no Line but what is join'd to some Surface, and no Surface but what is join'd to a Body, yet one may consider a Body which is Long, Homogeneous, of an equal Thickness all over, and extremely Thin and Slender, as a Line; and allow it Weight, and a Center of Gravity, which we shall find out by the help of the following Propositions.

PROPOSITION I.

THEOREM.

The Center of Gravity of Two Quantities taken together is in the Right-line which passes thro' the Center of Gravity of each.

LET us suppose any Two Quantities, as the Two lines AB, CD, whose middle points E,F, are their Centers of Gravity. I say, That in such a case the Center of Gravity of those Two lines AB, CD, consider'd as One Quantity, or as if they were united by the Right-line EF, which passes thro' their Centers of Gravity, is in some point of the said line EF, as G.

DE-

DEMONSTRATION.

For if the Center of Gravity which is common to the Two lines AB, CD, shou'd be in any point out of the line EF, as at H, having drawn the line EHI, you must consider that since the Two Quantities AB, CD, are in *Æquilibrio* about the point H, and also AE and EB about the point E, the Two CI, and DI must likewise remain in *Æquilibrio* about the point I, which being impossible, because CF and DF are suppos'd in *Æquilibrio* about the point F, it must also be impossible, that the Two AB, CD, shou'd be in *Æquilibrio* about the point H. Whence it is plain, that their common Center of Gravity cannot be out of the Line EF. *Q. E. D.* *Plate 17. Fig. 72.*

PROPOSITION II.

THEOREM.

The common Center of Gravity of Two Quantities divides the Right-line which joins their Centers of Gravity into Two Parts, which are reciprocally proportional to the Quantities.

LET us suppose any Two Quantities, as the Two lines AB, CD, whose Centers of Gravity are E, F, and their common Center of Gravity G. I say, That in such a case EG: is to FG: : as CD: to AB. *Fig. 72.*

DEMONSTRATION.

If the Weight of AB be reduc'd to its Center of Gravity E, and likewise the Weight of CD to its Center of Gravity F, the line EGF may be look'd upon as a Balance, whose fix'd Point is G, at the ends of which are suspended Weights equal to the Two Quantities AB, CD, which remain in *Æquilibrio* about the point G: And as in such a case the Weights wou'd be in a reciprocal *Ratio* of their distances EG, FG; it follows, That the Quantities AB, CD, will likewise be in a reciprocal *Ratio* of the parts EG, FG. *Q. E. D.*

COROLLARY.

It is evident from this Proposition, that if the Quantities AB, CD, were equal in Weight, the parts EG, FG, wou'd also be equal to one another; that is, the common Center of Gra-

Plate 17. Fig. 72. Gravity G of the Two equal Quantities AB, CD, will be exactly in the middle of the Right-line which joins their Centers of Gravity E, F.

PROPOSITION III.

THEOREM.

If several Quantities of Equal Weight, and equally distant from one another, are so dispos'd, that their Centers of Gravity are in a Right-line; their common Center of Gravity shall be in the middle of that Right-line.

Fig. 73. LET AB,CD,EF,GH, be equal Quantities equally distant from one another, whose Centers of Gravity I,K,L,M, are in the Right-line IM. I say, That in such a case, O the middle Point of that line IM, is the common Center of Gravity of all those Quantities taken together.

DEMONSTRATION.

Because by *Coroll. Prop.* 2. N the middle Point of IK, is the common Center of Gravity of the Two equal Quantities AB,CD, and likewise P the middle of LM, is the common Center of Gravity of the Two equal Quantities EF,GH; by reducing the whole Weight of the Two equal Quantities AB, CD, to their common Center of Gravity N, and the whole Weight of the Two equal Quantities EF,GH, to their common Center of Gravity P, you may consider NP as a Balance laden with equal Weights at its ends NP, whose middle point O will consequently be the common Center of Gravity. *Q. E. D.*

COROLLARY.

From this Proposition it is evident, that if the propos'd Quantities are odd in Number, their common Center of Gravity must be the Center of Gravity of that Quantity which is in the middle.

PRO-

PROPOSITION IV.

THEOREM.

The Center of Gravity of the difference of Two Quantities is in the Right-line drawn thro' both their Centers of Gravity.

LET the Quantities propos'd be AB, AD, whose Difference is CD, and whose Centers of Gravity are F,E. I say, That in such a case, the Center of Gravity of the Difference CD consider'd as taken away from the Quantity AB, is in some point of the line EF produc'd, as for Example, at G. *Plate 17. Fig. 74.*

DEMONSTRATION.

For if that Center of Gravity was in any point of any other line, as in the point H of the line FH, the Center of Gravity E of the whole Quantity AD wou'd not be in the Right-line FH, which passes thro' F, H, the Centers of Gravity of the Two Quantities AB, CD, of which it is compounded, which is contrary to what has been demonstrated in *Prop.* 1. Whence it follows, that the Center of Gravity G of CD the Difference of the Two Quantities propos'd AB, AD, cannot be out of the line EF. *Q. E. D.*

PROPOSITION V.

THEOREM.

The Center of Gravity of the Difference of Two Quantities divides the Right-line drawn thro' their Centers of Gravity, into Two Parts reciprocally proportional to the parts of the greatest of those Two Quantities.

LET AB, AD, be the Two Quantities, and E, F, their Centers of Gravity, G the Center of Gravity of their Difference CD. I say, That in such a case, GE: is to EF :: as AB: is to CD. *Fig. 74.*

DEMONSTRATION.

For since the Quantities AB, CD, are in *Æquilibrio* about the point E, if the Weight of the first AB be reduc'd to its Center of Gravity F, and the Weight of the second CD be re-

Plate 17. *Fig.* 74. reduc'd to its Center of Gravity G, the line FEG may be consider'd as a Balance, whose fix'd Point is at E, and whose Ends are laden with Weights equal to the Quantities AB, CD: And as those Weights are in *Æquilibrio* about the point E, they must be in a reciprocal *Ratio* of their distances; that is, AB; must be to CD: : as EG: is to EF. *Q. E. D.*

PROPOSITION VI.

PROBLEM.

How to find the common Center of Gravity of Two given Quantities, whose respective Centers of Gravity are known.

Fig. 74. TO find the common Center of Gravity of AB, CD, Two given Quantities; or the Center of Gravity of AD their Sum, by means of F, G, their particular Centers of Gravity; draw the line FG, and divide it at E in such manner that its Whole Length AD: may be to its Part AB: : as the Line FG: is to its Part GE, which will be done by finding a fourth Quantity (*viz.* GE) proportional to the Two Quantities AD: AB: : and to the Line FG: And the point E will be the Center of Gravity of the Two given Quantities AB, CD.

DEMONSTRATION.

Since, *by Construction*, the Four Quantities AD, AB, FG, EG, are proportional, it will be known *by dividing*, that these Four CD, AB, EF, EG, are also proportional; that is, that the Two Quantities AB, CD, are in a reciprocal *Ratio* of their distances EF, EG, and that consequently the point E is the common Center of Gravity of the Two given Quantities AB, CD, or the Center of Gravity of their sum AD. *Q. E. I. & D.*

PROPOSITION VII.

THEOREM.

How to find the common Center of Gravity of the difference of Two given Quantities, whose respective Centers of Gravity are known.

Fig. 74. TO find the Center of Gravity of CD the Difference of AB, AD, the given Quantities whose Center of Gravity F, E, are known; draw the line EF and continue it to-

wards

wards G, in such manner that CD: may be to AB:: as EF: is to EG, and G will be the Center of Gravity of the Difference CD, because the Quantities AB, CD, are in a reciprocal *Ratio* of the Lines EF, EG. *Plate 17: Fig. 74.*

PROPOSITION VIII.

PROBLEM.

How to find the Center of Gravity of a Right-line.

TO find the Center of Gravity of the Right-line AB, divide it into Two equal Parts at E, and E the middle point will be its Center of Gravity. *Fig. 72.*

DEMONSTRATION.

For since we consider a Right-line as a Quantity Homogeneous and of equal Thickness all over, its Center of Gravity must be the same as its Center of Magnitude. Thus E the middle point will be the Center of Gravity of the propos'd line AB. *Q. E. I. & D.*

PROPOSITION IX.

PROBLEM.

How to find the common Center of Gravity of Two Right-lines.

SEveral different Cases may happen according to the different Position of the Two lines propos'd. *Fig. 80.*

First, if the Two given Right-lines touch one another directly, as AB, BC, they may be consider'd as One line AC, and their Sum AC is to be divided into Two equal Parts at H, which, by *Prop.* 8. will be its Center of Gravity, and consequently the common Center of Gravity of the Two given lines AB, BC.

Secondly, if the Two Lines given are in a Right-line, but do not touch one another, as AB, CD, divide each of them into Two equal Parts at the points E, F, which, by *Prop.* 8. will be their Centers of Gravity, and having drawn the line BC, to the Three lines AB + CD: CD:: EF: find a Fourth proportional, *viz.* EO, and the point O, by *Prop.* 6. will be the common Center of Gravity of the Two propos'd Lines AB,CD, consider'd as if they were join'd together by the Right-line BC, to which you must attribute no Gravity.

Plate 17. *Fig.* 75. Whatever Position else the Two given Lines be in, their common Center of Gravity may always be found by *Prop.* 6. and the general Rule is this. Having, by *Prop.* 8. found E, F, the Centers of Gravity of the Two given Lines AB, AC, and having join'd 'em by the line EF, divide it at O in such manner that the Four lines AB, AC, OF, OE, may be proportional, which will happen, if to the Three lines AB+AC: AC: : EF: be found a fourth Proportional, as EO, or to the Three AB+AC: AB:: EF: a fourth Proportional as FO, and O will be the Center of Gravity requir'd.

PROPOSITION X.

PROBLEM.

How to find the common Center of Gravity of several Right lines given.

Fig. 75. BY means of the foregoing Problem it is easy to find the common Center of Gravity of as many Right-lines as you please. As if AB, AC, and CD were propos'd; first find O the common Center of Gravity of the Two first Lines AB, AC, as we have just taught: Then find I the common Center of Gravity of the third CD, and of the sum of the Two first AB, AC, which will consequently be the common Center of Gravity of the Three given Lines AB, AC, CD.

If there was a fourth Line, then you ought to find the common Center of Gravity of that fourth Line, and the sum of the Three first, and That will be the Center of Four given Lines, and so of the rest.

But to come to the Practice, divide the given Lines AB AC, CD, each into Two equal Parts at the points E, F, G, and having drawn EF, to the Three lines AB + AC: AB:: EF: find a fourth Proportional FO; then join GO, and again to the Three lines AB+ AC+ CD: CD:: GO: find a fourth Proportional OI, that you may have at I the common Center of Gravity of the Three given Lines AB, AC, CD, consider'd as join'd together by the Two lines EF, GO, which are to be look'd upon as without Weight.

SCHOLIUM.

The different Position and Proportion of the given Lines, may afford us shorter Ways of solving this Problem: As if the line CD shou'd be equal to the sum of the Two others AB, AC; then it wou'd only be necessary to divide the line GO

GO into Two equal Parts at I. This is a short Method founded upon the Proportion of the Lines, and in the following Problem you will have One founded upon the Disposition of the Lines. *Plate 17. Fig. 75.*

PROPOSITION XI.

PROBLEM.

How to find the Center of Gravity of the Periphery of a Triangle.

TO find the common Center of Gravity of the Three Sides of the given Triangle ABC, you must work as in *Prop.* 10. whence we have taken this Method. *Fig. 76.*

Divide the sides AB, AC, BC, each into Two equal Parts, at the points E, D, F, and make the Triangle EDF. Divide Two of hte Angles of this New Triangle each into Two equal Parts, by the lines DH, FG, and the point I, where these Two Lines intersect, will be the common Center of Gravity of the Three Lines propos'd AB, AC, BC, which shut up the Triangle ABC.

DEMONSTRATION.

Since CD: is to its double CA:: as CF, is to its double CB; it follows by *6. 6.* that the Triangles ABC, DFC, are Similar, and by 4. *6.* that AB: is to DF:: as BC: is to CF; and because BC is Twice CF, AB must also be Twice DF, and consequently DF must be equal to AE or BE. One may in the same manner demonstrate, that AD and EF are Two equal lines, and it is also plain by 33. 1. Moreover GD: has the same *Ratio* to GE:: as DF: has to EF, by 3. *6.* or as AE: to AD, or AB: to AC, because of the Similar Triangles ABC, AED. Whence it follows, that the point G is the Center of Gravity of the Two Lines AB, AC, which being consider'd as One Line, it will be known by *Prop.* 1. That the common Center of Gravity of this Sum and the third Line BC; that is, the common Center of Gravity of these Three lines AB, AC, BC, is in some point of the line FG: And in the same manner one may demonstrate, that it is in some point of the line DH, and consequently that it is in the common Section I of the Two Lines FG, DH. *Q. E. I. & D.*

PROPOSITION XII.

PROBLEM.

How to find the Center of Gravity of the Periphery of a Quadrilateral Figure.

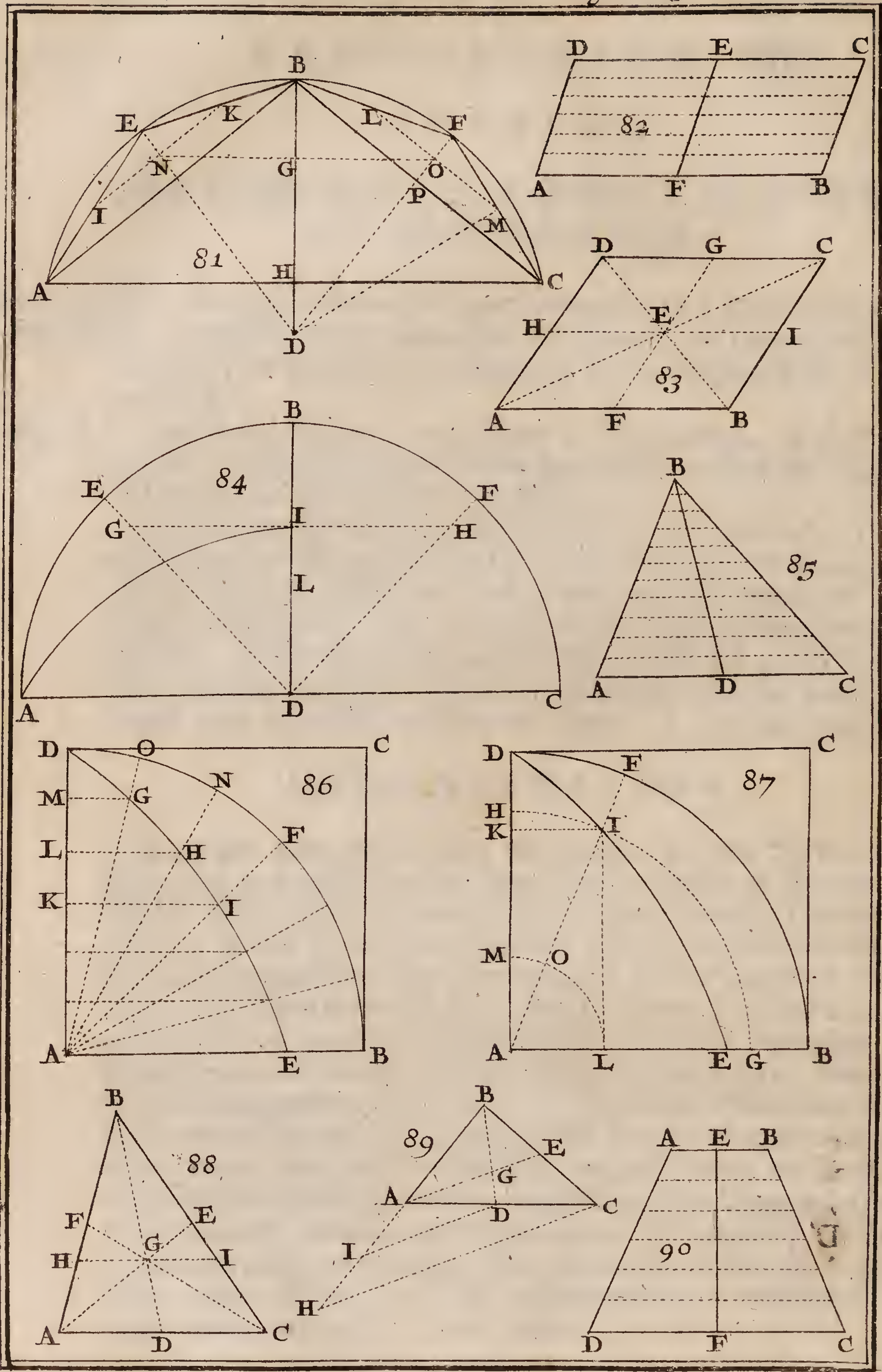
Plate 17. *Fig.* 77. If the Quadrilateral Figure propos'd be a Parallelogram, as ABCD, it is plain that the Center of Gravity of its Periphery is the point I, where its Two Diagonals AC, BD, intersect.

Fig. 78. But if the Figure propos'd be a Trapezium, as ABCD, the 10*th Prop.* has given us this short Way to find the Center of Gravity of the Periphery ABCD.

Divide the Four Sides AB, BC, CD, AD, each into Two equal Parts at the points E,F,G,H, and likewise the Four Angles A, B, C, D, also into Two equal parts, by means of the Lines AI, BK, CL, DM, and make the Quadrilateral Figure EFGH. Then set off HI on EN, FK on EO, FL on GP, and HM on GQ, and draw the lines NP, OQ; and the point R, where they intersect, will be the Center of Gravity requir'd.

DEMONSTRATION.

Since the Line AI divides the Angle A into Two equal parts, by 3. 6. AH: has the same *Ratio* to AE: : as IH: has to IE, or NE: to NH, wherefore N is the common Center of Gravity of the Two lines AB, AD: After the same manner one may demonstrate, that the point O is the Center of Gravity of the Two lines AB, BC; that the point P is the Center of Gravity of the Two lines BC, CD, and that the point M is the Center of Gravity of the Two lines AD, CD. Now it has been demonstrated in *Prop.* 1. that if the Two lines AB, AD, whose Center of Gravity is N, be look'd upon as One single Line; and the Two BC, CD, whose Center of Gravity is P, be also consider'd as One single Line; the Common Center of Gravity of those two Sums, or of the Periphery ABCD, is in some point of the line NP: And that likewise it is in some point of the line OQ, and therefore it must be at R where they intersect. *Q. E. I. & D.*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PROPOSITION XIII.

PROBLEM.

How to find the Center of Gravity of the Periphery of a Polygon.

IF the Polygon be Regular, it is plain that its Center of Gravity is the ſame with the Center of the Figure, or of the inſcrib'd or circumſcrib'd Circle.

But if the Polygon be Irregular, it will be eaſy by *Prop.* 10. to find the Center of Gravity of its Periphery, and you may commonly find out ſome ſhort manner of doing it, as you have ſeen in the Two foregoing Propoſitions.

PROPOSITION XIV.

THEOREM.

An Arch of a Circle being divided into any Evenly even Number of equal Arches; the Ratio *of the ſum of the Chords of all thoſe Arches : is to half the Chord of the great Arch: : as the* Ratio *of the Sine of the Complement of One of thoſe little Arches is to the diſtance between the Center of the Circle, and the common Center of Gravity of the Chords of all thoſe little Arches.*

LET us firſt divide the Arch ABC, whoſe Center is D, *Plate* 17.
and Chord AC, into Two equal Arches AB,BC, whoſe *Fig.* 79.
Chords are BA, BC, which we will divide into Two equal parts, at the points E,F, which are their Centers of Gravity, by *Prop.* 8. then let us draw EF, which will be cut at Right-angles, and into Two equal parts, by the *Radius* DB at G, which Point will be the common Center of Gravity of the Two equal lines BA,BC, by *Prop.* 2. Let us alſo draw the line DF, which will be the Sine of the Complement of half the Arch BC. This being done and ſuppos'd, I ſay, That the *Ratio* of half the Sum of the Chords BA, BC: to half the Chord AC: : or the *Ratio* of BC: to CH: : is the ſame as the *Ratio* of DF: to DG, which is evident by reaſon of the Two Similar Rectangular Triangles BCH,DGF.

Let us now again divide each of the Two equal Arches *Plate* 18.
AB, BC, into Two equal parts, in ſuch manner that the *Fig.* 81.
Whole Arch ABC be divided into Four equal parts at the

Plate 18. *Fig.* 81. points E,B,F, and draw the Four equal Chords AE,EB,BF,FC, which we will divide each into Two equal parts at the points I,K,L,M, which by *Prop.* 8. will be their Centers of Gravity; and then join the lines IK,LM, to divide them still into Two equal parts at N,O, which points we must join by the line NO, which by the *Radius* DB will be divided into Two equal parts at Right-angles at the point G, which by *Prop.* 10. will be the common Center of Gravity of the Four lines AE,EB,BF,FC. Let us draw also the *Radius* DF, which will go thro' the point O, and cut at Right-angles into Two equal parts the Chord BC at the point P: And the line DM, which will be the Sine of the Complement of half the Arch FC. This being done and suppos'd, I say again, That the Sum of the Chords AE,EB,BF,FC: has the same *Ratio* to half the Chord AC:: or BF + CF: has the same *Ratio* to CF:: as DM: has to DG.

DEMONSTRATION.

For it is plain that the Two Rectangular Triangles DOM, CPE, are Similar, and by 4. 6. DM: has the same *Ratio* to DO:: as CF: has to CP; or 2CF: to 2CP; that is, as CF + BF: has to CB. It is also plain, That the Two Rectangular Triangles DGO,BHC, are Similar, and consequently CB: is to CH:: as DO: to DG. Whence it follows by *Equality*, that CF+BF: is to CH:: as DM: is to DG. *Q. E. D.* The Demonstration will be the same when the Number of subdivisions is greater. Whence it is easy to conclude, That the Sum of the Chords of the Arches which are made by the subdivision of the great Arch ABC: is to its Chord AC:: as the Sine of the Complement of half of One of these Arches: is to the distance between the Center of the Circle, and the common Center of Gravity of the Chords of all the little Arches. *Q. E. D.*

SCHOLIUM.

It is evident, that the more subdivisions there are, the nearer will the Sine of the Complement DM be to the length of the *Radius*, and the nearer also will the Chords of all the little Arches be to the Circumference ABC: So that if you imagine the Arch ABC to be divided into an Infinite Number of little Arches, the Sine of the Complement DM will be equal to the *Radius* or Whole Sine, and the Sum of the Chords of all those little Arches will be exactly equal to the Arch ABC. Whence it is easy to conclude, That ABC: has the same *Ratio* to its Chord AC:: as the *Radius* DB: has to DG the distance between the Center D, and G the Center of Gravity of the propos'd Arch. Hence it is also easy to conclude, That

That *the* Radius *of a Circle is a mean Proportional between the Quarter of its Circumference, and the Distance from its Center to the Center of Gravity of half the Circumference.*

PROPOSITION XV.

PROBLEM.

How to find the Center of Gravity of a given Arch.

TO find the Center of Gravity of the Arch ABC, whose Center is D, divide it into Two equal parts at B by means of the *Radius* DB, which will also divide into Two equal parts at Right-angles the Chord AC at the point H; and to the Arch ABC: its Chord AC: : and the *Radius* DB: find a fourth Proportional, *viz.* DI, to have at I the Center of Gravity of the propos'd Arch ABC, as it is plain from what has been demonstrated in the foregoing Proposition. *Plate Fig.*

SCHOLIUM.

It is plain, that if the Arch ABC was a Semi-circle; to the Circumference ABC: to the Diameter AC: : and to the *Radius* DB: a fourth Proportional DI ought to be found; or taking the halves of the Two first Lines, it wou'd be requir'd to find a third Proportional to AB, or BC: (which is a Quarter of the Whole Circumference of the Circle) And to the *Radius* DB: : which wou'd be DI, to have at I the Center of Gravity of the Circumference ABC the Semi-circle propos'd. *Plate 18. Fig. 84.*

Whence it follows, That that Center I belongs to the *Quadratrix-Line*, which wou'd pass thro the point A, for the chief Property of that line is, that AB the Quarter of the Whole Circumference of the Circle: has the same *Ratio* to the *Radius* AD: : as the said *Radius* AD or BD has to DI, as we shall demonstrate at the end of this Section. If then thro' the point A be drawn the *Quadratrix-Line* AI, you will have at I, the Center of Gravity of the Semi-circle. We do not speak of the Whole Circumference of the Circle, because it is plain that the Center of Gravity is then the same with the Center of the Circle.

PRO-

PROPOSITION XVI.

PROBLEM.

Knowing the Center of Gravity of an Arch, How to find that of a double Arch.

Plate 18. Fig. 84. LET the Arch AB, whose Center of Gravity is D be given, as also its Center of Gravity G upon the *Radius* DE, which divides the Arch AB into Two equal parts at E; now it is requir'd to find the Center of Gravity of the double Arch ABC, upon the *Radius* BD, which divides it into Two equal parts at B.

Having set off BE, or BF, and drawn the *Radius* DF, make DH equal to DG, to have at G the Center of Gravity of the Arch BC: And as G is the Center of Gravity of the Arch AB, the Line GH will contain the common Center of Gravity of the Two Arches BA, BC, by *Prop.* 1. which being equal, I the middle Point will be their common Center of Gravity, and consequently the Center of Gravity of ABC the double Arch.

SCHOLIUM.

Knowing the Center of Gravity of an Arch, one may, by an Operation contrary to the foregoing, find the Center of Gravity of its half: for having I the Center of Gravity of the Arch ABC, to find that of its half AB, you need only divide it into Two equal parts by the *Radius* DE; and from I draw IG perpendicular to the *Radius* DB, which will on the *Radius* DE give G, the Center of Gravity requir'd.

PROPOSITION XVII.

PROBLEM.

How to find the common Center of Gravity of a given Arch, and of its Chords.

Plate 17. Fig. 79. TO find the common Center of Gravity of the Arch ABC, and of its Chord AC, first find out I the Center of Gravity of the Arch ABC, and H the Center of Gravity of the Chord AC; then by *Prop.* 1. it is plain that the common Center of Gravity is somewhere in the Line HI; wherefore the Line HI must be divided into Two parts

at

at L, in ſuch manner, that ABC: may be to the Chord AC:: or DF to DI:: as HL : is to LI; now this diviſion will be made, by finding a fourth proportional to the Three lines DF+DI: DF: : HI: which will be HL; and L will be the Center of Gravity requir'd. *Plate* 17. *Fig.* 79.

If the Arch ABC be a Semi-circle; having found out the Center of Gravity I of the Semi-circle ABC; find a fourth proportional to the Three lines DB+DI: DI: : DB: which will be DL, or elſe IL which is a third proportional to the Two DB+DI: DI:: to have at L the common Center of Gravity of the Circumference of the Semi-circle ABC, and the Diameter AC.

Of the Quadratrix-Line.

THIS Line has been ſo call'd, becauſe it contributes to the ſquaring of the Circle, as we ſhall ſhew after we have explain'd the Generation and Deſcription of this Curve in the following manner. *Plate* 18. *Fig.* 84.

Let ABFD be the fourth part of a Circle within the Square ABCD, and A One of the Angles of that Square, its Center. By Thought move the Side or *Radius* AD about the Center A, from D to B, with an Equal and Uniform Motion thro' all the points of Circumference BFD: And at the ſame time let the ſide CD move from D to A, remaining always parallel to its oppoſite ſide AB, with a Motion likewiſe Equal and Uniform, thro' all the points of the ſide AD, conceiving the ſide AD to be divided into as many equal parts as the Circumference BFD; and then that ſide CD moving thus parallel to it ſelf, and the *Radius* AD moving at the ſame time about the Center A, will cut one another ſucceſſively in ſuch points, as will make the *Quadratrix-Line* DIE, whoſe Center is A, whoſe *Vertex* is D, whoſe Axis is AD, and Baſe AE, the end of which, *viz.* E, cannot be exactly terminated, becauſe when the ſide CD is by its Equal and Uniform Motion come upon the ſide AB, the ſide AD is likewiſe come upon the ſaid AB by its Uniform Motion, whence it happens that theſe Two lines fall upon one another without any Interſection. *Fig.* 86.

Thus much for the Generation of this Curve, whence may eaſily be drawn the Method of deſcribing it upon Paper, with a Rule and Compaſſes, which may be done by finding ſeveral points of it, and afterwards joining them by a Curve, which will be the eaſier to deſcribe the nearer thoſe points are to one another. After the following Manner you may find as many of 'em as you pleaſe.

Ha-

Plate 18. Fig. 86. Having drawn at pleaſure the Two Perpendiculars AB, AD, deſcribe at what diſtance you will from the Right-angle A the Arch BFD, and divide it into as many equal Parts as you pleaſe, as for Example, into Six, divide likewiſe its *Radius* AD into Six equal Parts, and from all thoſe points draw Right-lines parallel to the other *Radius* AB. Draw alſo from the Center A thro' the points of Diviſion of the Arch BD, the ſame Number of Right-lines, or *Radii*, which will cut the firſt lines in Points, which you muſt join by the Curve DIE, which will be *Dinoſtratus's Quadratrix-Line*, whoſe Deſcription will be the more Exact, the more Points you have found, that is, the more equal Parts the Arch BD, and its *Radius* AD are divided into; but one cannot determine the point E which ſhou'd terminate the Baſe AE, becauſe in that Place there is no Section of Lines,otherways one might find out the ſquaring of the Circle ; for if the point E was found, a Right-line equal to the Arch BFD wou'd be found Geometrically, becauſe that Circumference is a third Proportional to the Baſe AE, and the *Radius* AB ; but we muſt demonſtrate it.

PREPARATION.

Fig. 87. To demonſtrate that the Arch BD is a third Proportional to the Two lines AE,AB, or that the Baſe AE is a third Proportional to the Arch BD, and its *Radius* AB ; it will ſuffice to demonſtrate that a line greater than the Baſe AE, as AG, or leſs as AL, cannot be a third Proportional to the Arch BD, and its *Radius* AB. For this end from the Center A draw thro' the Two points L,G, the Arches LM,GH, and thro' the point I where the *Quadratrix* is cut by the Arch GH, draw the *Radius* AF, and the Line IK perpendicular to the *Radius* AD. Draw alſo from the point L, LI perpendicular to the *Radius* AB, and thro' the point I, where it cuts the *Quadratrix* DE, draw the *Radius* AF, and the Line IK parallel to the *Radius* AB. From the Center A, deſcribe the Arch GH thro' the point I.

DEMONSTRATION.

If the Three lines BD,AB, and AG were proportional; that is, if we had this Analogy, BD: AB:: AB: AG, by putting inſtead of the Two laſt Terms AB, AG, the Arches BD, GH, which are in the ſame *Ratio*, becauſe they are Similar, we ſhou'd have this other Analogy BD: AB:: BD: GH,where the Antecedents being Equal, the Conſequents ſhou'd be Equal; that is, the line AB ſhou'd be equal to the Arch GH. This being ſuppos'd, one muſt conſider, that the Arches BD, GH, be-

being Similar, as well as the Two BF, GI, we shall likewise have this Analogy, BD: BF :: GH : GI, and if instead of the Two first Terms BD, BF, we put the lines AD, AK, which are in the same *Ratio by the Generation of the Quadratrix*, we shall have this other Analogy AD : AK :: GH : GI; and because we have said that the Antecedent AD, or AB, must be equal to the Antecedent GH, the Consequent AK or LI, must also be equal to the Consequent GI, which being Impossible, it is also Impossible that the Three Lines BD, AB, AG shou'd be proportional. *Which is one of the Two Things to be demonstrated.* Plate 18. Fig. 87.

If the Three lines BD, AB, AL, were Proportional, so as we might have this Analogy, BD: AB :: AB : AL, by putting instead of the Two last Terms AB, AL, the Two Similar Arches BD, LM, which are in the same *Ratio* as their *Radij*, we shou'd have this other Analogy BD : AB :: BD : LM, where it appears as before, that the Arch LM wou'd be equal to the Line AB, or AD. This being suppos'd, one must consider, that BD, LM being Similar Arches as well as BF, LO, we shall have this Analogy LM: LO :: BD: BF, and if instead of the Two last Terms BD, BF, we put the Two AD, AK, which are in the same *Ratio, by the Generation of the Quadratrix*, we shall have this other Analogy LM: LO :: AD : AK, in which the Antecedent LM has been demonstrated equal to the Antecedent AD, whence the Consequent LO must also be equal to the Consequent AK or LI, which being Impossible, it is also Impossible that the Three Lines BD, AB, AL, shou'd be proportional. *Which was left to be demonstrated.*

SCHOLIUM.

As we are only to speak occasionally of this *Quadratrix* Line (call'd only the *Quadratrix*) we must not mention any more of its different Properties: We shall only say here, that by the means of it you may divide a given Arch into as many equal parts as you will; as if you wou'd divide the Arch DF into Three equal parts, you must draw the *Radius* AF, and thro' the Point I where it cuts the *Quadratrix* DE, draw the Line IK parallel to the *Radius* AB, or perpendicular to the *Radius* AD, after which having divided the Line DK into Three equal parts at the Points L, M, the Two Lines LM, and MG must be, thro' those Points LM, drawn parallel to the Line IK, which Lines will on the *Quadratrix* DE, give the Two Points H, G, thro' which you must draw from the Center A the Right-lines AN, AO, which will divide DF the Arch propos'd, into Three equal parts. Fig. 86.

But

But this Division may be made with the same facility by means of another Curve, invented by Mr. *Tschirnhaus* a *German* Gentleman, the Description of which Curve Line we will teach, as also give the Demonstration of Two fine Theorems which He has given us concerning this Line, the Last of which has been wrong stated, when we spake of it in our *Mathematical Dictionary*, where we mistook one *Radius* for another: and therefore for the satisfaction of this Learned Mathematician, we shall give the Demonstration of his Two Theorems, after we have taught the Description of his Curve, which is as follows.

Plate 19. *Fig.* 91. Let the Quadrant ABCD be describ'd as before within the Square ABLD. Having divided the Circumference BCD, and its *Radius* AD, Each into an equal Number of Parts such as you think fit, as for Example, into Six, draw thro' the Points of Division of the Arch BCD Lines parallel to the *Radius* AD, and thro' the Point of Division of the *Radius* AD, Lines parallel to the other *Radius* AB, and the Point where those parallels intersect being taken equally from D will form the Curve BED, by means of which you may divide an Arch into as many equal parts as you please, thus;

To divide the Arch CD into Three parts, *for Example*, Draw thro' the Point C, the line CE parallel to the *Radius* AD, and thro' the Point E, where this parallel CE cuts the Curve BED, draw EF parallel to the other *Radius* AB. Divide the line DF into Three equal parts at the Points G, H, and thro' those Points G, H, draw the lines GK, HI, parallel to the line EF, to have upon the Curve BED, the Two Points I, K, thro' which you must draw IN, KM, parallel to the *Radius* AD, which will divide AD the Arch given, into Three equal parts at the Points M,N.

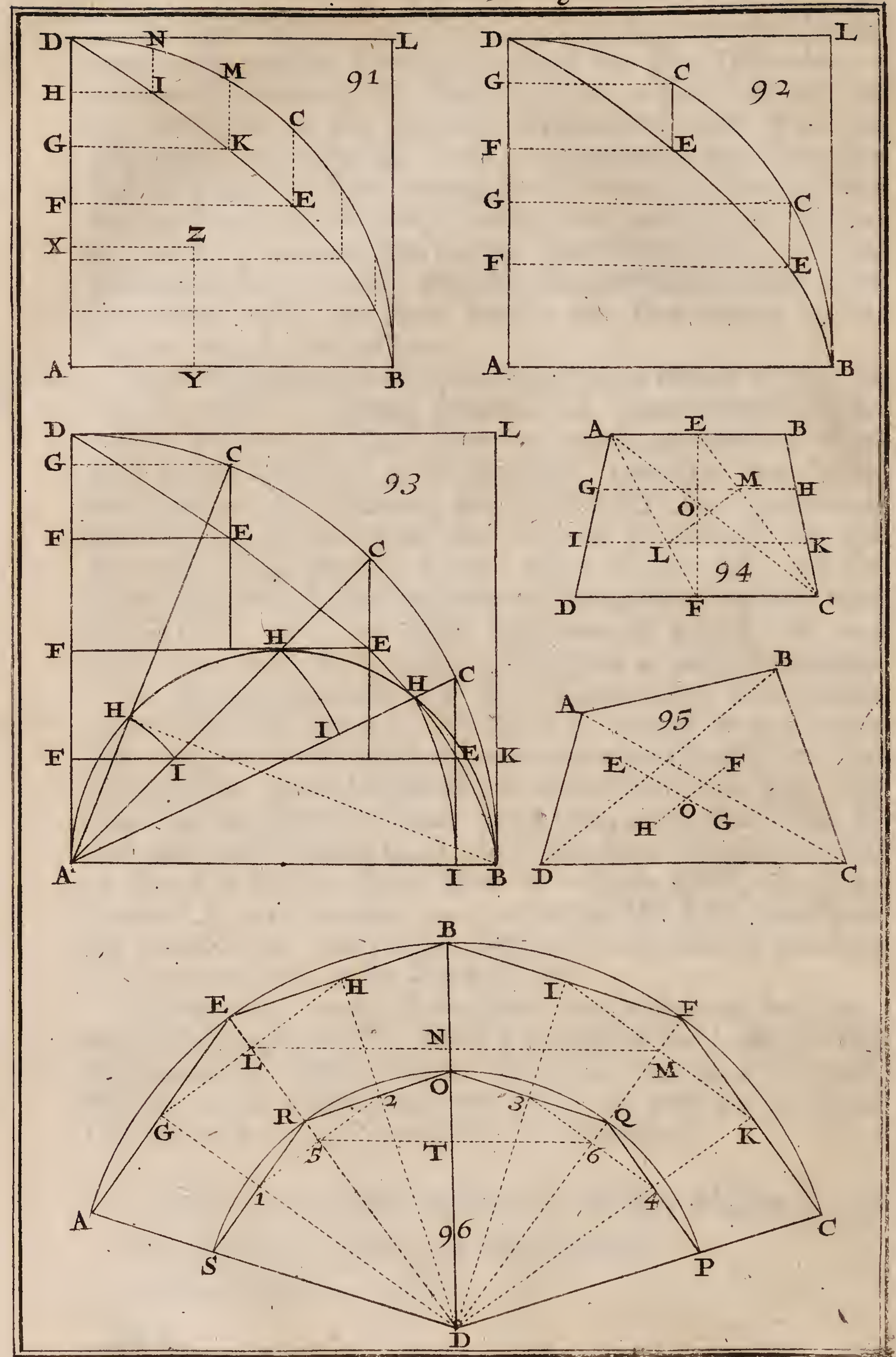
To come now to the Two Theorems which we have promis'd, I thought my self oblig'd to publish the Letter which the Reverend Father *Nicholas* the Jesuit honour'd me with, thereby to do him justice, and shew as well his excellent Genius, as his great Penetration in Geometry.

A Letter from the Reverend Father Nicholas, *of the Society of* Jesus, *to the Author.*

Toulouse April 14. 1691.

SIR,

'THO' I have not the Honour to be known of you; 'I thought the following Demonstrations, which I 'have found out concerning a Subject, which you gave me 'the first Occasion of applying my self to, wou'd not be ungrate-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'ungrateful to you. It is a Fortnight since I first had your 'Learned Dictionary; I have read it thro' with a great deal 'of Pleasure; so great a Work cou'd not be perform'd by a 'Person inferior in Learning to you. As I am a great Lover 'of Geometry, to which I have very much apply'd my self, 'and of which I have wrote several Treatises, which may in 'a little time be publish'd; I took a particular delight in 'Speculative Geometry: in treating of it, you speak in few 'Words of several Curves, but always very justly and well. 'Amongst others in your 99. and 100 *Pages* you speak of a 'New Curve fit to divide a given Angle according to a given '*Ratio*, which you say was invented by Mr. *Tschirnhaus*; I 'must own that I had never heard of this Curve, and this 'excited my Curiosity to examine it: What engag'd me the 'more to it, is that you say that Mr. *Tschirnhaus* has advanc'd 'Two Theorems concerning it, which he has not demon- *Plate* 19. *Fig.* 91. 'strated; the First is, that *When ABCD is the Quarter of a 'Circle, the Space ABED: is to the Square ABLD:: as the 'Radius AB: is to the Circumference BCD:* And the other, '*that the Solid which is produc'd by the Circumvolution of 'the Figure ABED about the Axis AB: is to the Circum- 'scrib'd Cylinder*:: as 1: is to 2. Thereupon you say, that 'this Second Theorem wou'd be True, and the First very 'near True, if the Curve BED was a *Parabola*. Now as 'BED of Mr. *Tschirnhaus* comes pretty near a *Parabola*, it 'follows that his Two Theorems are very near True. 'Seeing then that you doubted of the Whole Truth of these 'Two Theorems, and that with a great deal of reason, be- 'cause they were not demonstrated, I was willing to examine 'thoroughly into the matter, to see if what Mr. *Tschirnhaus* 'advances be True or False in the Geometrical strictness. I 'found the First Theorem to be True, and the Last False: 'Being thus oblig'd to examine into the Nature of this Curve, 'I have, in my opinion, discover'd and demonstrated the 'finest Properties of it, whether the Dimensions of the Space 'ABED, or of its Parts; or the Solids which may be pro- 'duc'd by the Circumvolution of this Figure about AB, or 'AD; or the Center of Gravity of the said Figure. I have 'also found out the Tangents in any Points whatever of the 'Curve BED; as for the Point B, you must only draw from 'B a Line parallel to AD, but for the other Points C, D, 'you must suppose the Squaring of the Circle. I have also 'shewn, that the Curve BED may be continued *in Infinitum* 'both upwards and downwards, and that it goes winding, 'and is compris'd between Two Parallels. I have also abso- 'lutely Squar'd the Figure which is comprehended by the 'Curve BD continued till its Axis becomes Twice the *Radius*,

AB.

Plate 19. *Fig.* 91. 'AB. I have demonstrated what Respect this Curve has to 'the Cycloid, and some other Things, concerning which, I 'have wrote a Treatise of Thirty Propositions, which I 'shall be very proud of sending to you, if you have a mind 'to see it. You may judge of it by this Specimen which I 'send you, *viz.* Two Demonstrations, the One concerning 'the Space ABED, and the Other concerning the Solid which 'is made by the Circumvolution of the Space ABED about 'the line AD.

Plate 19. *Fig.* 92. 'Let then the Curve BED be Generated by the Quadrant 'ABCD, after the manner explain'd in the Mathematical 'Dictionary, *Page* 99. dividing the *Radius* AD into any Num-'ber of equal Parts at the points F, and the Arch BCD into 'as many equal Parts at the point C, and drawing from the 'points F the lines FE parallel to AB, and from the points C 'the lines CE parallel to AD, and describing the Curve BED, 'thro' the points E where those Lines meet.

'From this Generation one may, at the very first, see that the 'Property of the Curve BED is, that if you draw what Ordinate 'you will, as EF, to the Line AD, and from the Point E, EC 'parallel to AD, in such manner that it cuts the Arch at C, 'then will AD: be to DF: : as the Arch BD: is to the Arch 'DC: Whence it follows, that as DF: is to DF: : so is the Arch 'DC : to the Arch DC. It is also evident, that the Line EF is 'equal to CG the Right Sine of the Arch CD, and therefore 'as EF: EF: : so are the Sines CG: CG.

'This being suppos'd, I say, *That the Figure ABED: is to the 'Square of AB: : as the* Radius *AD: is to the Arch BCD.*

'Let the Quadrant ABCD be turn'd round about AD; in 'such a case each Sine CG will describe a Circle, and the 'Circumference of those Circles will be to one another as 'their *Radii* CG, CG, that is, as EF, EF. Since then the 'Arches DC, DC, to which we may suppose the Circumfe-'rences applied, are in the same *Ratio* as the lines DF, DF, to 'which we may suppose the lines EF, EF, to be applied; it 'follows, *by the Method of Indivisibles*, (and it might be 'easily demonstrated by the Method of the Ancients) that 'the Sum of the Lines EF, EF; that is, the Figure ABED; is 'to the sum of the Circumferences, that is, to the Surface of 'the Hemisphere, in a *Ratio* made up of the *Ratio* of the line 'AD (which is the Height of the Figure ABED) to the Arch 'BC C (which determines the Height of the Surface of the 'Hemisphere) and that of One of the *Radii* EF, to its Cir-'cumference. As I speak to a Great Geometrician, I think 'it needless to explain my self any farther.

'Then I reason thus; the Figure ABED: is to the Square 'A B: : in a *Ratio* made up of these Two

of

Of the Ratio *of the Figure ABED: to the Surface of the Hemisphere,* *Plate* 19, *Fig.* 92.
And of that of the Hemisphere: to the Square AB.

'Now the first *Ratio* of these Two is, as we have before said, 'compounded of these Two,

Of the Ratio *of the Line AD: to the Arch BCD,*
And of the Ratio *of the* Radius: *to its Circumference,*

'and the *Ratio* of the Surface of the Hemisphere to the Square 'AB, is the same as that of the Circumference to its *Radius*, 'as it is easy to demonstrate by the Principles of *Archimedes*. 'Therefore the *Ratio* of the Figure ABED, to the Square AB 'is made up of these Three,

Of that of the Line AD: to the Arch BCD,
Of that of the Radius: *to the Circumference,*
Of that of the Circumference: to the Radius.

'Now these Two last make up the *Ratio of Equality*. The '*Ratio* then of the Figure ABED: to the Square AB:: is the 'same as that of the *Radius* AD: to the Arch BCD. *Q. E.D.* 'Thus is Mr. *Tschirnhaus*'s first Theorem true.

'The second Demonstration which I send you, Sir, is con'cerning the Solid produc'd by the Circumvolution of the 'Figure ABED about AD.

'Let us then suppose the said Figure ABED to be turn'd 'round about the Line AD. I say, That the Solid which is 'produc'd by this Circumvolution; is to the circumscrib'd 'Cylinder:: as 1: to 2.

'1. Describe the Semi-circle AHB upon the Line AB, as *Fig.* 93. 'its Diameter. 2. Let the Right-angle BAD be divided into 'any Number of equal parts by the lines AC,AC,AC, which 'must meet the Circumference AHB, at the points H, H, H. 'The Arches DC, CC, CB, will then be equal. 3. From the 'points C, C, C, draw the lines CE, CE, CE, parallel to AD, 'which meet the Curve BED, at the points E,E,E, and thro' 'the points E,E,E, draw the lines EF,EF,EF, Ordinates to 'AD. The line AD will be divided at the points FFF into 'as many equal parts as the Arch BD by the Property of this 'Curve. 4. Make an end of the Rectangles FE,FE,AE, which 'will be inscrib'd in the Figure ABED. 5. From the Center 'A, taking the Chords AH, AH, AH for your *Radii*, draw 'the Sectors AHI, AHI, AHI. 6. Lastly, from one of the 'points C, draw the Sine CG, and from its correspondent 'point H, draw the line HB.

'This being suppos'd, each Chord AH is equal to each 'Ordinate which answers to it; for if you take, *for Example*, 'the least Chord AH, it will be easily demonstrated that it is 'equal to the Sine CG, because the Rectangular Triangles

'AHB, ACG are Equal and Similar, having the Angles HAB, 'ACG, equal (by reason of the Parallels AB, CG,) and the 'sides AB, AC, likewise equal. Now the Sine CG is equal 'to the Ordinate EF. Therefore the little Chord AH is 'equal to the little Ordinate EF; and the same thing may be 'demonstrated of the others.

Plate 19. Fig. 93. 'Let us now compare the Sectors AHI the one with the 'other, *for Example*, the least Sector AHI with that next to it. 'As the Angles HAI are equal, *by Constr.* the Sectors must be 'Similar; thus the little Sector AHI, is to the Next AHI, in 'a Duple *Ratio* of the little Chord AH, to the next Chord 'AH; that is, in a Duple *Ratio* of the little Ordinate EF 'to the next Ordinate EF; that is, as the Circle of the little '*Radius* EF to the Circle of the next *Radius* EF; that is, as 'the Cylinder made by the Circumvolution of the Rectangle 'FE, about FF, to the next Cylinder made by the next Rect-'angle FE: For these Cylinders having their Heights FF, FF, 'equal, are to one another as their Bases, that is, as the Circle 'of the little *Radius* EF, is to the Circle of the next *Radius* EF.

'Thus we shall prove that all the Sectors AHI, are to one 'another, as the Cylinders made by the Circumvolution of 'the Rectangles FE, AE, about AD, are to one another. 'Whence it follows, that all the Sectors together are to the 'greatest Sector, as all the Cylinders together are to the 'greatest Cylinder made by the Circumvolution of the Rect-'angle AE about AF. Now the greatest Sector AHI is to the 'Sector ABC which is compris'd under the same Angle BAC, 'in a Duple *Ratio* of the great Chord AH, to the *Radius* 'AB; that is, of the greatest Ordinate EF, to the Line FK, '(producing FE till it meets at K the line BL, which touches 'the Circle at B.) Therefore the greatest Sector AHI: is to 'the Sector ABC, which answers to it:: as the Cylinder 'made by the Rectangle AE: to the Cylinder made by the 'Rectangle AK. Lastly, the Sector ABC: is to the Qua-'drant ABD:: as the Arch BC: to the Arch BD; that is, as 'the Line AF: to the Line AD; that is, as the Cylinder made 'by the Rectangle AK: to the Cylinder made by the Rect-'angle AL, about AD.

'It follows from all this, that, *ex æquo*, all the Sectors to-'gether AHI: are to the Quadrant ABD:: as all the Cylinders 'made by the Rectangles EF, AE: are to the Cylinder made 'by the Square AL. Now it is plain that the Sectors may be 'so multiplied, that *desinent in Semicirculum AHB*, and the 'Cylinders may be so multiplied, that *desinent in Solidum factum* '*ex Figura ABED circa AD, in orbem ducta*. Therefore the Semi-'circle AHB: is to the Quadrant ABCD:: as the Solid pro-'duc'd by the Circumvolution of the Figure ABED about

AD;

'AD : is to the circumscrib'd Cylinder made by the Circumvolution of the Square AL about the said line AD. *Plate* 19. *Fig.* 93.
'Now the Semi-circle AHB is half of the Quadrant ABCD, 'as it is easy to demonstrate. Therefore the Solid made by 'the Circumvolution of the Figure ABED about AD, is 'half of the Cylinder circumscrib'd. *Q. E. D.*

'I send you this second Demonstration, Sir, because I 'fancy that it must be this Solid made by a Revolution about 'AD, that Mr. *Tschirnhaus* speaks of, because it has so exactly the *Ratio* of 1 to 2, to the Circumscrib'd Cylinder; 'and besides it is easy to mistake these Solids made by the Figure ABED one for another, by reason that the Two *Radii* 'AB, AD, are equal. Be so kind as to consult Mr. *Tschirnhaus* 'about it, and send me word, whether my Conjecture be 'true. If you find that he speaks of the Solid made about 'AB, and that he says as you have wrote, that such a Solid : 'is to the Circumscrib'd Cylinder :: as 1 : to 2, his Theorem 'is certainly false; for it wou'd then follow that the Solid 'made about AB shou'd be equal to the Solid made about 'AD, which I have demonstrated to be false. I will send 'you the Demonstration when you please; it is suppos'd in 'my Method, that the Center of Gravity of the Figure 'ABED is known: and thus I determine its place.

'Let the point Z be the Center of Gravity of the Figure 'ABED; and let the Two lines XZ, YZ, be drawn thro' Z *Fig.* 91.
'parallel to AB, AD. I say, That the line AB is so divided 'at Y, that AY is equal to the fourth part of the Arch BD, 'and that AD is so divided at X, that AD : is to DX :: as 'the Arch BD : is to the *Radius* AD.

'This Letter grows too long, therefore I'll make an end, 'assuring you, Sir, that the Excellent Pieces which you have 'publish'd, have given me such an Esteem of your Merit, 'that I shall look upon it as a great Obligation to keep a Correspondence with you concerning Geometrical Subjects; 'This will be so advantageous to me, that I cannot but 'earnestly wish it. When you will vouchsafe me the Honour 'of a Letter, give it to Friar *Brotes*, who lives at the *Maison* '*Professe*, and he will take care to help me to it, and likewise 'to deliver you mine. I expect with Impatience the great 'Treatise of *Algebra*, which you have promis'd to publish; 'coming from you it cannot but be Extraordinary Good. For 'my part I am going on with a Treatise of *Conchoides* and '*Cissoides*, which is already very forward, and which I have 'discontinued only this Fortnight, that I have been meditating upon this Curve of Mr. *Tschirnhaus*. I am, *&c.*

We shall at the End of the next Section, give the Demonstration of the foregoing Method, to find the Center of

Gravity of the Figure ABED, in another Letter by the Reverend Father *Nicholas*, by which it will appear more than by the foregoing, how Strong his Genius is, and how Deep are his Speculations in Geometry.

SECTION II.

PROBLEM.

Of the Center of Gravity of Plains.

THO' there is no Plain but what is joyn'd to a Body; yet one may consider a Body which is Flat, Homogeneous, of an equal and insensible Thickness all over, as a Plain, if we only take notice of its Length and Breadth; and we may allow it Weight and a Center of Gravity, which we will teach how to find in the following Propositions.

PROPOSITION I.

THEOREM.

The Center of Gravity of a Parallelogram is in some one Point of that Line, which passes thro' the middle of the Two Opposite sides.

Plate 18. *Fig.* 82. IF AB and CD the Two opposite sides of the Parallelogram ABCD, be divided into Two equal Parts at the Points E, F; I say, That the Center of Gravity of that Parallelogram ABCD, is in some Point of the Line EF.

DEMONSTRATION.

If you imagine within the Figure ABCD, an infinite Number of Lines parallel to one another, and likewise to the sides AB, CD, they will be equal to one another, and equally divided, and the Center of Gravity of each will be in the line EF, because that Center is in the middle of each of 'em *by Defin.* 6. Wherefore the Common Center of Gravity of all those Lines taken together, or of the Parallelogram ABCD, must also be in the Line EF. *Q. E. D.*

PROPOSITION II.

PROBLEM.

How to find the Center of Gravity of a given Parallelogram.

Fig. 83. LEt ABCD be the Parallelogram given, whose Center of Gravity is to be found. Draw the Two Diagonals AC, BD, and E the Point where they intersect will be the Center Gravity of ABCD, the Parallelogram propos'd.

DE-

DEMONSTRATION.

If the Sides be divided into Two equal Parts at the points F, I, G, H, it will be known *by Prop.* 1. that the Center of Gravity of the Parallelogram ABCD, is in the line FG, and also in the Line HI. Whence it is easy to conclude, that it is in their common Section; that is, in the Point E. *Q. E. I. & D.* *Plate* 18. *Fig.* 83.

PROPOSITION III.

THEOREM.

The Center of Gravity of a Triangle, is in a Line which passes thro' one of its Angles, and the side which is opposite to it.

IF the side AC of the Triangle ABC be divided into Two equal Parts at the Point D, and that from the Opposite Angle B be drawn the Line BD; I say, that the Center of Gravity of the Triangle ABC is in the Line BD.

DEMONSTRATION.

If you imagine within the Triangle ABC an infinite Number of Lines parallel to one another, and likewise to the side AC, they will all be divided into Two equal Parts by the Line BD, and consequently the Center of Gravity of each will be in the Line BD. Wherefore the common Center of Gravity of all those Lines taken together, or of the Triangle ABC, will be in the Line BD. *Q. E. D.* *Fig.* 85.

COROLLARY.

It is evident from this Proposition, that if a Right-line be drawn from one of the Angles of a Triangle, as the Angle B of the Triangle ABC, thro' its Center of Gravity G, that Right-line as here BD, will divide the opposite side AC, into Two equal Parts at the Point D. *Fig.* 88.

PROPOSITION IV.

PROBLEM.

How to find the Center of Gravity of a given Triangle.

LET ABC be the Triangle given, whose Center of Gravity is to be found. Divide Two of the sides, as AB,

Plate 18. Fig. 88. AC, each into Two equal parts at the Points F, D, and from the opposite Angles C, B, draw the Right-lines CF, BD, and the Point G where they intersect will be the Center of Gravity requir'd, since *by Prop.* 3. it must be in each of the Lines BD, CF.

COROLLARY.

Hence it follows, that if from the Three Angles of a Triangle, as many Right-lines be drawn thro' the middle of their opposite sides, those Three Lines will intersect within the Triangle in the same Point, *viz.* in the Center of Gravity of the Triangle.

SCHOLIUM.

Fig. 89. One may find out the Center of Gravity of the given Triangle ABC, another way, because the Part DG is equal to half the other Part BG, or to the Third Part of the whole Line BD. For if from the Points C, D, the Lines CH, DI, which meet the side AB produc'd at I, and H, be drawn parallel to the Line AE, it will be known that the Triangles ABE, HBC, are Equiangular and Similar, and consequently, that the Two Lines AB, AH, are equal, because EB and EC are equal: And likewise that because of the Similar Triangles ADI, ACH, and of the Two equal Lines DA, DC, the Two IA, IH are also equal, and consequently that the Line AI is equal to half the Line AH or AB, or to the Third Part of the whole Line BI; and because BGA, BDI are Similar, the Line DG will also be equal to the third Part of the Line BD. *Q. E. D.*

Fig. 88. If then the Line DG be taken equal to the third Part of the Line BD, you will have at G the Center of Gravity of the Triangle ABC; which you may have also another way, *viz.* by taking AH equal to the third Part of the side AB, and likewise CI equal to the third Part of the side BC, and joyning HI, whose middle Point G will be the Center of Gravity requir'd.

PROPOSITION V.

THEOREM.

The Center of Gravity of a Trapezoid is in the Right-line, which divides each of its parallel Sides equally.

If the Two parallel sides AB, CD, of the Trapezoid ABCD be divided each into Two equal Parts at the Points E, F; I say, That the Center of Gravity of that Trapezoid is in some Point of the Line EF.

DE-

DEMONSTRATION.

If by Thought you draw an infinite Number of Lines within the Trapezoid ABCD, which are parallel to one another and to the Two sides AB, CD, they will all be equally divided by the Line EF, and the Center of Gravity of each will be consequently in the Line EF. Wherefore their Common Center of Gravity, that is, the Center of Gravity of the Trapezoid will be also in the Line EF. *Q. E. D.* *Plate* 18. *Fig.* 90.

PROPOSITION VI.

PROBLEM.

How to find the Center of Gravity of a given Trapezium.

If the Trapezium propos'd be a Trapezoid, as ABCD, whose Two Opposite sides AB, CD, are parallel; each of those sides AB, CD, must be divided into Two equal Parts at E, F, and the Two others AD, BC, into Three equal Parts at I, G, H, K, then if Right-lines be drawn as you see in the Figure, the Point L will by *Prop.* 6. be the Center of Gravity of the Triangle ACD, and the Point M the Center of Gravity of the Triangle ACB; wherefore by *Prop.* 1. *Sect.* 1. the common Center of Gravity of these Two Triangles ACD, ACB, that is, the Center of Gravity of the Trapezoid ABCD will be in the Line LM: and as it is also in the Line EF, *by Prop.* 5. it will be at O their Point of intersection. *Plate* 19. *Fig.* 94.

But if the Trapezium has no parallel sides, as ABCD, you must draw the Diagonals AC, BD, and *by Prop.* 4. you will find the Center of Gravity E of the Triangle ABD, and the Center of Gravity G of the Triangle DBC, and then you may *by Prop.* 1. *Sect.* 1. know that the common Center of Gravity of those Two Triangles ABC, DBC, or the Center of Gravity of the Trapezium ABCD is in the Line EG. After the same manner you may know, that if the Center of Gravity F of the Triangle ABC, and the Center of Gravity H of the Triangle DAC be found, the common Center of Gravity of these Two Triangles ABC, DAC, or the Center of Gravity of the Trapezium ABCD is in the Line FH; whence it is easy to conclude that it is in the Point O, where EG and FH intersect. *Fig.* 95.

PROPOSITION VII.

PROBLEM.

How to find the Center of Gravity of a given Polygon.

Plate 20. *Fig.* 97. IF the propos'd Polygon be Regular, it is plain that its Center of Gravity is the ſame as the Center of the inſcrib'd, or circumſcrib'd Circle, that is, the ſame as the Center of the Polygon, and there needs no particular demonſtration of it.

But if the given Polygon be Irregular, as the Pentagon ABCDE, it muſt be reduc'd into Triangles by the Diagonals DA, DB, which may be drawn from what Angles you pleaſe, and *by Prop.* 4. the Center of Gravity I of the Triangle ADE muſt be found, and *by Prop.* 6. the Center of Gravity G of the Trapezium ABCD, and then it will be known *by Prop.* 1. *Sect.* 1. that the Center of the Pentagon ABCDE is in the Line IG. Likewiſe H the Center of Gravity of the Triangle BDC, and F the Center of Gravity of the Trapezium ABDE muſt be found, and it will be known after the ſame manner, that the Center of Gravity of the Pentagon ABCDE is in the Line FH. Whence it is eaſy to conlude, that it is in the Point O, where the Lines IG and FH interſect.

COROLLARY.

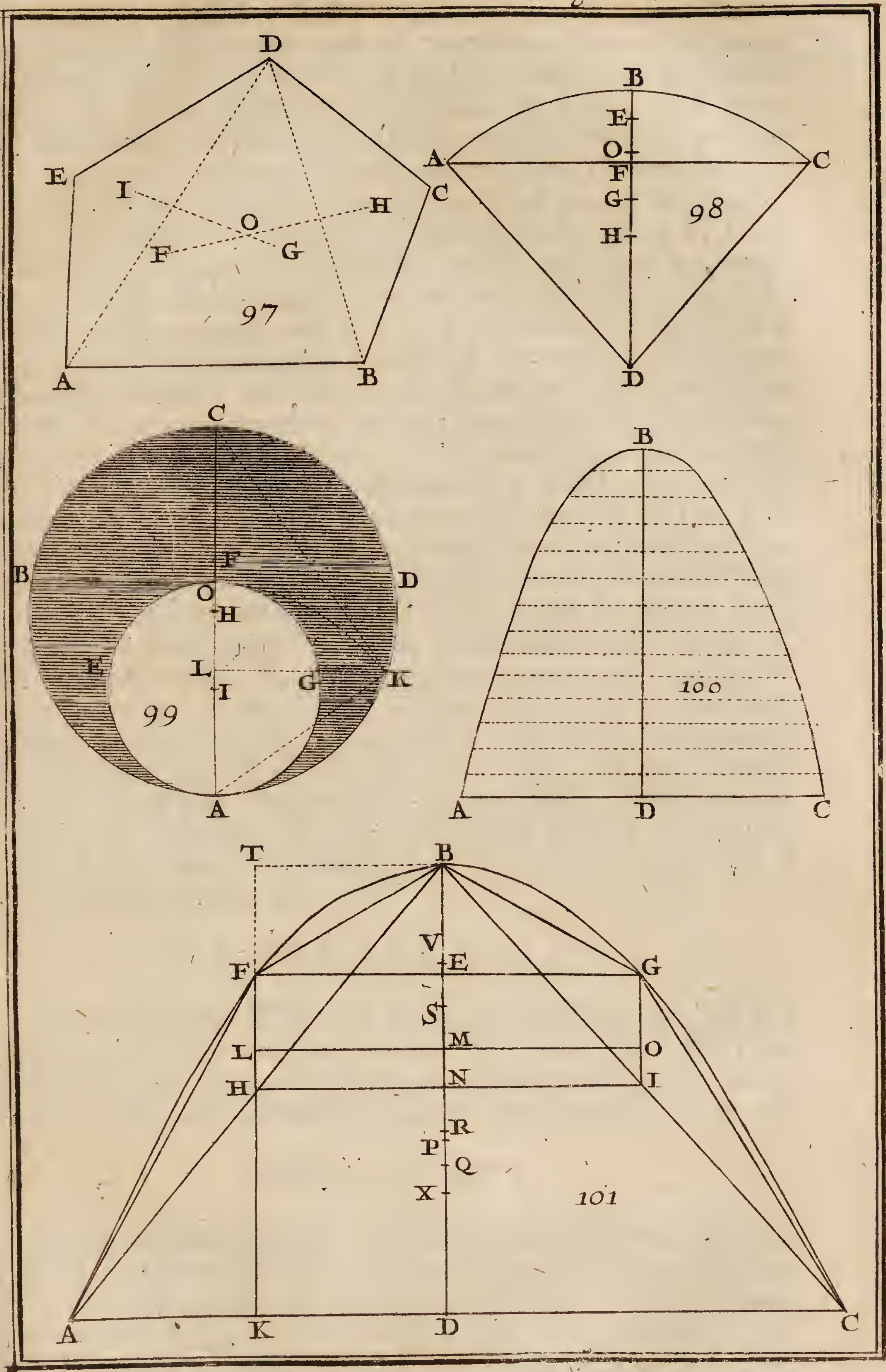
Thus has O, the Center of Gravity of the propos'd Pentagon ABCDE, been found; and after the ſame manner may the Center of Gravity of any other Polygon be found, *viz.* if it be always Twice reduc'd into Two parts, to joyn their Centers of Gravity by Two Right-lines, which will in their Point of Interſection give the Center of Gravity of the Figure propos'd.

PROPOSITION VIII.

THEOREM.

An Arch being divided into any Evenly even Number of ſmall equal Arches, the Center of Gravity of the Figure made by the Chords of all thoſe little Arches and the Two Radii *drawn from the Two Extremities, is diſtant from the common Center of Gravity of all thoſe Chords, One third part of the diſtance from the ſaid common Center of Gravity of the Chords to the Center of the Circle.*

Plate 19. *Fig.* 96. DIvide the Arch ABC, whoſe Center is D, into what Evenly even Number of equal parts you pleaſe, as into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into Four at the Points E,B,F, and having drawn the Chords AE, EB, BF, FC, divide each of them into Two equal parts at the Points G, H, I, K, which will be their Centers of Gravity, and if the Right-lines GH, IK, be joyn'd and their middles L, M, by the Line ML, its middle point N will be the common Center of Gravity of the Four Chords AE, EB, BF, FC. Then make CP equal to the third part of the *Radius* CD, and from the Center D draw thro' the point P, POS the Circumference of a Circle, which will give as many little Chords equal to one another, *viz.* SR, RO, OQ, QP, whose middle points are 1, 2, 3, 4. by means of which you will find T the common Center of Gravity of those Four Chords as before: and this second Center of Gravity will also be the Center of Gravity of the Rectilineal Figure AEBFCDA; for since CP is the third Part of CD, or FQ the third part of FD, and consequently K4 the third Part of KD, the middle point 4 of the line PQ, is the Center of Gravity of the Triangle CDF, *by Prop.*4. and likewise the point 3 will be the Center of Gravity of the Triangle FDB, and consequently the middle point 6 of the line 3, 4, is the common Center of Gravity of the Two equal Triangles CDF, FDB, or the Center of the Trapezium BDCF. After the same manner you may know that the point 5 is the Center of Gravity of the Trapezium ADBE equal to the foregoing BDCF, and that consequently the middle point T of the line 5, 6, is the common Center of Gravity of the Two equal *Trapezia* ADBE, BCDF, or the Center of Gravity of the rectilineal Figure ADCFBE. This being done and suppos'd, I say, That the line NT is the third part of the line ND. *Plate* 19. *Fig.* 96.

DEMONSTRATION.

Because the line CP is the third part of CD, or FQ the third part of FD, and the line K 4 the third part of KD; the line M 6 must also be the third part of MD, and consequently the line NT will be the third part of the line ND. *Q. E. D.*

COROLLARY.

From this Proposition it is evident, that the Center of Gravity of the Sector ADCB, is distant from the Center of Gravity of its Circumference ABC, the third part of the distance between the Center of Gravity of the Circumference and the Center of the Circle. For if, by Thought, you divide the Arch ABC into an infinite Number of equal parts, N the Center of Gravity of those Innumerable Chords taken together will be the same as that of the Circumference ABC, and

Plate 19. Fig. 96. and T the Center of Gravity of the Figure ADCFBE will be the ſame as that of the Sector ADCB. Whence it follows, That the diſtance of the Center of Gravity of a Sector, is equal to the Two third parts of the diſtance of the Center of Gravity of its Circumference, if thoſe Two diſtances be reckon'd from the Center of the Circle.

PROPOSITION IX.

PROBLEM.

How to find the Center of Gravity of a given Sector of a Circle.

Fig. 96. TO find the Center of Gravity of the Sector ADBC, whoſe Center is D, you muſt *by Prop.* 15. *Sect.* 1. find the Center of Gravity N of the Circumference ABC, and make the line NT equal to the third part of the line ND, that you may at T have the Center of Gravity of the propos'd Sector ABCD, as it is evident, *by Coroll. Prop.* 8.

SCHOLIUM.

If the propos'd Sector be a Semi-Circle, one may make uſe of this ſhort Method to find its Center of Gravity. *To a Line equal to a Quarter of the Circumference of the Circle : to the* Radius :: *and to the Two third parts of the* Radius : *find a Fourth proportional, which will be the diſtance from the Center of Gravity of the Semi-circle propos'd to the Center of the ſaid Semi-circle.*

We ſhall not give a Rule for finding the Center of Gravity of a whole Circle, becauſe it is ſo plain that the Center of Gravity is the ſame as the Center of the Circle, that it needs not be demonſtrated.

PROPOSITION X.

PROBLEM.

How to find the Center of Gravity of a Segment of a Circle.

TO find the Center of Gravity of the Segment ACB, whoſe Center is D, you muſt, *by Prop.* 15. *Sect.* 1. find the Center of Gravity E of the Circumference ABC, and having taken EG equal to the third part of ED, upon the *Radius* BD, (which divides into Two equal parts at Right Angles the

Chord

Chord AC at the point F) to have at G the Center of Gravity of the Sector ADCB, *by Prop.*9. and having also made FG equal to the third part of FD, to have at H the Center of Gravity of the Triangle ADC, *by Prop.* 4. to the Segment ACB : to the Triangle ACD :: and to GH the distance from the Center of Gravity of the Sector, to the Center of Gravity of the Triangle : find a fourth Proportional, which will be GO, that you may have at O the Center of Gravity of the given Segment ACB, the Demonstration of which is evident *by Prop.*7. *Sect.* 1. *Plate* 20. *Fig.* 98.

PROPOSITION XI.

PROBLEM.

How to find the Center of Gravity of a Lunula.

WE call *Lunula* a Plain terminated by the Circumferences of Two Circles, which touch one another on the inside, as the Figure comprehended between the Circumferences of the Two Circles AEOG, ABCD, which touch on the inside at the point A, thro' which and also the Centers H, I, of the Two Circles the Line AC is drawn, to find in it the Center of Gravity of the propos'd *Lunula* after this manner. *Fig.* 99.

It is plain, that to find the Center of Gravity of this *Lunula*, we need only, *by Prop.* 7. *Sect.*1. find the Center of Gravity of the difference of the Two Circles AEFG, ABCD. But to come to the Practice ; find a fourth Proportional to the *Lunula* : the Little Circle AEFG :: (or to the difference of the Squares of the Diameters AC, AO : to the Square of the little Diameter AO ::) and to the distance IH of the Centers I, H : which will be HF, and at F you will have the Center of Gravity of the propos'd *Lunula*.

SCHOLIUM.

If in the great Circle you inscribe the Line AK equal to the little Diameter AO, and draw the line CK, the Angle AKC will be a Right, *by* 31.3. and *by* 47.1. the Square CK will be the first Term of the foregoing Proportion, and the Square AK or AO, the second : and if instead of those Two Squares you wou'd have Two lines in the same *Ratio*, you must from K draw KL perpendicular to the Diameter AC, and then the lines AC, CL, will be in the same *Ratio* as the Two Squares CK, AK, by reason of the Three Proportionals AC, CK, and CL ; as it is evident *by* 8, 6, &c.

PRO-

PROPOSITION XII.

THEOREM.

The Center of Gravity of a Conick Section is in its Diameter.

Plate 20. Fig. 100. LET us, for Example, propose the Parabola ABC, terminated by AC Ordinate to the Diameter BD, which divides it into Two equal parts at D. I say, That the Center of Gravity of the Parabola ABC is in some point of the Diameter BD.

DEMONSTRATION.

If you imagine within the Parabola ABC an infinite Number of Lines parallel to one another, and also to the Ordinate AC, they will all be Ordinates to the Diameter BD; that is, they will all be divided into Two equal parts by the Diameter BD, and the Center of Gravity of each of them will consequently be in that Diameter BD; wherefore the common Center of Gravity of all those lines, or the Center of Gravity of the Parabola ABC, will be in the Diameter BD. *Q.E.D.*

PROPOSITION XIII.

THEOREM.

If upon any Number of Ordinates to the same Diameter of a Conick Section Triangles be describ'd, Each of which has its Vertex at the top of the Conick Section, every one of those Triangles, and every Trapezium, which will be in the Conick Section, will have its Center of Gravity in the Diameter of the said Conick Section.

Plate 20. Fig. 101. LET ABC, *for Example*, be the Parabola propos'd, whose Diameter is BD, to which we must draw, *for Example*, the Two Ordinates AC, FG, to have the Two Triangles ABC, FBG, and the Trapezium AFGC. I say, That the Center of Gravity of that Trapezium, and that of each of the said Triangles is in the Diameter BD.

DE-

DEMONSTRATION.

Becaufe the Two Bafes AC, FG, of the Triangles ABC, FBG are each divided into Two equal parts by the Diameter BD, the Center of Gravity of each of thofe Triangles will *by Prop.* 3. be in the Diameter BD. *Which is one of the Two Things to be demonftrated.* Becaufe AC, FG, the Two Oppofite and Parallel fides of the Trapezium AFGC will be divided each into Two equal parts by the Diameter BPD, the Center of Gravity of this Trapezium or Trapezoid AFGC will *by Prop.* 5. be in the faid Diameter BD. *Which was left to be demonftrated.* *Plate* 20. *Fig.* 101.

COROLLARY.

It is evident from this Propofition, that the Rectilineal Figure AFBGC, which arifes from the great Number of Ordinates to the Diameter BD, has alfo its Center of Gravity in the Diameter BD, becaufe that Rectilineal Figure is made up of Triangles and *Trapezia*, all which Figures have their Centers of Gravity in the Diameter BD.

Whence it follows, that the more Ordinates are drawn in the Conick Section, the more fides will this Rectilineal Figure have, and confequently it will come nearer and nearer to the Conick Section, infomuch that it will be equal to it when the Number of the Ordinates becomes Infinite: And as this Figure has always its Center of Gravity in the Diameter BD; what has been already demonftrated muft needs follow, *viz.* That the Conick Section ABC has alfo it Center of Gravity in the Diameter BD.

PROPOSITION XIV.

THEOREM.

The Centers of Gravity of any Two Parabola's divide likewife their Diameters.

TO demonftrate that in any Two Parabola's of the fame Kind the Centers of Gravity divide the Diameters; it will fuffice to make the Conftruction in, and to Reafon concerning, One Parabola; and the Conclufions will agree with any other Parabola.

On AC, Ordinate to the Diameter BD of the Parabola ABC, defcribe the Triangle ABC, and divide the fides AB, BC

Plate 20. *Fig.* 101. BC, each into Two equal parts at H, I, to draw the line HI. Draw besides, thro' the points H, I, the lines FK, GI, parallel to the Diameter BD, and joyn FG, and the Four AF,FB,BG, GC; Take HL equal to the third part of HF; and likewise IO equal to the third part of IG, and lastly, DQ equal to the third part of BD to have at L the Center of Gravity of the Triangle ABF, at O that of the Triangle CBG, and at Q that of the Triangle ABC; and the point M will be the common Center of Gravity of the Two Triangles ABF, CBG: Wherefore the common Center of Gravity of the Three Triangles ABF, ABC, CBG, or the Center of Gravity of the Pentagon AFBGC will be in the line MQ, *by Prop.* 13. and to find it, MQ must be divided at R in such manner, that the Sum of the Triangles ABF, CBG: may be to the Triangle ABC:: reciprocally as QR: is to RM, and the point R will be the Center of Gravity of the Pentagon AFBGC. If the same Operation be perform'd in any other Parabola, one may in Either reason thus.

By the Property of the Parabola, the Square of AD; is to the Square of EF or KD its equal:; as BD; is to BE; and because AD is Twice KD, the Square of AD will be Four times the Square of KD, and consequently the line BD will also be Four times the Line BE.

Because AD is Twice KD, so also must AB be Twice BH, and consequently the Line BN will be Twice BE. Whence it is easy to conclude, that the Two lines BE, EN, are equal to one another; and that EN as well as BE, is the fourth part of BD.

Because MN is the third part of EN, and that EN is the fourth part of BD, it follows, that MN is the Twelfth part of BD, to which if you add ND the Half of BD, you will have MD equal to $\frac{7}{12}$ of BD, and consequently BM equal to $\frac{5}{12}$ of BD: and if from MD you take away DQ equal to the third part of BD, the Remainder will be MQ equal to the fourth part of BD. Thus the Three lines BE, EN, MQ, are equal.

If the line BT be drawn parallel to the Ordinate EF, and meets the line FH produc'd, at T; it will easily be known, that as the Two lines EB, EN are equal, so also will FT, and FH be equal to one another, and consequently the Triangles FBT, FBH, be likewise equal. Whence it follows, that the Triangle BTH, or its equal AKH, is double the Triangle BFH: and because the Triangle AFB is also double the Triangle BFH, by reason of the Base AB, which is double the Base BH, it fol-

follows that the Triangle AKH is equal to the Triangle ABF: and besides, because the Triangle AKH is the fourth part of the Triangle ADB, since the Base AD is double the Base AK, and the Height BD double the Height KH, it follows also, that the Triangle ADB is Four times the Triangle ABF, and consequently that the whole Triangle ABC is Four times the Sum of the Two equal Triangles ABF, CBG. Whence it follows, that the line MR is Four times the line QR, because their *Ratio* is equal to the *Ratio* of the Triangle ABC, to the Sum of the Two ABF, CBG. Wherefore if MQ be divided into Five equal parts, the Line QR will be one of them, and the line MR Four: and because QM is one Quarter of BD, the line BD will have Twenty parts, so that QR will be $\frac{1}{20}$ of BD, and MR $\frac{1}{5}$ of the same BD. *Plate 20. Fig. 101.*

If to the line QR equal to $\frac{1}{20}$ of BD, be added DQ which is $\frac{1}{3}$ of BD, and to MR equal to $\frac{1}{5}$ of BD be added BM equal to $\frac{5}{12}$ of BD; you will have DR equal to $\frac{23}{60}$ of BD, and BR equal to $\frac{37}{60}$ of BD. Thus you see that BR: is to RD:: as 37: to 23, which will be demonstrated after the same manner in all other Parabola's of the first Kind, such as is that which we speak of here, and it must always be understood so, when we use the Word *Parabola* alone.

Since then the Center of Gravity R of this Rectilineal Figure AFB GC divides likewise the Diameter of each Parabola, it will also divide the Diameter in a Rectilineal Figure of more sides, and consequently in a Rectilineal Figure of an infinite Number of sides, in which case it will be the same with a Parabola, and therefore its Center of Gravity must needs be the same as the Center of Gravity of a Parabola; that is, the point R will coincide with P the Center of Gravity of the Parabola, which consequently divides the Diameter BD proportionally. *Q. E. D.*

COROLLARY.

From what has been said it follows, That if once the Center of Gravity of one Parabola be found, it will be easy to find out that of another Parabola of the same Kind; because it always divides the Diameter into Two parts which are Proportional. What remains then is to shew how to find the Center of Gravity of a Parabola.

PRO-

PROPOSITION XV.

PROBLEM.

How to find the Center of Gravity of a given Parabola.

Fig. 101. TO find P the Center of Gravity of the Parabola ABC, whose Base is AC Ordinate to the Diameter BD; it will suffice to find out what Proportion BP, and DP bear to one another, because it is the same in all Parabola's, *by Prop.* 14.

Make a Construction like the former, except that the Points L, O, must be the Centers of Gravity of the Two Parabola's AFB, CGP, whose common Center of Gravity will consequently be in the point M. Then draw ES equal to the third part of BE, or of EN its equal.

This being done and suppos'd, it is plain *by Prop.* 14. that the Diameter BD, FG, of the Two Parabola's ABC, AFB, have the same *Ratio* to one another as PD and LH; and as it has been demonstrated that BD is Four times EN, or FG its equal; it follows that the part PD is also Four times LH, or MN its equal; and that the other part BP is also Four times the other part LF, or ME its equal, which being taken from BP, there will remain the Two lines BE, MP, which being taken together are the Triple of EM; and because ES is the third part of EN, or of EB its equal, you will have MS equal to the third part of PM, because of the equal lines EB, EN, and EM equal to the third part of BE + MP. Since then BD is Four times BE, and BE Three times ES, the line BS will be the third Part of BD; and because DQ is also the third part of BD, the lines DS, SQ, and QD must needs be equal.

Since then P is the Center of Gravity of the Parabola ABC, Q that of the Triangle ABC, and M the common Center of Gravity of the Two Parabola's AFB, BGC, the distance QP: will be to the distance PM :: reciprocally as the Sum of the Two Parabola's AFB, BGC : to the Triangle ABC. But because that Sum is the third part of the Triangle ABC by reason of the *Ratio* of the Triangle ABC to the Parabola ABC, which is as 3 to 4, as it is easy to know by what has been said in our Treatise of Geometry, it follows that the distance QP is the Third part of the distance PM. But it has been demonstrated before that MS is the third part of PM; therefore the lines QP, and MS are equal.

Thus to find the Center of Gravity P, of the Parabola ABC, you need only to make QP equal to MS. Now to find what Proportion BP and PD bear to one another, one must con-

consider that since MP is Three times QP, or MS, the whole line QS, or QD its equal, will be Five times the line QP: and because the line DQ is the third part of BD, it follows, that PQ is equal to $\frac{1}{15}$ of BD, to which line PQ if DQ the third part of BD be added, you will have DP equal to $\frac{2}{5}$ of BD, and consequently BP equal $\frac{3}{5}$ of BD. Thus you see, that the part BP ; is to the part PD : : as 3 : is to 2. Whence we draw this General Method to find the Center of Gravity of a given Parabola. *Divide the Diameter of the given Parabola into Five equal parts, and take Three from the Top, or Two from the Base, and you will have the Center of Gravity of the propos'd Parabola.* *Plate 20. Fig. 101.*

PROPOSITION XVI.

PROBLEM.

How to find the Center of Gravity of a Parabola truncata.

We call *Parabola truncata*, a part of a Parabola which is terminated by Two parallel lines, as AFGC, whose Two lines AC, FG, are divided into Two equal parts by the Diameter BD of the whole Parabola ABC. We may, on that Diameter BD, find the Center of Gravity of the Parabola truncata AFGC, having *by Prop.* 15. found the Center of Gravity V of FBG the Parabola added, and P the Center of Gravity of the great Parabola ABC; by finding a fourth Proportional to the Parabola truncata AFGC : the Parabola which is added FBG : : and the line VP : which will be PX, and the point X will be the Center of Gravity of the Parabola truncata AFGC, as it is plain *by Prop.* 7. *Sect* 1. *Fig. 101.*

SCHOLIUM.

One may more easily find this Center of Gravity X, if instead of the Two first Terms of the foregoing Analogy (*viz.* the Parabola truncata AFGC, and the Parabola which was added FBG,) you shou'd Substitute the Triangles, ADB, FEB : and the Triangle FEB which are in the same *Ratio.* Or else, without lengthning or finishing the Parabola truncata AFGC, one may instead of the Two foregoing Terms,

Substitute the difference of the Cubes of the Two Ordinates AD, EF: and the Cube of the Ordinate EF, which are likewise in the same *Ratio*, &c.

PROPOSITION XVII.

THEOREM.

If a Circle be describ'd about an Ellipsis and any Perpendicular be drawn to the great Axis, the Segments of the Circle and those of the Ellipsis will have the same Center of Gravity.

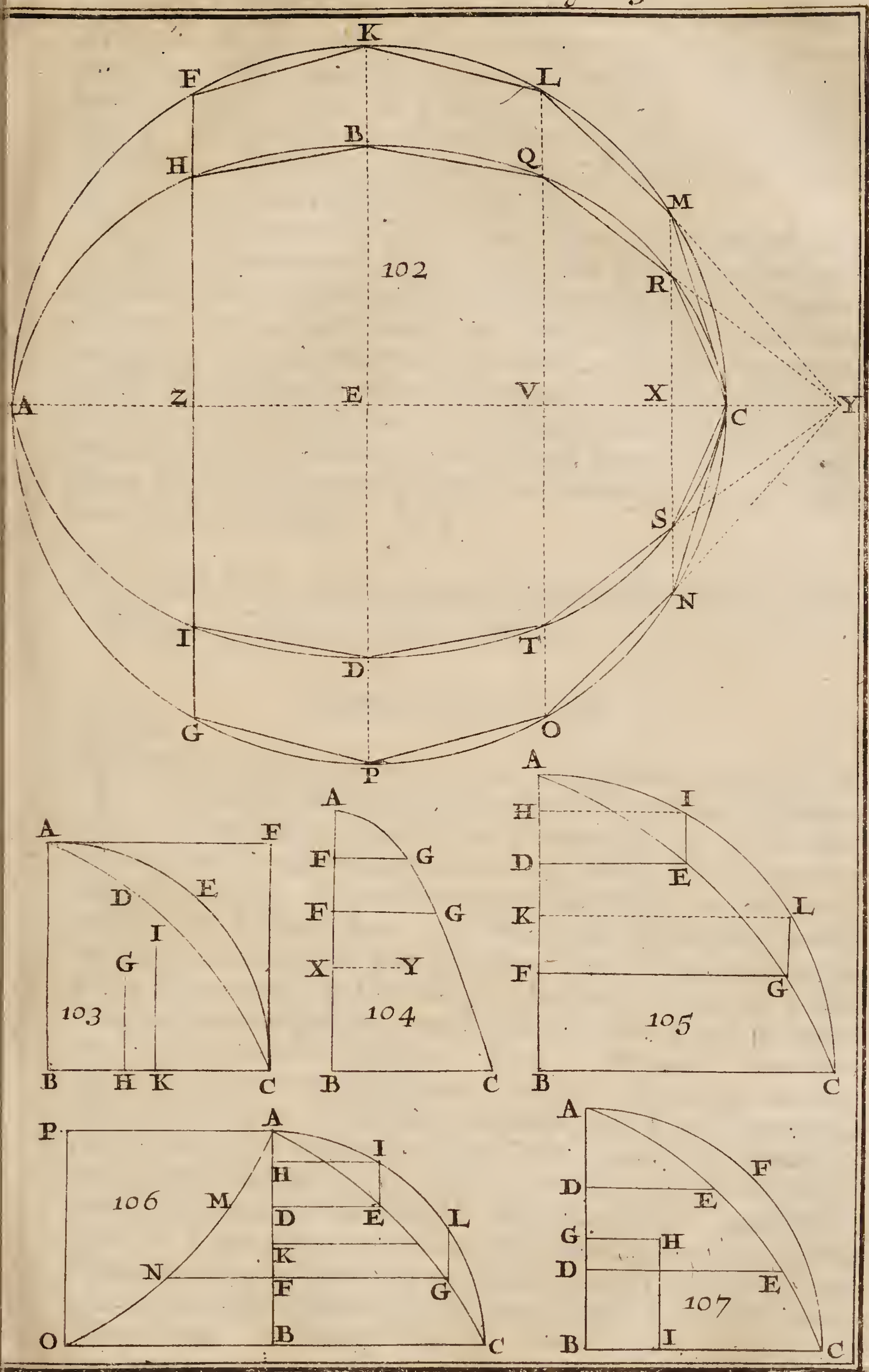
Plate 21. *Fig.* 102. I Say, That if thro' the point Z taken at pleasure on AC the great Axis of the Ellipsis ABCD, the Perpendicular FG be drawn, which determines the Segment of the Ellipsis HCI, and FCG the Segment of the Circle describ'd round the Diameter AC; those Two Segments have the same Center of Gravity.

PREPARATION.

Divide the Arch FCG into any Evenly even Number of equal parts, *as for Example*, into Eight, at the points KLMCNOP, to inscribe a Polygon in it, and joyn the opposite points equally distant from the line FG, by the lines KP, LO, MN, which will give on the Ellipsis the points B, Q, R, S, T, D, to have in the Ellipsis another Polygon of as many sides. Produce the Four lines LM, QR, ON, TS, till they cut one another in the same point of the Axis AC produc'd as far as it will be needful, as to Y; which will happen because of the Two lines MR, RX, which are equal to the Two NS, SX, and proportional to the Two LQ, QV, which Two last are equal to OT, TV, as it is evident by what has been said concerning the Ellipsis in our Treatise of Geometry.

DEMONSTRATION.

Having made this Preparation it will be easily known that the Two Isosceles Triangles MCN, RCS, having the same Height CX, have the same Center of Gravity; because it can only be in CX the common Height, which divides the Bases MN, RS into Two equal parts. One may after the same manner know that the Two Triangles LYO, QYT, have the same Center of Gravity, as well as the Two MYN, RYS. One may also know that the Two Trapezoids LMNO, QRST, have the same Center of Gravity; for since the Two Triangles LYO, QYT, have the same Center of Gra-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Gravity, if from them be taken the Two Triangles MYN, RYS, which have alſo the ſame Center of Gravity, the remainders which are the Trapezoids LMNO, QRST, will have the ſame Center of Gravity, as alſo the Two KFGP, HBDI. Whence it is eaſy to conclude, that a Polygon inſcrib'd in a Circle has the ſame Center of Gravity as a Polygon inſcrib'd in an Ellipſis. *Q. E. D.*

But if you conceive the Arch FCG to be divided into an infinite Number of equal Parts, the Correſpondent part of the Ellipſis will be alſo divided into an infinite Number of parts, and in ſuch a caſe the Polygon of the Circle will be equal to the Segment of the Circle, and the Polygon of the Ellipſis will be equal to the Segment of the Ellipſis: And as we have juſt demonſtrated that theſe Two Polygons have the ſame Center of Gravity, it follows, that thoſe Two Segments have alſo the ſame Center of Gravity, as well as the Circle and the Ellipſis. *Q. E. D.*

A Letter from the Reverend Father Nicholas, *of the Society of* Jeſus, *to the Author.*

Toulouſe, June *1ſt. 1691.*

SIR,

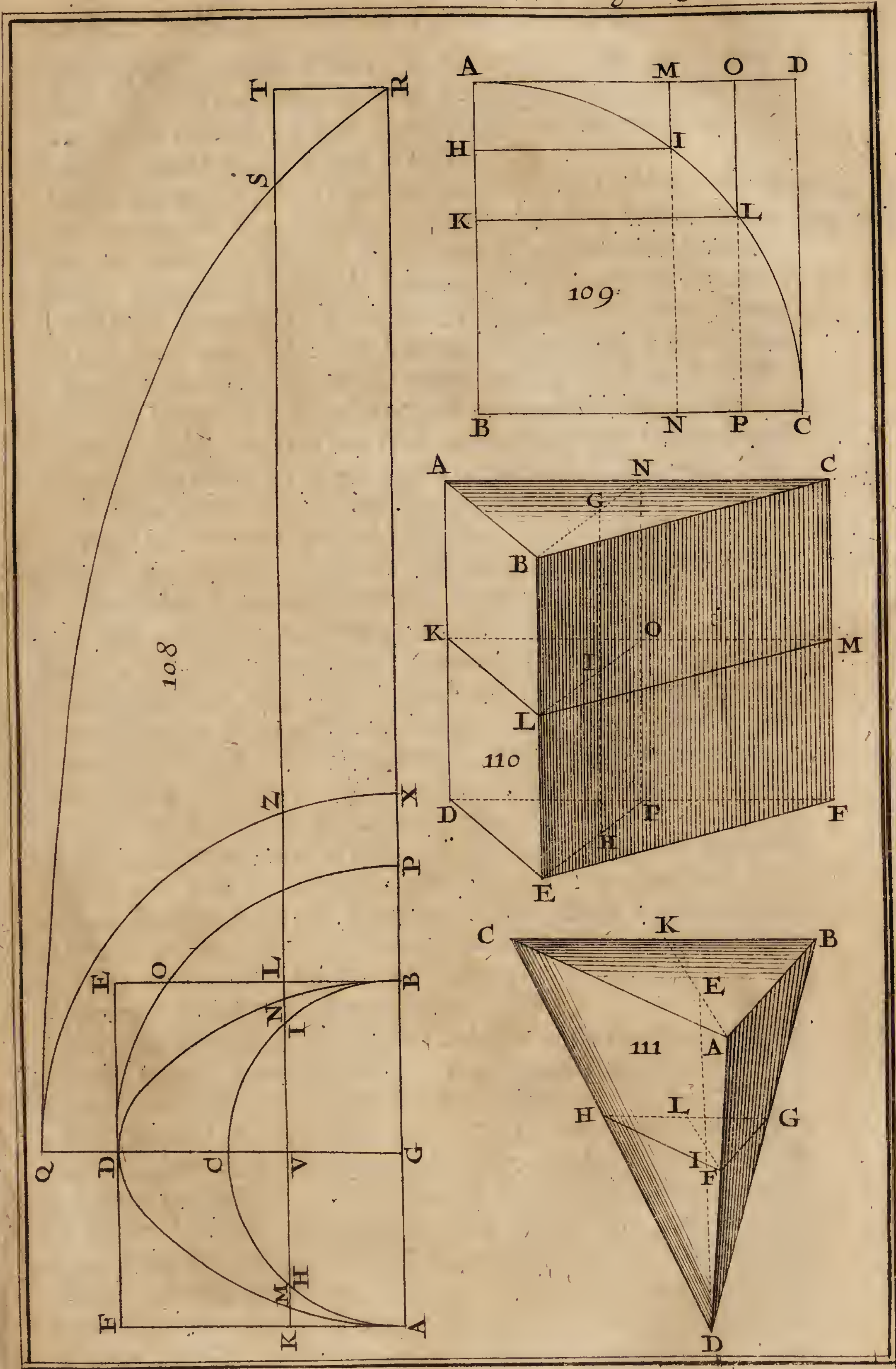
'YOUR Letter of the 5th of laſt Month came ſafe to 'my hands: I am oblig'd to You for all the Civilities 'in it, but eſpecially return You Thanks for the kind offer 'that You make me, to take care of my Books, if I have 'em 'Printed at *Paris*. It is a Favour which I have not deſerv'd, 'and beſides, I know how precious Your Time muſt needs 'be. How is it poſſible that You ſhould be but Eight Months 'about Your excellent Dictionary? It is a prodigious Work, 'which I think might very well take up Two Years. I never 'ſaw either *Slucius*'s *Meſolabus*, *Wallis*'s *Mechanicks*, or the 'Engliſh *Commercium Epiſtolicum*; I believe I ſhall find the laſt 'Book in Mr. *Fermat*'s Library, becauſe that great Mathema-'tician his Father, whom doubtleſs You know, did ſometime 'ago furniſh Matter for part of that Book. I'll ſee what he 'has concerning the Conchoid; and ſince *Slucius* and *Wallis* 'ſpeak of it but lightly and ſuccinctly, as You ſent me word, 'I find that what I have wrote about it is quite different from 'what they have done. My Work, which is divided into 'Three Books, is already finiſh'd. In it I treat, not only of 'the Conchoid of *Nicomedes*, which is the only one that has 'been known hitherto; but of all the other Conchoids, 'which may be made by other Figures, as by the Triangle, 'the Ellipſis, the Parabola, the Hyperbola, *&c.* I examine

'all their Tangents, their Quadrature, the Solids which are 'made as well round the Axis, as round the Base, and round 'the Center of Gravity. Among other things, I have de-'monstrated that fine Proposition, which Father *Lalouvere* 'has advanc'd in the second Appendix of his Cycloid with-'out giving the Demonstration of it; and where he says, 'that the Quadrature of the Conchoid of *Nicomedes* de-'pends upon the Quadrature of the Circle, and that of the 'Hyperbola. I should join to this Treatise of Conchoids 'another of Cissoids, where I treat not only of the Dioclean, 'which is the Cissoid of the Semi-circle, and concerning 'which I have made several fine Discoveries, but also of the 'Cissoids which may be form'd by other Figures. I join 'those Two Treatises together, because of the wonderful 'Affinity that I have discover'd betwixt the Conchoids and 'the Cissoids, so that the same Principles have serv'd me for 'both. I take *Wallis* to be the Person who has spoke more at 'large concerning the Cissoid in a Book which he Writ *de* '*Cycloide, &c.* Printed at *Oxford*, in 1659. But I have gone 'much beyond what that Book mentions.

'I come now to the Question that you propos'd to me 'concerning the Zones of the Hemisphere, of the Semi-'sphæroid and of the Cylinder. As for those of the Hemi-'sphere and of the Cylinder, it is evident that they are equal, 'and it may be easily demonstrated, because the Surface of 'the Hemisphere being equal to the Surface of the circum-'scrib'd Cylinder (taking away the Bases) and the parts of 'the Hemispherical Surface, as well as the parts of the Sur-'face of the Cylinder, being to one another as the parts of 'the Axis; it follows, that the Zones of the Hemisphere, 'and those of the Cylinder, are equal to one another. I 'need not say any more to You about this, because it is not 'properly what You ask, and You say that it has already been 'treated of. But You would have it demonstrated, that the 'Zone of the Semi-spheroid is also equal to the Two other 'Zones; to which I answer, that it can't be demonstrated; 'because the Zone of the Semi-spheroid is less than the 'other Two. For to make use of the Figure that You have 'sent me, I say that the Zone AMNB of the Semi-spheroid 'is less than the Zone AKLB of the Cylinder.

Plate 22. *Fig.* 108. 'Taking GD the greatest Semi-Axis of the Ellipsis ADB 'for my *Radius*, I describe the Quadrant GDP. From the 'point BI draw the Perpendicular BO, which meets the Arch 'DP at the point O. Let GQ be a third Proportional to the 'Two BO, GD. Let the line GP be produc'd to R, in such 'manner that GR may be equal to the whole Circumference 'of the Circle ACB. If you conceive the fourth part of an

Ellipsis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'Ellipsis GQR, whose Center is G, and which goes thro' 'Q, R; I say, That the line MN being produc'd so far that 'it may meet that Ellipsis at S; GVSR, the Segment of the 'Ellipsis is equal to the Zone AMNB.

'I have demonstrated this Proposition in a Treatise which *Plate* 22. 'I have wrote *de Superficiebus Rotundis*, which I shall some *Fig.* 108. 'Time or other publish. I do not send you this Demon-'stration, because it depends upon a great many Principles, 'and I shou'd be oblig'd to write out a great Part of the Trea-'tise, but you may take it upon my Word; for I have read 'my Book over again to be the more certain of it, and I have 'found the Demonstration very just and exact, and conform-'able to what Mr. *Hugens* has said concerning the Conoid and 'Spheroid Surfaces in his Book *de Horologio Oscillatorio*, *Pag.* '75. *& Seqq.* and *Wallis* in the forementionʼd Book *de Cycloide*, '&c. *Pag.* 98. *& Seqq.* tho' my Method is very different from '*Wallis*'s: As for Mr. *Hugens*'s Manner, he has not publish'd 'it. Now this Proposition being suppos'd, it is not hard to 'shew, that the Zone AMNB is less than the Cylindrical Zone 'AKLB. For the Cylindrical Zone is equal to a Rectangle 'whose Height is AK, or GV, and whose Base is equal to the 'Circumference of the Circle ACB. It is then equal to the 'Rectangle GVTR. The Rectangle GVTR therefore being 'greater than the Elliptical Segment GVSR, which is equal 'to the Zone AMNB; it follows, that the Cylindrical Zone 'AKLB is greater than the Zone of the Semi-spheroid AMNB.

'You are much in the right, Sir, to say that this Pro-'position concerning the Equality of the Zone of the Semi-'spheroid to the other Two Zones wou'd be of great Use; 'for if it was true, we shou'd have the Quadrature of the 'Circle as it is easy to demonstrate.

'Let the Quadrant GOX be drawn with the *Radius* GQ, 'and cut by the Line VS at Z. It is plain that the Elliptical 'Segment GVSR : is to GVZX the Segment of the Circle :: 'as GR : is to GX (Archim. *Prop. 6. de Conoid.*) Now GR is 'equal to the Circumference of the Circle ACB. GVSR 'therefore (which is the Elliptical Segment) being equal to 'the Zone AMNB, as we have said, if that Zone were equal 'to the Cylindrical Zone AKLB, which may be reduc'd to a 'Circle, it wou'd follow, that a known Circle : wou'd be 'to the Circular Segment GVZX :: as the Circumference of 'the Circle ACB : is to the known Right-line GX, and thus 'wou'd the Circular Segment GVZX be squar'd, which wou'd 'be sufficient for the squaring of the Circle. But as the 'Spheroidical Zone is not equal to the Cylindrical Zone, the 'Quadrature of the Circle is still to seek.

'Since you seem desirous to know what Method I make use 'of to find the Center of Gravity in Mr. *Tschirnhaus*'s Figure, 'you will find it in the Paper inclos'd, which I have left in 'Latin, because I copied it from a Latin Treatise, which I 'compos'd lately upon this Figure. I believe you won't be 'dissatisfied with it.

'If I can be serviceable to You in any thing, I beg You 'would employ me; I shall think it an Honour to oblige 'You, and take a particular Satisfaction in it. Permit me 'also to desire the Favour of your Friendship, which I hope 'You will not refuse to him who is sincerely

Yours, &c.

Methodus ad inveniendum Centrum gravitatis in nova Quadratrice D. Tschirnhaus.

Plate 21. Fig. 103. 'ESto nova Quadratrix ABCD genita ex Quadrante circuli 'ABCE, sitque punctum G Centrum gravitatis Figuræ 'ABCD. Ex G demittatur in BC perpendicularis GH. Dico 'BH esse æqualem quartæ parti arcus Quadrantis AEC. 'Compleatur Quadratum BF, sitque Quadrati BF Centrum 'gravitatis I, & per I demittatur in BC perpendicularis IK.

'Omne Solidum Rotundum genitum ex conversione ali'cujus Figuræ circa lineam rectam æquatur Solido recto 'cujus basis est ipsa Figura, altitudo autem æqualis viæ Rota'tionis, sive circumferentiæ descriptæ à Centro gravitatis in 'illa Rotatione Figuræ (ex principio generali quod traditum 'est a *Guldino* in Centrobaricis, & demonstratum à *Tacqueto* 'Lib. 5. Cylindricorum & Annularium.) Ergo Solidum ro'tundum genitum ex conversione Figuræ ABCD circa AB, 'æquatur Solido recto, cujus Basis est ipsa Figura ABCD, 'altitudo autem æqualis circumferentiæ Radii BH: & Cylin'der genitus ex conversione quadrati BF circa eamdem AB, 'æquatur Cylindro recto, cujus basis est ipsum quadratum 'BF, altitudo autem æqualis circumferentiæ Radii BK. Igitur 'Rotundum genitum ex Figura ABCD: se habet ad Cy'lindrum genitum ex Quadrato BF:: ut Solidum rectum, 'cujus basis Figura ABCD, altitudo circumferentia Radii 'BH: ad Solidum rectum, cujus basis Quadratum BF, altitudo 'circumferentia Radii BK. Solida autem recta sunt inter se 'in Ratione compositâ basium & altitudinum. Quare Ro'tundum ex Figura ABCD est ad Cylindrum ex Quadrato 'BF, in Ratione compositâ Figuræ ABCD, ad Quadratum 'BF, & circumferentiæ Radii BH ad circumferentiam Radii 'BK. Demonstratum autem est Figuram ABCD esse ad Qua'dratum BF, ut Radius BC est ad arcum Quadrantis AEC: '& circumferentia Radii BH est ad circumferentiam Radii 'BK,

'BK, ut ipse Radius BH est ad Radium BK. Ergo Rotundum 'ex Figura ABCD: est ad Cylindrum ex Quadrato BF :: in 'Ratione composita Radii BC ad arcum Quadrantis AEC, & 'rectæ BH ad rectam BK; sive secto arcu AEC bifarium, in E, 'in Ratione composita dimidiæ BC ad arcum AE & BH ad BK. *Plate* 21. *Fig.* 103.

'Cum autem KI transeat *ex hypot.* per Centrum gravi-'tatis Quadrati BF, BK est dimidia ipsius BC; Ergo Rotun-'dum ex Figura ABCD ad Cylindrum ex Quadrato BF, est 'in Ratione composita ex Rationibus BK ad AE, & BH ad 'BK, sive in Ratione BH ad AE, quæ ex illis composita est. 'Demonstratum est autem idem Rotundum ex Figurâ ABCD 'circa AB esse ad Cylindrum ex Quadrato BF circa eandem 'AB, ut 1 ad 2. Ergo BH est ad arcum AE, ut 1 ad 2: & 'cum arcus AE sit dimidia pars arcus Quadrantis AEC, BH, 'est ad arcum Quadrantis AEC, ut 1 ad 4. *Quod erat demon-'strandum.*

'Hinc habemus determinatam distantiam G Centri gravi-'tatis Figuræ ABCD à recta AB: sed paulo difficilius est de-'terminare distantiam ejusdem Centri gravitatis à recta BC, 'nec possumus uti Methodo priori, cum ad huc ignotum sit 'Rotundum ex Figura ABCD circa BC rotatâ. Aliâ igitur 'viâ nobis progrediendum est, quam sequentibus Proposi-'tionibus explicabimus.

'Supponimus primo Principium hoc universale ad inve-'nienda Centra gravitatis utilissimum.

'Si sit quæcumque Figura plana ABC contenta duabus 'rectis AB,BC, angulum rectum comprehendentibus, & linea 'AGC: sive recta sive curva: sint autem ex singulis punctis 'F, rectæ AB, ordinatæ FG, parallelæ BC; & intelligantur 'singula Segmenta AFG, AFG, erigi perpendiculariter supra 'singulas ordinatas FG, FG, & Segmentum ABC erigi simi-'liter supra ordinatam BC; ex hujusmodi Segmentis ita ere-'ctis, & insistentibus perpendiculariter Plano ABC, consti-'tuetur Solidum, cujus basis erit ipsa Figura ABC erecta, alti-'tudo autem AB. *Fig.* 104.

'Dico hujusmodi Solidum, quod est summa Segmento-'rum erectorum, esse ad aliud Solidum rectum, cujus basis 'est ipsa Figura ABC altitudo AB, ut BX recta, est ad rectam 'BA, posito quod XY recta parallela BC transeat per Cen-'trum gravitatis Figuræ ABC.

'Hoc Principium jam demonstratum est à D. *Pascal* sub 'nomine *Dettonville*, latentis in Tractatu quem edidit de Cy-'cloide; quod enim nos vocamus hic summam Segmentorum 'AFG, AFG, apud illum est summa Triangularis eorundem 'Segmentorum; quare superfluum est addere aliam ejusdem 'Principii demonstrationem Geometricam, quam invenimus, 'deduximusque ex Principio *Guldini* supra posito.

Plate 21. *Fig.* 104. 'Hinc autem conſtat ſi figura ABC ſupponatur eſſe nova 'Quadratrix D. Tſchirnhaus & ſupponatur XY parallela BC 'tranſire per illius Centrum gravitatis Y, ut habeatur Ratio 'BX, ad BA, ac proinde ipſa BX, quærendam eſſe Rationem 'ſummæ Segmentorum AFG, AFG, ABC, erectorum ſupra 'rectas FG, FG, BC, ad Solidum rectum, cujus baſis eſt ipſa 'Figura ABC, altitudo autem AB. Hanc autem rationem 'ex Lemmatibus ſequentibus deducemus.

LEMMA I.

Fig. 105. 'Eſto nova Quadratrix ABCE genita ex Quadrante circuli 'ABCI. Ducatur autem in Quadratrice quæcumque ordinata 'DE parallela BC, & ex E recta EI parallela AB occurrens ar-'cui Quadrantis in I: ſitque IH Sinus rectus arcus AI, ac proin-'de AH Sinus verſus ejuſdem arcus. Dico Segmentum ADE 'eſſe ad Quadratricem ABCE, ut AH eſt ad Radium AB.

'In Propoſitione quâ demonſtravimus, Quadratricem 'ABCE, eſſe ad Quadratum circumſcriptum, ut Radius AB, 'eſt ad arcum Quadrantis; Oſtenſum eſt Quadratricem 'ABCE eſſe ad ſuperficiem Hemiſphæricam genitam ex arcu 'Quadrantis AIC in Ratione compoſitâ Radii AB, ad arcum 'Quadrantis AIC, & Radii circuli ad circumferentiam. Eo-'dem autem plane modo oſtendetur Segmentum Quadrantis 'ADE, eſſe ad portionem ſuperficiei Sphæricæ deſcriptam ab 'arcu AI, in ratione compoſita AD ad arcum AI, & Radii 'ad ſuam circumferentiam. Cum ergo Rationes AD, ad 'arcum AI, & AB ad arcum AIC, ſint æquales ex pro-'prietate & generatione curvæ AEC, ac proinde ſit eadem Ra-'tio compoſita ex Rationibus AD ad arcum AI, & Radii ad 'ſuam circumferentiam, quæ componitur ex Rationibus AB 'ad arcum AIC, & Radii ad ſuam circumferentiam, ſequitur 'Segmentum ADE eſſe ad portionem ſuperficiei Sphæricæ ge-'nitam ex arcu AI, ut tota Quadratrix ABCE, eſt ad ſuperfi-'ciem Hemiſphæricam genitam ex arcu Quadrantis AIC, & 'permutando. Cum igitur portio ſuperficiei Sphæricæ genita 'ex arcu AI, ſit ad ſuperficiem Hemiſphæricam genitam ex 'arcu AIC, ut Sinus verſus AH eſt ad Radium, ut conſtat *ex* '*Archimede*, Segmentum ADE eſt ad Quadratricem ABCE, 'ut AH ad AB. *Quod erat demonſtrandum.*

COROLLARIUM.

'Hinc ſequitur, ductâ aliâ quâcumque Ordinata FG, & ex 'G, GL parallela AB, atque ex L, LK, Sinu recto arcus AL, 'Segmentum ADE, eſſe ad Segmentum AFG, ut AH Sinus 'verſus arcus AI, eſt ad AK Sinum verſum arcus AL: quod 'facilè colligetur *ex æquo*, comparando utrumque Segmen-

'tum

'tum ADE, AFG, cum totâ Quadratrice ABCE. Unde Segmenta Quadratricis sunt semper inter se ut Sinus versi arcuum Quadrantis proportionalium altitudinibus Segmentorum.

LEMMA II.

'Si concipiatur Sinus versus AH applicari in D, sive poni DM æqualis ipsi AH, ad angulos rectos AB, & Sinus versus AK applicari in F, sive poni FN æqualis AK, & Sinus totus AB applicari in B, sive poni BO ipsi æqualis, & ita applicentur omnes Sinus versi in punctis rectæ AB, in quibus secatur proportionaliter cum arcubus illorum Sinuum versorum, fiet nova Figura ABO, quæ vocetur *Figura plana Sinuum versorum.* *Fig: 106.*

'Dico hujusmodi Figuram planam Sinuum versorum ABO esse ad Rectangulum BP circumscriptum, ut summa Segmentorum ADE, AFG, ABC, erectorum, est ad Solidum rectum circumscriptum, cujus nimirum basis est ipsa Figura ABCE erecta, altitudo autem AB.

'Nam summa Segmentorum ADE, AFG, &c. erectorum nihil est aliud quam Solidum, cujus Sectiones sunt ipsa Segmenta ADE, AFG, &c. erecta perpendiculariter supra DE, FG, &c. ac proinde sibi ipsis parallela. Hujusmodi autem Segmenta sunt semper inter se ut Sinus versi AH, AK, (*Lem.* 1.) sive, ut ipsis æquales DM, FN. Cum igitur Sectiones Solidi illius sint semper proportionales cum Sectionibus Figuræ planæ ABO, sitque eadem distantia tam inter Sectiones Solidi, quam inter Sectiones Figuræ planæ; ex Methodo Indivisibilium, quæ facile etiam reduci potest ad Methodum Antiquorum, Solidum quod est summa Segmentorum ADE, AFG, &c. erectorum, est ad Solidum rectum circumscriptum, cujus nimirum basis est ipsa Figura ABCE erecta, altitudo AB, ut Figura plana Sinuum versorum ABO, est ad Rectangulum BP circumscriptum. *Quod erat demonstrandum.*

COROLLARIUM.

'Habebimus igitur fractionem Solidi, quod est summa Segmentorum ADE, AFG erectorum, ad Solidum rectum circumscriptum, si habeamus Rationem Figuræ planæ Sinuum versorum ABO, ad Rectangulum BP circumscriptum; hanc autem ultimam Rationem sic indagabimus.

LEM-

LEMMA III.

Plate 21. Fig. 106. 'Si intelligantur singuli Sinus versi AH, AK, &c. erigi 'perpendiculariter in punctis I, L, & supra arcum Quadrantis 'AIC, ex illis Sinubus ita erectis & insistentibus perpendicu-'lariter supra arcum AIC, fiet quædam superficies curva, 'cujus basis erit ipse arcus AIC, altitudo autem AB, vocetur 'hæc superficies *Figura curva Sinuum versorum.*

'Dico Figuram hujusmodi Curvam Sinuum versorum esse 'ad superficiem Cylindricam circumscriptam, cujus basis est 'arcus Quadrantis AIC, altitudo AB, ut Figura plana Si-'nuum versorum ABO, est ad Rectangulum circumscriptum 'BP.

'Ex proprietate Quadratricis ABCE, recta AB secatur in 'D, F, &c. in eadem ratione ac arcus AIC in I, L, &c. Cum 'igitur Ordinatæ DM, FN, &c. sint *ex hypothesi* æquales 'Sinubus versis AH, AK, &c. qui eriguntur in I, L, &c. ex 'Methodo Indivisibilium summa Sinuum versorum AH, AK, '&c. erectorum in I, L, &c. sive Figura Curva Sinuum verso-'rum, est ad superficiem Cylindricam circumscriptam cujus 'basis arcus AIC, altitudo AB, ut Figura plana Sinuum ver-'sorum ABO, est ad Rectangulum BP circumscriptum. Quod 'erat demonstrandum.

'Restat igitur nobis inquirenda Ratio quam habet Figura 'curva Sinuum versorum erectorum supra arcum Quadrantis 'ad superficiem Cylindricam circumscriptam; hanc autem 'habebimus ex Lemmate sequenti.

LEMMA IV.

'Figura Curva Sinuum versorum erectorum supra arcum 'Quadrantis : est ad superficiem Cylindricam circumscriptam, 'cujus basis est arcus Quadrantis, altitudo vero æqualis Ra-'dio : : ut differentia Radii & arcus Quadrantis : est ad arcum 'Quadrantis.

Plate 22. Fig. 109. 'Esto Quadrans circuli ABC, per singula puncta I, L, &c. 'arcus AIC, intelligantur ductæ rectæ MN, OP, parallelæ 'AB, occurrentes BC in N, P, & (completo Quadrato BD) 'rectæ AD, in M, O, atque ex iisdem punctis I, L, &c. ductis 'IH, LK, ordinatis ad AB, erunt AH, AK, Sinus versi ar-'cuum AI, AL, & illis æqualis IM, LO.

'Consideremus tres summas rectarum, primam Rectarum 'MN, OP, &c. erectarum in punctis I, L, &c. Secundam 'Rectarum IN, LP, erectarum etiam in I, L, &c. Tertiam de-'nique Rectarum IM, LO, &c. erectarum pariter in I, L, &c. 'Patet secundam & tertiam summmam simul sumptas esse æ-

'quales

'quales primæ, cum IM+IN æquetur MN, & LO+LP, æ- *Plate* 22.
'quetur OP, & sic de cæteris. Unde tertia summa est diffe- *Fig.* 109.
'rentia primæ & secundæ summæ.

'Jam prima summa Rectarum MN, OP, &c. æqualium 'inter se & erectarum in I, L, &c. est superficies Cylindrica 'cujus basis arcus Quadrantis AIC, altitudo vero æqualis 'Radio AB. Ergo est æqualis Rectangulo cujus unum latus 'est æquale arcui AIC, alterum vero Radio AB.

'Secunda vero summa Rectarum IN, LP, erectarum in I, 'L, &c. est æqualis Quadrato BD, quod sic ostendemus. In-'telligatur ex Quadrato ABC circa BC converso generari 'Hemisphærium, singulæ IN, LP, generant circulos, quorum 'Radii sunt ipsæ IN, LP, & quoniam circumferentiæ sunt in-'ter se ut Radii, summa Radiorum IN, LP, erectorum in I, L, 'est ad summam circumferentiarum eorumdem Radiorum, 'sive ad superficiem Hemisphæricam, ut una circumferentia 'est ad Radium, ex Methodo Indivisibilium: est autem *ex* '*Archim.* superficies Hemisphærica dupla circuli maximi, 'sive æqualis Rectangulo contento sub Radio AB, & periphe-'riâ Radii ejusdem AB. Ergo summa Rectarum IN, LP, &c. 'erectarum in I, L, &c. est ad Rectangulum contentum sub 'Radio AB, & peripheriâ ejusdem Radii AB, ut Radius AB est 'ad suam peripheriam. Sed in eadem Ratione Radii AB ad 'suam peripheriam est Quadratum BD ad idem Rectangulum 'contentum sub AB & peripheriâ Radii AB, ut patet. Ergo 'summa rectarum IN, LP, &c. & Quadratum BD, habent eam-'dem Rationem ad idem Rectangulum, ac proinde summa 'Rectarum IN, LP, &c. erectarum est æqualis Quadrato BD.

'Cum igitur ostensum sit, Tertiam summam Rectarum 'IM, LO, &c. esse differentiam primæ Rectarum MN, OP, '& secundæ Rectarum IN, LP, &c. prima autem summa sit 'æqualis Rectangulo contento sub AB & arcu AIC, secunda 'vero Rectarum IN, LP, &c. sit æqualis Quadrato BD, sive 'Rectangulo sub AB & AB, patet tertiam summam Recta-'rum IM, LO, &c. esse differentiam Rectanguli contenti sub 'arcu AIC, & sub Radio AB, & Rectanguli sub AB & AB. 'Cum autem horum Rectangulorum eadem sit altitudo AB, 'eorum differentia æquatur Rectangulo cujus altitudo eadem 'AB, basis vero differentia basium, nempe differentia Radii 'AB, & arcus AIC. Ergo summa rectarum IM, LO, &c. 'erectarum in I, L, &c. æquatur Rectangulo cujus altitudo est 'AB, basis autem differentia AB, & arcus AIC. Ergo est 'ad Rectangulum cujus eadem altitudo AB, basis arcus AIC, 'ut basis ad basim, sive ut differentia Radii AB, & arcus Qua-'drantis AIC, ad arcum Quadrantis AIC. Est autem Rectan-'gulum cujus altitudo AB, basis arcus AIC, æquale superficiei 'Cylindricæ ejusdem altitudinis & basis. Ergo summa recta-

'rum

Plate 21. Fig. 107. 'rum IM, LO, &c. ſive Sinuum verſorum AH, AK, &c. 'erectorum in I, L, &c. eſt ad ſuperficiem Cylindricam cir'cumſcriptam, ut differentia Radii & arcus Quadrantis, ad 'arcum Quadrantis. *Quod erat demonſtrandum.*

'His poſitis jam facile determinabimus Centrum gravitatis 'quæſitum. Sit enim nova Quadratrix ABCE, genita ex 'Quadrante ABCF, ſitque illius Centrum gravitatis H Ex 'H in AB, demittatur perpendicularis HG. Dico AG eſſe 'ad AB, ut Radius AB, eſt ad arcum Quadrantis AFC.

'Eſt enim ex principio ſupra poſito, BG ad BA, ut ſumma 'Segmentorum ADE, ADE, ABC, ad Solidum rectum cir'cumſcriptum: ut autem prædicta ſumma ad Solidum re'ctum, ita Figura plana Sinuum verſorum ad Rectangulum 'circumſcriptum (*Lem.* 2.) & ut Figura plana Sinuum ver'ſorum ad Rectangulum circumſcriptum, ita (*Lem.* 3.) Fi'gura curva Sinuum verſorum ad ſuperficiem Cylindricam, 'circumſcriptam, & ut figura curva Sinuum verſorum ad 'ſuperficiem Cylindricam, ita (*Lem.* 4.) differentia Radii & 'arcus AFC, ad arcum AFC. Ergo BG eſt ad BA, ut diffe'rentia AB Radii & arcus AFC, ad arcum AFC. Ergo AG 'differentia antecedentis BG & conſequentis AB, eſt ad con'ſequens AB, ut AB differentia ſecundi antecedentis & con'ſequentis eſt ad ſecundum conſequens nempe ad arcum 'AFC. *Quod erat demonſtrandum.*

COROLLARIUM.

'Hinc determinata eſt diſtantia Centri gravitatis H à rectâ 'BC: determinavimus autem initio diſtantiam ejuſdem 'Centri gravitatis H à rectâ AB. Ergo determinatum eſt 'Centrum gravitatis H. *Quod erat faciendum.*

SECTION III.

Of the Center of Gravity of Solids.

THIS Section ſeems to be more uſeful than the Foregoing, becauſe it treats of ſuch Bodies as we make uſe of every Day, and the Two Foregoing only treat of Lines and Plains, which do not exiſt by themſelves, unleſs in the Imagination. Nevertheleſs thoſe Two Sections are of great Uſe as being the Foundation of this; beſides all that has been ſaid of Lines and Plains may be applied to Bodies like them of an equal Thickneſs every where, as you will ſee better in the Sequel.

FRO-

PROPOSITION I.

THEOREM.

If a Prism be cut by a Plain parallel to the Two opposite Plains, the Section will be a Plain Equal and Similar to each of those opposite Plains, and its Center of Gravity will be in the Right-line which passes thro' the Center of Gravity of the said Two opposite Plains.

LET ADEFC, *for Example*, be a Triangular Prism propos'd, whose Two Opposite, Similar, Equal and Parallel plains are the Triangles ABC, DEF, whose Centers of Gravity are G, H. I say, That if such a Prism be cut by a Plain parallel to either of those Triangles, so that the Section be, *for Example*, the Triangle KLM, that Triangle KLM will be Equal and Similar to each of the opposite Triangles ABC, DEF, and its Center of Gravity I will be in the Right-line GH. *Plate 22. Fig. 110.*

DEMONSTRATION.

Since the Two plains ABC, KLM, are Parallel, and that they are both cut by the plain ABED, their Sections AB, KL will be parallel, *by* 16. 11. For the same Reason the Two lines BC, LM, are Equal and Parallel, as well as the Two AC, KM. Thus all the sides of One Triangle will be equal to all the sides of the other, the One to the other, wherefore *by* 8. 1. they will be Equal, Equiangular, and Similar. *Which is the first of the Two things to be demonstrated.*

Because the Three Triangles ABC, KLM, DEF, are Equal and Similar, their Centers of Gravity G, I, H, will have the same Position, and therefore must be in the line GIH. *Which is the other thing we had to demonstrate.*

SCHOLIUM.

To leave nothing doubtful in this Demonstration, we will demonstrate that the Centers of Gravity G, I, H, are in a Similar Position; that is, that if the Triangles ABC, KLM, DEF, were laid one upon another, their Centers of Gravity, G, I, H, wou'd coincide; tho' this is self-evident.

It is plain from what has been said elsewhere, that the Three lines BGN, LIO, KHP, divide their opposite equal sides

Plate 22. *Fig.* 110. sides AC, KM, DF, into Two equal parts at the points N, O, P, so that the Three lines AN, KO, DP, will be equal to one another, whence the Three Triangles ABN, KLO, DEP will likewise be equal to one another, and also the Three Angles ABN, KLO, DEP, and the Three lines BN, LO, EP, and consequently their thirds NG, OI, PH. Hence it follows, that the Three lines BG, LI, EH, are also equal to one another, and as they are parallel, their Ends G, H, I, must have the same Position and be in the same Right-line.

PROPOSITION II.

THEOREM.

The Center of Gravity of a Prism is in the middle of the Right line which goes thro' the Centers of Gravity of Two opposite Plains.

I Say, That the Center of Gravity of the Prism ADEFC is at I the middle of the Right-line GH, which goes thro' the Centers of Gravity G, H, of the Two opposite Plains ABC, DEF.

DEMONSTRATION.

As we have demonstrated in *Prop.* 1. that in whatsoever part a Prism be cut by a Plain parallel to its opposite Plains, the Section will also have its Center of Gravity in the line GH; And as that Prism may be cut thus by an infinite Number of such Plains, it may be consider'd as made up of an infinite Number of Plains parallel to one another, and to the Two opposite Plains, whose Centers of Gravity are in the line GH; and by the *Method of Indivisibles*, you immediately conclude, That the Sum of that infinite Number of Plains, or the Prism ADEFC has its Center of Gravity in the line GH, and consequently in its middle point I. *Q.E.D.*

COROLLARY.

It follows evidently from this Proposition, that a Triangular Prism has its Center of Gravity in a Plain, which goes thro' One of the Angles and the middle of the opposite side. For since that Center of Gravity is in the line GH, it is also in the Plain BH, or BEPN, which goes thro' the Angle B, and the middle N of the opposite side AC.

It follows also that the Center of Gravity of any Parallelepipedon, or of a Cylinder is in the middle of its Axis.

It follows likewise that the Center of Gravity of a Prism, whose Two opposite Plains are Trapezoids, will be in a Plain, which divides the parallel sides of those Two Trapezoids into Two equal Parts. *Plate 22. Fig. 110.*

PROPOSITION III.

THEOREM.

If a Pyramid be cut by a Plain parallel to its Base, the Section will be a Plain Similar to that Base, and its Center of Gravity will be in the Right-line which goes thro' the Center of Gravity of the Base and the Vertex of the Pyramid.

LET ABCD, *for Exam.* be a Triangular Pyramid, whose Base is the Triangle ABC, (which has its Center of Gravity at E) and whose Vertex must consequently be the point D. I say, That if such a Pyramid be cut by a Plain, parallel to the Base ABC, so that the Section be, *for Example*, the Triangle FGH; this Triangle FGH will be Similar to the Triangle ABC, and its Center of Gravity I, will be in the Right-line DE. *Fig. 111.*

DEMONSTRATION.

Since the Two Plains ABC, FGH, are parallel, and both cut by the Plain CAD, their Sections CA, HF, will be parallel, *by* 16. 11. For the same reason the Two lines AB, FG, will be parallel, as also the Two CB, HG, which makes these Lines Proportional, and their Angles Equal, and consequently the Triangles ABC, FGH. *Which is One of the Things to be demonstrated.*

Now to demonstrate that I, the Center of Gravity of the Triangle FGH is in the line DE, one must consider that in the Similar Triangles CAK, HFL, the Lines FL, AK, or FI, AE, are parallel, and that as FH: is to AC:: so FL: is to AK. But the *Ratio* of the said lines FH, AC, is the same with that of the lines DF, DA. Wherefore DF: is to DA:: as FL: is to AK. Now FL: is to AK:: as FI: is to AE, because the lines FI, AE, are each Two Third Parts of their lines FL, AK; therefore as DF: is to DA:: so FI: is to AE; and since the the Right-lines FI, AE, are parallel, it follows that their ends I, E, are in the same Position, and consequently in the Right-line DE. *Which is the last Thing to be demonstrated.*

SCHO-

SCHOLIUM.

What has been demonstrated of a Triangular Pyramid may be demonstrated of a Pyramid of more sides, and even of a Cone, which is properly a Pyramid of an infinite Number of sides; and likewise of any other Pyramid, whose Base is terminated by one continued Line.

PROPOSITION. IV.

THEOREM.

The Center of Gravity of a Pyramid is in the Right-line which goes thro' One of its Angular Points, and thro' the Center of Gravity of the Plain opposite to that Point.

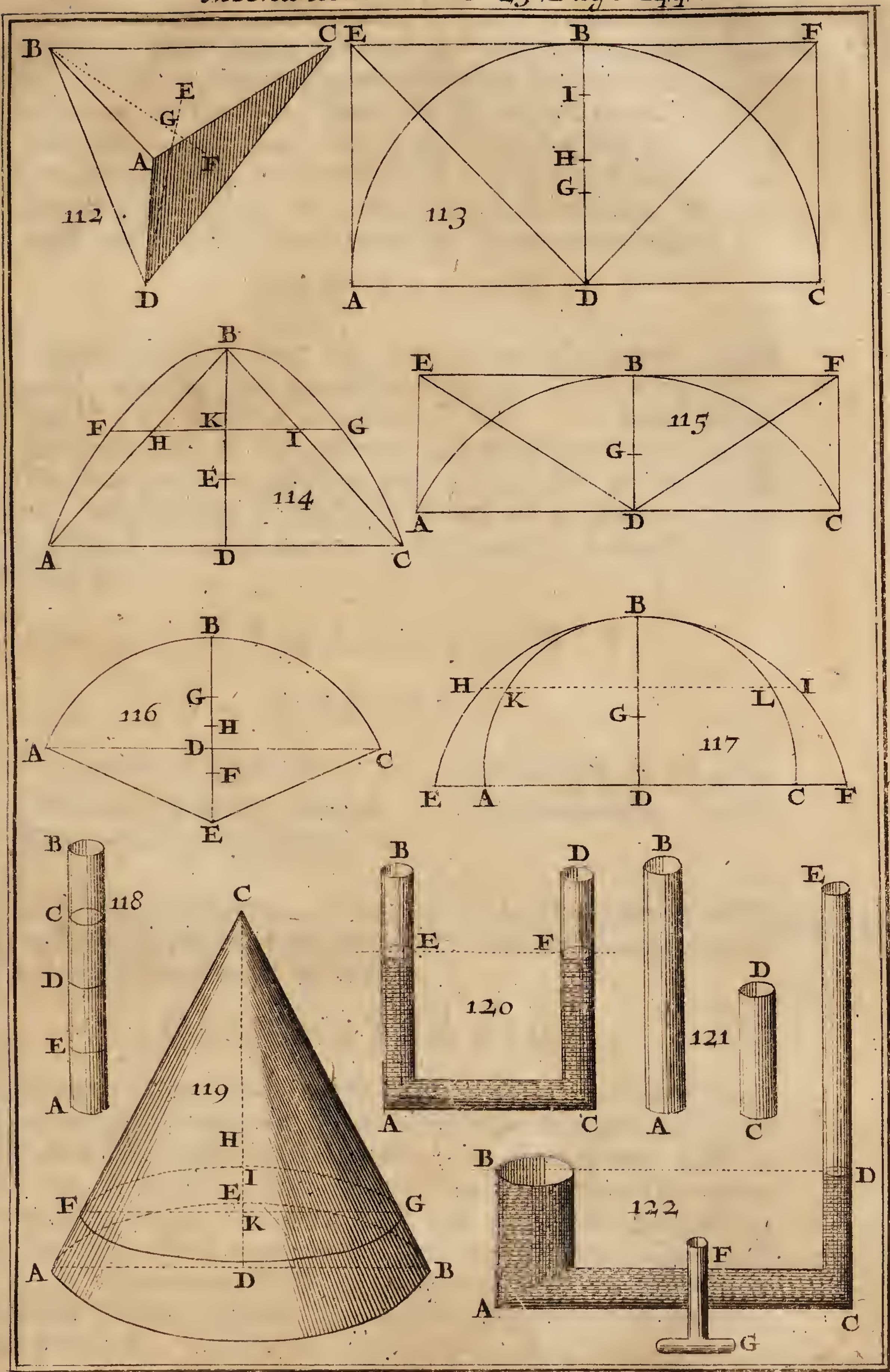
Plate 22. *Fig.* 111. I Say, that the Center of Gravity of the Pyramid ABCD, is in some point of the Right-line DE, which goes thro' the point or solid Angle D, and thro' E the Center of Gravity of the opposite Plain ABC.

DEMONSTRATION.

As it has been demonstrated in *Prop.* 3. that in what ever part the Pyramid ABCD be cut by a Plain parallel to One of its Bases, as to the Base ABC, whose Center of Gravity is E, the Section will have its Center of Gravity in the line DE: and as that Pyramid may thus be cut in an infinite Number of Places, it may be look'd upon as made up of an infinite Number of Plains parallel to one another, and likewise to the Base ABC, whose Centers of Gravity are in the line DE: Whence it follows, That the Sum of that infinite Number of Plains, or the Pyramid ABCD has also its Center of Gravity in the line DE. *Q. E. D.*

COROLLARY.

Plate 23. *Fig.* 112. It is evident from this Proposition, that the Center of Gravity of a Pyramid is in the Concourse of Two Right-lines drawn thro' the solid Angles of the Pyramid and the Centers of Gravity of the Plains opposite to 'em. For as it has been demonstrated, that the Center of Gravity of the Pyramid ABCD is in the line DE, which goes thro' the solid Angle D, and E the Center of Gravity of the opposite Plain ABC; one may after the same manner demonstrate that that Center of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of Gravity is also in the line BF, which goes thro' the solid Angle B and the Center of Gravity F of the opposite Plain CAD. Whence one may conclude, that it is in G the common Section of the Two lines DE, BF, whose parts GE, GD, GF, GB, are proportional, the Least GE being the Third part of the Greatest GD, or the Least GF being the Third part of the Greatest GB, which we cou'd here demonstrate; but that we must end this Section to come to Things more useful. *Plate* 23. *Fig.* 112.

SCHOLIUM.

You must observe, that what has been said concerning the Pyramid, may be also understood of the Cone, which is a Pyramid of an Infinite Number of sides. To make short, we shall not stand to demonstrate that the Center of Gravity of a Regular Body, is the same with that of the circumscrib'd Sphere, and that the Center of Gravity of a Sphere or Spheroid, is the same as its Center of Magnitude; because that is self-evident.

PROPOSITION V.

THEOREM.

The Center of Gravity of an Hemisphere is in that Semi-Diameter, which is Perpendicular to the Diameter of its Base.

I Say, That the Center of Gravity of the Hemisphere ABC, is in some Point of the Semi-diameter BD perpendicular to AC the Diameter of its Base. *Fig.* 113.

DEMONSTRATION.

For as the Hemisphere ABC is form'd by the Circumvolution of the Semi-circle ABC about the immoveable Line BD, one may easily conceive that the Hemisphere ABC is made up of an Infinite Number of Circles, whose Diameters are parallel to the Diameter AC, and whose Centers are in the Axis BD: and as those Centers are also their Centers of Gravity, it follows, That the common Center of Gravity of the Sum of that Infinite Number of Circles, or of the Hemisphere ABC, is in the said Axis BD. *Q. E. D.*

SCHOLIUM

One may after the same manner demonstrate, that the Center of Gravity of the Segment of a Sphere, or of a Spheroid,

 of

Plate 23. Fig. 113. of a Paraboloid, or of a *Conus truncatus*, is in their Axis, because all those Bodies are made of an infinite Number of Circles parallel to their Bases, as it is evident by their Generation, *&c.*

PROPOSITION VI.

THEOREM.

The Center of Gravity of an Hemisphere divides its Axis into Two Parts, of which That which is nearest to the Surface, is to the Other as 5, *to* 3.

I Say, That in the Hemisphere ABC, whose Center is D, and Axis BD; the Center of Gravity G divides the Axis BD into Two parts GB, GD, in such manner, that GB : is to GD : : as 5 : is to 3; so that if the Axis BD be divided into Eight equal parts, GB will be 5, and GD 3, of those parts.

PREPARATION.

Describe about the Semi-circle ABC, which is the Profil of the Hemisphere, the Rectangle AF, and joyn DE, DF. If, by thought, the whole Plain AF be mov'd about BD the Semi-diameter, which must remain unmoveable as an Axis, the Rectangle AF will by its Circumvolution describe a Cylinder; the Semi-circle ABC an Hemisphere; and the Rectangular Triangle EDF a Cone, whose Base will be equal to that of the Cylinder, and to that of the Hemisphere.

DEMONSTRATION.

Because the Three foregoing Solids have the same Height, *viz.* the common Axis BD, and equal Bases; the Cylinder AF will be Three times the Cone EDF, and the Hemisphere ABC will be Twice the same Cone EDF, as it is evident from what has been said and demonstrated in our *Treatise of Geometry*. So that if the Cylinder AF be suppos'd to consist of Three parts, the Cone EDF will be One of 'em; and the Hemisphere ABC, Two of 'em: And if the Cone EDF be taken from the Cylinder AF, that is, 1 from 3, there will remain Two for the Solid Concave AEDFCD, which consequently will be equal to the Hemisphere ABC, and to Twice the Cone EDF. Now the Center of Gravity of the Cylinder AF is at H the middle Point of the Axis BD, *by Prop.* 2. and that of the Cone EDF is at the point I, distant from the point

B

B the fourth part of the Axis BD, *by Prop.* 4. Since then H is the common Center of Gravity of the Cone EDF, and of the Solid Concave AEDFCD, those Two Solids will be in a Reciprocal *Ratio* of their distances HG, HI, and as they are in a Duple *Ratio*, the distance HI, will be double the distance HG. Whence we may learn, That to find out G the Center of Gravity of the Hemisphere ABC, HG must be taken equal to Half of HI. Now as BI is $\frac{1}{4}$ of BD, HI must also be $\frac{1}{4}$ of BD, and consequently HG $\frac{1}{8}$ of BD, to which if BH equal to Half of BD be added, you will have BG equal to $\frac{5}{8}$ of BD, and consequently GD equal to $\frac{3}{8}$ of BD; which shews, that BG : is to GD :: as 5 : is to 3. *Q.E.D.* *Plate* 23. *Fig.* 113.

COROLLARY.

From this Theorem it follows, That if the Axis of an Hemisphere be divided into Eight equal parts, and you take Five of 'em counting from the Superficies, or Three counting from the Center, you will have the Center of Gravity of the Hemisphere propos'd.

SCHOLIUM.

Because the foregoing Demonstration supposes that the Center of Gravity of a Cone divides its Axis into Two parts, of which the nearest to the Vertex contains the other Three times, or is equal to $\frac{3}{4}$ of the Axis, which we have not demonstrated; we will make you more certain of it by shewing in the following Problem, the Manner of finding the Center of Gravity of a Cone, according to Mr. *Fermat*'s Method, which may be applied to any other Figure.

PROPOSITION VII.

PROBLEM.

How to find the Center of Gravity of a Cone.

TO find the Center of Gravity I upon the Axis CD of the Cone ABC, cut that Cone by a Plain parallel and infinitely near to the Basis AB; in such manner that the Section be, *for Example*, the Circle FG, whose Diameter is cut into Two eqnal Parts at E by the Axis CD; and by this *Fig.* 119.

Plate 23. *Fig.* 119. Section the Cone ABC will be divided into Two parts, which make the Cone FCG, whose Center of Gravity we will suppose H, and the *Conus truncatus* AFGB, whose Center of Gravity we will suppose at K.

This Preparation being made; let a be the Axis CD, x the distance CI from the Center of Gravity I, of the Cone ABC to its Vertex C, and you will have $a - x$ for the distance DI from the same Center of Gravity I to the Base AB: and if o be put for the distance DE of the Plain cutting FG at the Base AB, (this Letter o representing a Cypher, because the part DE is suppos'd infinitely Little, and consequently the *Conus truncatus* AFGB infinitely Little, which makes the Two lines IK, ID, equal, so that they may be express'd by the same *Algebraical* Characters, *viz.* $a - x$,) you will have $a - o$ for the Axis CE of the Cone FCG.

$$CD = a$$
$$CI = x$$
$$DI = a - x = IK.$$
$$DE = o$$
$$CE = a - o$$
$$CH = x - \frac{xo}{a}$$
$$HI = \frac{xo}{a}$$

Because the Center of Gravity I of the Cone ABC, divides its Axis CD after the same manner that the Center of Gravity H of the Cone FGC divides its Axis CE, since that happens in all Pyramids, as it is easy to conclude by what has been said in *Prop.* 4. you will have this Analogy, CD: CI:: CE: CH; or $a : x :: a - o : $ CH, which will give $x - \frac{xo}{a}$, for the part CH, which being taken from the part CI, or from x you will have $\frac{xo}{a}$ for the part IH.

Because by the general Principle of the Balance, the *Conus truncatus* AFGB: is to the Cone FCG: : reciprocally as the distance HI: is to the distance IK, it will be known, *by Composition*, that the Cone ABC is to the Cone FCG, as HK is to IK, and if instead of those Two Cones which are Similar, (because of the Similar Triangles ABC, FGH,) you put the Cubes of their *Axes* CD, CE, which are in the same *Ratio*, you will know, that the Cube of CD: is to the Cube CE:: as HK: is to IK; so that in Analytical Terms you will have this Analogy, $a^3 : a^3 - 3aao + 3aoo - o^3 ::$ HK: IK, and, *by dividing*, this, $3aao - 3aoo + o^3 : a^3 - 3aao + 3aoo - o^3 ::$ HI: IK; and if instead of the Two last Terms HI, IK, you put the Letters which express them, *viz.* $a - x$, $\frac{xo}{a}$, or in whole Numbers $aa - ax$, xo, which are in the same *Ratio*, you will have this last Analogy, $3aao - 3aoo + o^3 : a^3 - 3aao + 3aoo - o^3 :: xo : aa - ax$, and consequently

quently this Equation, $3a^4o - 3a^3oo + aao^3 - 3a^3xo + 3aaxoo - axo^3 = a^3xo - 3aaxoo + 3axo^3 - xo^4$, or $3a^4o - 3a^3oo + aao^3 - 4a^3xo + 6aaxoo - 4axo^3 + xo^4 = 0$, which being divided by o, will be chang'd into this, $3a^4 - 3a^3o + aaoo - 4a^3x + 6aaxo - 4axoo + xo^3 = 0$, from which taking all the Terms where the Letter o is you will have this last Equation, $3a^4 - 4a^3x = 0$, in which you will find $x = \frac{3a}{4}$, which shews that the part CI is equal to 3 Quarters of the Axis CD, and therefore, That to find the Center of Gravity of the propos'd Cone ABC, the Axis CD must be divided into Four equal Parts, and Three of 'em must be taken from C to I, or One from D to I, and the point I will be the Center of Gravity requir'd. *Plate 23. Fig. 119.*

PROPOSITION VIII.

THEOREM.

The Center of Gravity of a Paraboloid is the same as that of the Triangle, whose Height is the Height of the Paraboloid, and whose Base is the Diameter of the Base of the same Paraboloid.

I Say, that the Center of Gravity of the Paraboloid or Parabolick Conoid ABC is the same as the Center of Gravity E of the Triangles ABC. *Fig. 114.*

DEMONSTRATION.

If to AC Ordinate to the Diameter BD, be drawn any Parallel as FG, which will also be an Ordinate to the Diameter BD, you will have the Two Similar Triangles HBI, ABC, and *by* 4.6. BK: will be to BD:: as HI: to AC; and because by the Nature of the Parabola, BK: has the same *Ratio* to BD:: as the Square FG: has to the Square AC; that is, as the Circle whose Diameter is FG: is to the Circle whose Diameter is AC; it follows, that the Circle HI: is to the Circle AC:: as the Circle FG: is to the Circle AC; which shews that those Circles whose Number is Infinite, which make up the Paraboloid ABC, are Proportional to as many Right-lines which make up the Triangle ABC. Whence it is easy to conclude, That their Centers of Gravity coincide. *Q.E.D.*

Plate 23. Fig. 114.

COROLLARY.

It is evident from this Propoſition, that E the Center of Gravity of the Paraboloid ABC, divides the Diameter BD into Two parts EB,ED, in ſuch manner that the firſt EB is double the ſecond ED, as in the Triangle; and therefore, that to find the Center of Gravity of ABC, the propos'd Paraboloid, its Axis BD muſt be divided into Three parts, and Two of the parts muſt be taken, counting from B to E; or One, counting from D to E, *&c.*

PROPOSITION IX.

PROBLEM.

How to find the Center of Gravity of the Segment of a Sphere.

Fig. 115. THE Center of Gravity of the Segment of a Sphere is found after the ſame manner as that of a Hemiſphere. Therefore to find the Center of Gravity of the ſolid Segment, whoſe Profil is ABC; the Perpendicular BD, which divides the Chord AC into Two equal parts, muſt be divided into Eight equal parts, and Three of 'em muſt be taken from D to G, or Five of 'em from B to G, to have in G the Center of Gravity requir'd.

DEMONSTRATION.

If the Rectangle AF be deſcrib'd about the Arch ABC, and the Right-lines DE, DF, be drawn, it is evident that the Plain AF, by a Circumvolution about the unmoveable Line BD will deſcribe a Cylinder, the Segment ABC, a Segment of a Sphere, and the Triangle EDF a Cone, after which the reſt may be demonſtrated, as in *Prop.* 6.

PROPOSITION X.

PROBLEM.

How to find the Center of Gravity of the Sector of a Sphere.

Fig. 116. TO find the Center of Gravity of ABCE the Sector of a Sphere, whoſe Center is E, and Axis BE, take upon that Axis BE the Part DF, equal to a fourth Part of DE, and the

the Part DG equal to $\frac{3}{8}$ of BD, to have G the Center of Gravity of the solid Segment ACB, and at F the Center of Gravity of the Cone AEC. Then to find the common Center of Gravity of the Two Solids ACB, AEC, or the Center of Gravity of the Sector AECB, you must only divide FG the distance of those Two Centers of Gravity F, G, into Two Parts FH, GH, reciprocally Proportional to the Two Solids ACB, AEC, which will be done, by finding a fourth Proportional to the Sector ABCE: the Cone AEC:: and the Line FG: *viz.* GH, *&c.* *Plate* 23. *Fig.* 116.

PROPOSITION. XI.

THEOREM.

If the Segment of a Sphere and the Segment of a Spheroid have the same Axis, and their Bases on the same Plain, they will also have the same Center of Gravity.

I Say, That the Centers of Gravity of ABC the Segment of a Sphere, and of EBF the Segment of a Spheroid, whose common Axis is BD, coincide; that is, that they have the same Center of Gravity, as G. *Fig.* 117.

DEMONSTRATION.

If within those Two Figures you draw as many Lines as you please, Parallel to the Line EF, or Perpendicular to the Axis BD, as HI; HI: will always be to KL:: as EF: is to AC, by the Nature of the Ellipsis, and the Circle of the Diameter HI: will be to the Circle of the Diameter KL:: as the Circle of the Diameter EF: is to the Circle of the Diameter AC. Thus you see, that the innumerable Circles, which make up the Segment ABC of a Sphere, are Proportional to the innumerable Respective Circles, which make up the Segment EBF of a Spheroid. Whence it is easy to conclude, That they have One common Center of Gravity. *Q.E.D.*

COROLLARY.

It follows from this Theorem, that if you divide BD the Axis of EBF the Segment of a Spheroid into Eight equal Parts, and take Three of 'em from D to G, or Five from B to G, you will at G have the Center of Gravity of the Segment of the Spheroid EFB.

The Third Book.

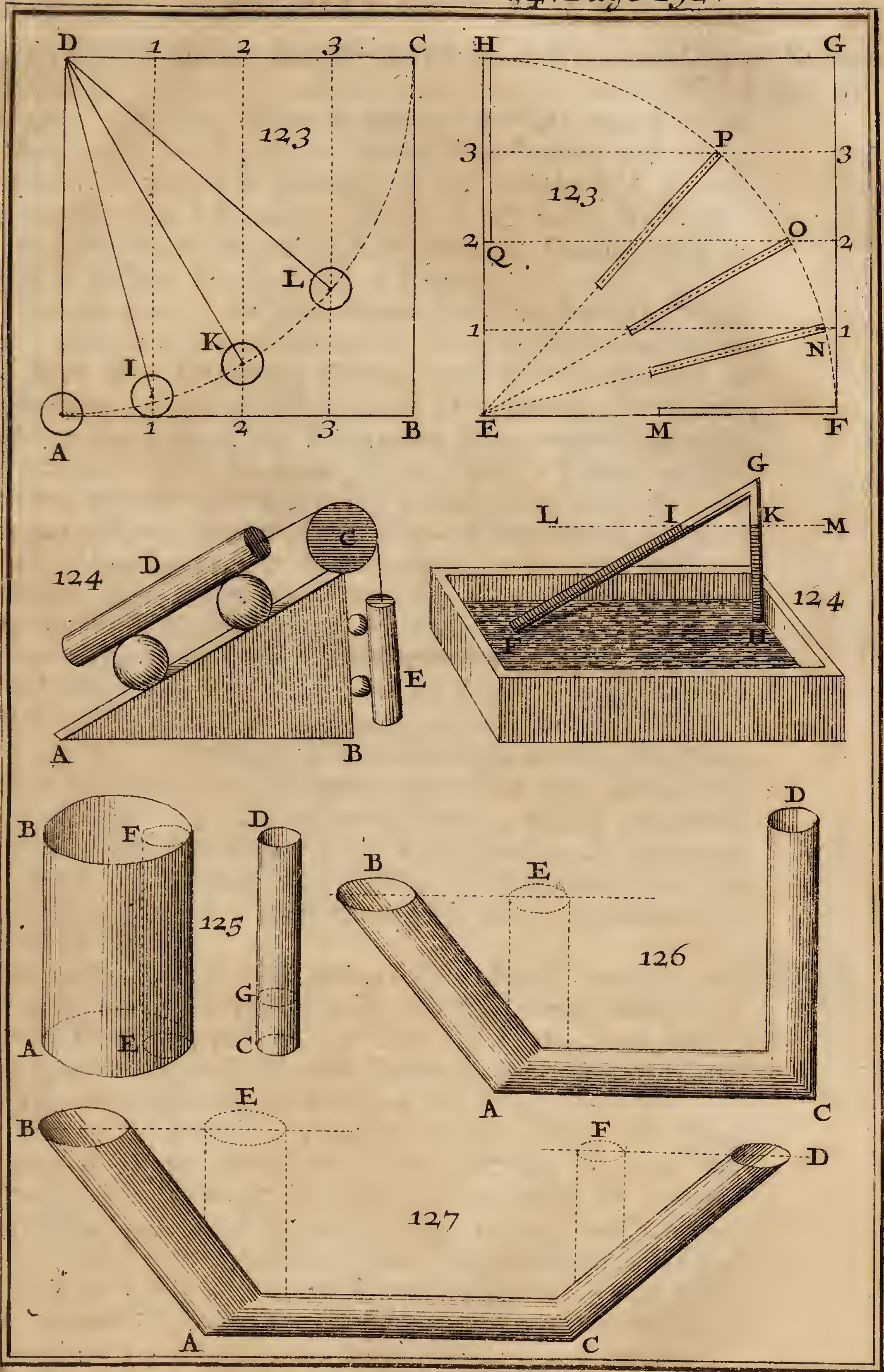
OF

HYDROSTATICKS.

HYDROSTATICKS is that part of *Mechanicks*, which considers the Weight of Liquids, and especially Water, or Solid Bodies immers'd into, or lay'd upon Liquids by comparing them with each other.

Although Liquids are Ponderous, neverthelesſs they have not of themselves a Center of Gravity, because their parts are not link'd together so as to sustain one another in *Æquilibrio* about one certain Point, unless they are shut up in a Vessel, and then one may observe a great deal of Conformity occurring in *Staticks* and *Hydrostaticks*, which we will here explain occasionally.

Plate 24. Fig. 123. As by the Principles of *Staticks*, we know the Weight encreases according to its distance from the Perpendicular, which is to say, that to remove the Weight A, which we suppose to be 4 Pound, upon the Arch AC, of the Quadrant ACD, whose Center is D, from the Perpendicular AD, from A to I, *for Example*, by the 4*th* part of Horizontal distance CD, There must be a Force equal to the fourth part of the Weight A, or the Force of 1 Pound; and a Force equal to the half of the Weight A, or the Force of 2 Pounds to remove it to K, half of the distance CD: and a Force equal to 3 quarters of the Weight A, or the Force of 3 Pound, to bring it to L, which is 3 quarters of the same distance CD: and at last a Force equal to the whole Weight A, or the Force of 4 Pounds, to bring it to C, the whole distance CD: just the contrary it shewn by experience in *Hydrostaticks*; namely, that the Weight increases its Gravity as it comes nearer to a Perpendicular, that is to say, that in removing a Cylinder of Water, as MF, from the Horizontal line EF, to bring it nearer to the Perpendicular EH, by raising it upon the Arch FH, of the Quadrant EFH, round its Center E, the Force of the Water contain'd in the Cylinder MF, increases in proportion to its elevation; in so much that if the Cylinder MF, was



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A C

was rais'd to N, the 4th part of the Height EH, it wou'd get the Force of 1 pound to raiſe a Sucker, or to make a Wheel turn, *&c.* The ſame Water being rais'd to O, which is half the Height EH, will gain the Force of 2 Pound, and being rais'd to P, which is 3 quarters of the Height EH, it will have the Force of 3 Pounds, and laſt of all, being rais'd perpendicularly up to the point A, ſo that its Poſition may be HQ, it will have the Force of 4 Pounds. *Plate* 24. *Fig.* 123.

In like manner, as we have demonſtrated in *Staticks, Prop.*4. *Chap.* 2. that when Two Weights are in *Æquilibrio* upon the Two ſides of a Triangular Plain, whoſe Baſe is parallel to the Horizon, by mutually balancing each other in Two lines parallel to the ſides of the Triangular Plain, which I ſuppoſe perpendicular to the Horizon, by means of a Pulley apply'd to the top of the Triangle: Their Powers or abſolute Gravities are proportionable to the ſides of the ſame Triangle; the ſame thing happens in *Hydroſtaticks*; for if a Pipe or Tube be ſo bent, as to repreſent Two ſides of a Triangle, (and being partly fill'd with Water, its Two Ends, which will be the Extremities of Two Cylinders of Water, be plung'd into a Veſſel full of Water, whoſe Surface being always parallel to the Horizon, may paſs for the Baſis of that Triangle, which I alſo ſuppoſe Perpendicular to the Horizon, or to the Surface of the Water contain'd in the Veſſel) the Length of thoſe Two Cylinders of Water, which are above the Surface of the Water in the Veſſel ſhall be alſo proportionable to the ſides of the Triangle, when the Water in each Tube or Cylinder ſhall be in *Æquilibrio*, that is to ſay, of the ſame Height.

Thus, becauſe according to the Principles of *Staticks*, we know that the Two Weights D, E, that are in *Æquilibrio* upon the Two ſides AC, BC, of the Triangular plain ABC, whoſe Baſe AB is parallel to the Horizon, their abſolute Gravities are to each other as the ſides AC, BC; ſo that if the ſide AC, is, *for Example*, Double the ſide BC, the abſolute Gravity of the Weight D, is alſo Double that of the Weight E; likewiſe it appears in *Hydroſtaticks*, that the Length of the Cylinder of Water FI : is to the Length of the Cylinder of Water HK:: as the ſide FH, of the Triangle FGH: is to the ſide GH; ſo that if the ſide FH, is Double the ſide GH, alſo the Length FI, is double the Length HK, when the Two Extremities IK, are level, that is to ſay, in the Horizontal line LM, which is evident, becauſe in that caſe the Two Triangles GIK, GFH, are Similar. *Fig.* 124.

CHAP.

CHAPTER I.

Concerning the Theorems.

THE Theorems, which we ſhall add here, are founded upon Experience, which may ſerve as a Demonſtration in ſuch caſes as theſe: Neverthelеſs we ſhall give their Demonſtration as clearly as we can.

THEOREM I.

A Ponderous Liquor, contain'd in a Cylinder perpendicular to the Horizon, Endeavours to get out at the Bottom, with a Force proportionable to its Height in the Tube.

Plate 23. *Fig.* 118. LET us ſuppoſe, that the Tube AB, be of an equal bigneſs throughout, and perpendicular to the Horizon; and that being fill'd, either wholly, or in part, with ſome Ponderous Liquor, *for Example*, Water, we ſhut up the open End A, in order to hinder it from running out. That being ſuppos'd, I ſay, that the Water contain'd in the Cylinder AB, endeavours to get out at the End A, with a Force proportionable to its Height: So that if the Tube AB is fill'd with Water, *for Ex.* up to C, and that the Height AC is divided into as many Equal parts as you pleaſe, as for inſtance, into Three at the points D, E, the Force with which the Water will endeavour to deſcend from C, through the Hole A, will be triple that with which it will endeavour to get out with from E thro' the ſame Hole A, if the Water ſhould reach but juſt up to E; becauſe in that caſe the Height AC, being triple the Height AE, the Cylinder of Water AC, wou'd be alſo triple the Cylinder of Water AE, and that conſequently, the Cylinder of Water AC wou'd be Three times Heavier than the Cylinder of Water AE, which wou'd give to the Water contain'd in the Tube AC, Three times more Force to deſcend, than to the Water contain'd in the Tube AE; it being certain that the Force which a heavy Body deſcends with, is proportionable to its Weight.

SCHOLIUM.

This Demonſtration ſuppoſes that Water deſcends by its own Gravity, as it is evident: But as it is alſo puſh'd on all ſides by the continual Motion its Fluidity cauſes, one part does

does not only preſs that part which anſwers perpendicularly under it, but, alſo thoſe on each ſide of it. From whence it follows, that if you pierce the Tube AB, thro' the ſide, the Water that is above the place you pierc'd will run out at it. *Plate 23. Fig. 118.*

COROLLARY I.

It evidently follows from this Propoſition, that if Two Cylinders of equal bigneſs, contain each of them a certain quantity of the ſame Liquor, *for Example*, Water, the Powers with which that Water will endeavour to run out of each of thoſe Tubes, will be to one another as the Heights of the Water in the ſaid Tubes, and conſequently if thoſe Heights are equal, the Water comes out of each Tube with equal Force.

COROLLARY II.

It follows alſo from this Propoſition, that the ſame ſort of Liquor being in Two Tubes of equal bigneſs, and perpendicular to the Horizon, which have communication by means of a Tube of the ſame bigneſs, and parallel to the Horizon; has always its upper parts in the ſame Level in each Tube: As much as to ſay, that if you pour ſome Liquor, as Water, into one of theſe Tubes, it will by means of the Tube of Communication diſperſe it ſelf into the other Tube, and will place it ſelf in each Tube at the ſame Height. *Fig. 120.*

As if one pour'd Water into the Tube AB, it wou'd run into the Tube AC, of the ſame Size, and in riſing up in CD, which is a Tube of the ſame Size alſo, it will place it ſelf at an equal Height in each Tube, that is to ſay, that the Water will ceaſe to riſe in the Tube CD, when it comes to the Height of that in the Tube AB; becauſe in that caſe the Two Cylinders of Water, as AE, CF are equal, and conſequently they have equal Gravity.

THEOREM II.

If Two Cylinders fill'd with the ſame Liquor are of an equal Height, and of an unequal bigneſs, and perpendicular to the Horizon; the Liquor endeavours to get out at the Bottom of each, with a Force proportionable to its Baſe.

I Say, That if a Ponderous Liquor, as Water, is at an equal Height as AB, CD, in the Tubes AB, CE, Perpendicular to the Horizon, and of unequal bigneſs, as *for Example*, if the Di-

Plate 23. *Fig.* 122. Diameter of the Base of the Tube AB, be Double the Diameter of the Base of the Tube CE, in which case the Base of the Tube AB, will be Quadruple to that of the Tube CE; the Force with which the Water will endeavour to get out at the Bottom of the largest Tube AB: will be to the Force with which it will endeavour to get out at the Bottom of the least CE:: as the Base of the largest Tube AB: is to the Base of the least CE; in so much that in the Supposition we have made, the Water will endeavour to get out at the Bottom of the largest Tube AB, with a Force Quadruple to that with which it will endeavour to get out at the Bottom of the least CE; because the largest Tube AB, will be Four times as big as the least CD, of the same Height, and consequently will have Four times its Gravity, which gives the Water Four times the Force to go out with.

COROLLARY.

It evidently follows from this Proposition, that if Two Cylinders of Water, perpendicular to the Horizon, are not only of unequal Bigness, but also of unequal Height, the Force with which the Water contain'd in One of these Tubes, will endeavour to get out at the Bottom with, will be to the Force with which the Water will endeavour to get out at the Bottom of the other Tube, in a *Ratio* compounded of the Proportions of the Bases and the Heights.

As here, we have suppos'd, that the Base of the Tube AB, is Quadruple to that of the Tube CE, whose Height, *for Example*, may be Triple that of the Cylinder AB, the Force with which the Water contain'd in the Tube AB, endeavours to get out at the Bottom, will be to that by which the Water contain'd in the Tube CE endeavours to get out at the Bottom, in the same Proportion, as 4 to 3, which is compounded of the *Ratio* of 4 to 1, or of the Base of the Tube AB, to that Base of the Tube CE, and of the *Ratio* of 1 to 3; or of that of the Height of the Tube AB, to that of the Height of the Tube CE.

SCHOLIUM.

If the Two Tubes AB, CE, communicate with each other, by a third Tube AC, parallel to the Horizon, it will make a Machine, which is call'd a Leaver of Water, which is such as that the Tube AB, will contain at a certain Height Four times as much as the Tube CE, at the same Height; because we have suppos'd the Base of the Tube AB, to be Four times that of the Tube CE: and by the same reason, the Water in

in deſcending 4 Inches, *for Example*, in the Tube CE, will rife but One Inch in the Tube AB. *Plate 23. Fig. 122.*

To the end that we may explain what we have been ſaying, we'll add a Cock FG, to the Tube of Communication AC, to open, when we have a mind to make the Water contain'd in the Tube CE riſe, and aſcend through the Tube AB, by paſſing through the Tube of Communication AC. But if we put Wine in the Tube AB, to a certain Height, and that we fill with Water the other Tube CE; by opening ſoftly the Cock FG, the Water in the Tube CE will drive up the Wine, and will make it riſe unmixt in the Tube AB, becauſe Wine has leſs Specifick Gravity than Water.

THEOREM III.

If Two Tubes of unequal Bigneſs communicate by means of a third Tube parallel to the Horizon; the Liquor pour'd into one of'em, will riſe up to a Level in the other Tube.

EACH of theſe Two Tubes may be perpendicular to the Horizon, or the One may be Inclin'd and the Other Perpendicular to the Horizon, or Both may be Inclin'd to the Horizon. In any of theſe Caſes, I ſay, that if any Liquor be pour'd into one of theſe Two Tubes, as high as you pleaſe, this Liquor will come to a Level, that is, will ſtand at equal Height in both Tubes.

Demonſtration of the Firſt Caſe.

Firſt, If the Two Tubes AB, CD, are perpendicular to the Horizon, and then you take from the biggeſt AB, the part EF equal to the Bigneſs of the Smalleſt CD, you will eaſily know that the Liquor in CD muſt be of the ſame Height, and remain in *Æquilibrio*, with the Liquor in the Tube EF, which you muſt conceive ſeparately from the Tube AB, becauſe thoſe Two Tubes CD, EF, being equal, the Liquor will have as much Force to riſe in the One as in the Other: Now tho' the Liquor of the Tube EF is not in a Pipe ſeparate from the whole Tube AB, nevertheleſs the Effect of it can neither be help'd nor hinder'd, by the Reſt of the Liquor of the Tube AB, becauſe it does not adhere to the Reſt, all the parts of any Liquor being ſo little united to one another, as not to be able to retain one another; and ſo the Effect of the Liquor in Tube EF, and that of the remaining part in AB, and conſequently the Effect of both together, that is, of the whole Cylinder AB is the ſame. Wherefore ſince the Liquor of EF is in *Æquilibrio* and level *Plate 24. Fig. 125.*

with

Plate 24. *Fig.* 125. with that in CD, all the Liquor in AB, tho' it be in greater Quantity, ought to remain in *Æquilibrio*, and at the same Height with that in CD. *Q. E. D.*

Demonstration of the Second Case.

Fig. 126. If the Tube AB be Inclin'd to the Horizon, and the other Tube CD be Perpendicular to the Horizon; imagine upon the Base of the Tube AB, the Tube AE perpendicular to the Horizon, and of the same Height as the Tube AB, to which it must consequently be equal, and equally heavy, when fill'd with the same Liquor, which will cause the Liquor at A to be equally press'd by that which is contain'd in the Inclin'd Pipe AB, and by that in the Perpendicular Pipe AE, whose effect therefore will be the same, as if it was in the Inclin'd Pipe AB; that is, the Liquor contain'd in the Inclin'd Prism AB, must as in the Perpendicular AE remain in *Æquilibrio*, and level with that of the perpendicular Pipe CD. *Q. E. D.*

Demonstration of the Third Case.

Fig. 127. Lastly, If both of the Tubes AB, CD, are Inclin'd to the Horizon, you must likewise imagine upon the Base of the Tube CD a Tube CF of the same Height to be perpendicular, after which you will by the foregoing Case know, that the effect of the Tube AB is the same as that of the Tube AE, and likewise that the Effect of the Tube CD is the same with that of the Tube CF; and as in the first Case the effect of the Two Perpendicular Tubes AE, CF, is the same; it is easy to conclude, that it ought likewise to be the same in the Two Inclin'd Tubes AB, CD; that is, the Liquor which is pour'd into one of them, will rise to a level in the other. *Which was left to be demonstrated.*

SCHOLIUM.

Thus we learn from this Proposition, the Reason of what Experience shews us every Day, (*viz.*) That Water rises as high as its Spring, when it runs thro' a Pipe which contains it, otherwise the Resistance of the Air, the Winds, and the Weight of the Water, wou'd hinder it from rising as high as its Spring, as may be seen in the Jetto's of Fountains.

LEMMA.

LEMMA.

If Two Cylinders of equal Thickneſs and Weight are of different Matter, their lengths will be to one another reciprocally as the Specifick Gravities of their Matters.

I Say, That if the Two Cylinders AB, CD, are of equal Thickneſs and Weight, but of different Matter; the Specifick Gravity of the Matter of the Cylinder AB: is to that of the Cylinder CD:: reciprocally as the length CD; is to the length AB; ſo that if the length AB be, *for Example*, twice the length CD, the Specifick Gravity of the Matter of the Cylinder CD will be twice as great as the Specifick Gravity of the Matter of the Cylinder AB; becauſe if theſe Two Cylinders AB, CD, were of the ſame Matter, the Cylinder AB being ſuppos'd double the Cylinder CD, wou'd Weigh twice as much: And as it is ſuppos'd to Weigh but juſt as much, its Matter muſt be proportionably of a leſs Specifick Gravity than the Matter of the Cylinder CD, *&c.* *Plate* 23. *Fig.* 121.

THEOREM IV.

Two different Liquors being pour'd into Two Tubes, that have a Communication by a Third Tube parallel to the Horizon, will have their Heights proportionable to their Specifick Gravities, when their Relative Weights come to be Equal.

I Say, that if in the Tube AB, which I ſuppoſe bigger than the Tube CD, there be, *for Example*, Water up to the Height AB, and in the Tube CD, Quickſilver up to CG, in ſuch manner, that thoſe Two Liquors may be in *Æquilibrio*; the Height AB of the Water: will be to the Height CG of the Quickſilver:: reciprocally as the Specifick Gravity of the Quickſilver; is to the Specifick Gravity of the Water. *Plate* 24. *Fig.* 125.

DEMONSTRATION.

If you imagine (*as in Prop.* 3.) within the biggeſt Tube AB, the Tube EF of the ſame bigneſs as the leaſt Tube CD, you will know that the Liquor in the Tube AB will have the ſame Effect in reſpect of the Tube CD, as if it was but in the Tube EF which is part of AB; and this Tube or Cylinder EF will be equal in Weight to the Cylinder CG; becauſe

the

Plate 24. *Fig.* 125. the Two Liquors which they contain, are suppos'd to be in *Æquilibrio*, which shews by the foregoing *Lemma*, that the Height of the Cylinder EF, which is the same as that of AB : is to the Height of the Cylinder CG :: as the Specifick Gravity of the Quick-Silver contain'd in the Cylinder CG : is to the Specifick Gravity of the Water contain'd in the Cylinder EF, which is suppos'd the same with that contain'd in the Cylinder AB. *Q. E. D.*

SCHOLIUM.

Fig. 124. What has been said in this, and the foregoing Proposition, may also be understood of the *Syphon*, which is a Recurve Tube, whose Two parts are call'd *Branches*; it being plain that a Recurve Tube is the same thing as Two Tubes, which have a communication by means of a third Tube, which in a *Syphon* is infinitely short, as G, by means of which the Two Branches GF, GH, communicate.

THEOREM V.

If a Cylinder of any heavy Liquor be inclin'd to the Horizon, the Relative Weight of that Liquor in its Tube : is to the Force by which it endeavours to go out at Bottom :: as the Length of that Tube : is to its Height.

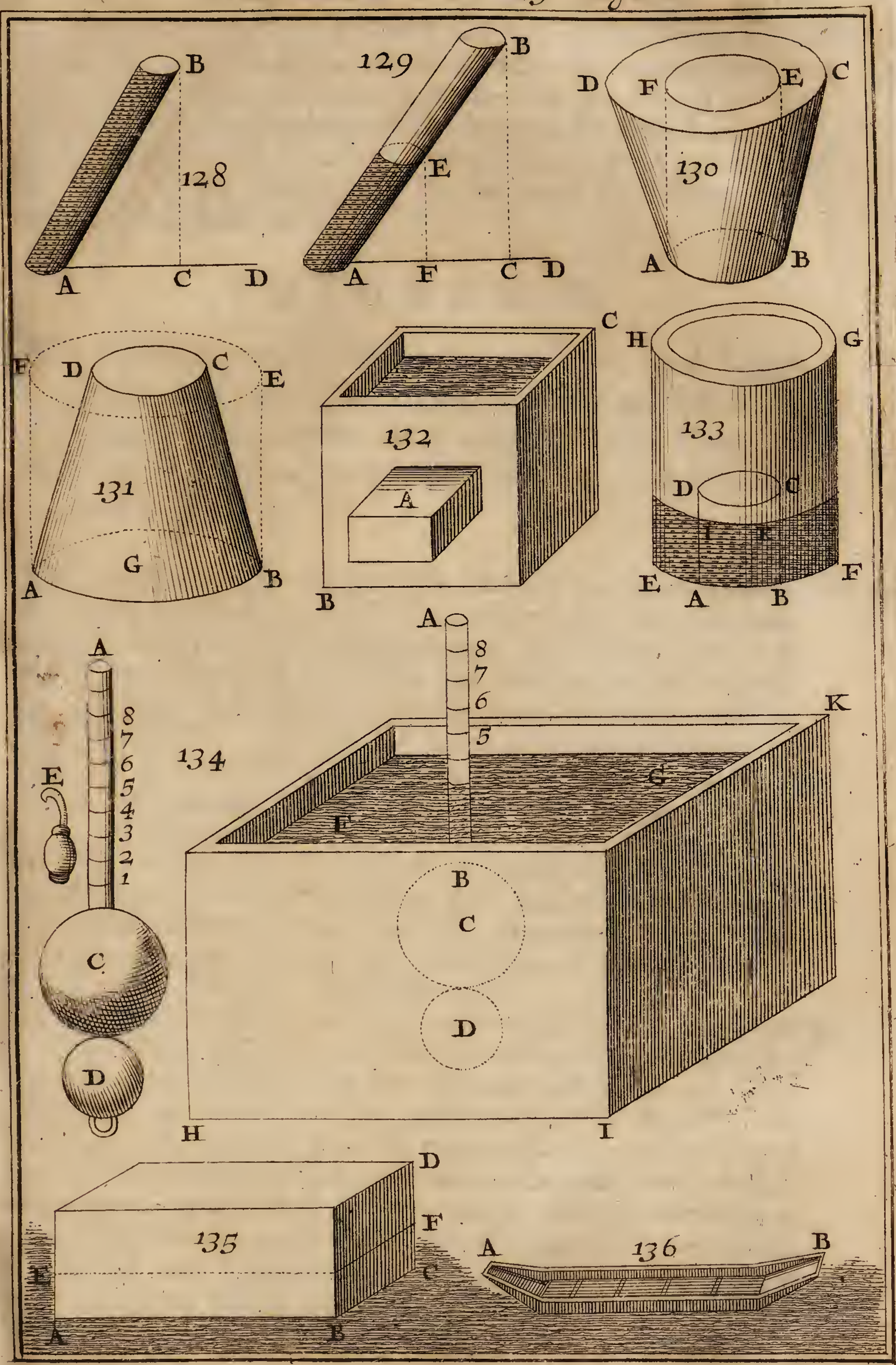
Plate 25. *Fig.* 128. I Say, That if the Tube or Cylinder AB inclin'd to the Horizon AD, be fill'd with a ponderous Liquor, *for Example*, with Water; the Relative Gravity of that Water in the Tube AB : is to the Force with which it endeavours to get out thro' A the Hole at Bottom :: as the Length AB : is to the Height BC.

DEMONSTRATION.

If the Water contain'd in the Tube AB, be consider'd as a Weight which endeavours to roll down an Inclin'd Plain, you will know *by Prop.* 1. *Chap.* 2. *Book.* 2. that the Relative Gravity of such a Weight : is to the Force which it has to descend along that Plain :: as the length of the said Plain : is to its Height; whence it is easy to conclude; That the Relative Gravity of the Water in the Tube AB : is to the Force with which it endeavours to descend thro' A, the Hole at Bottom :: as AB the length of the Tube : is to its Height BC. *Q. E. D.*

SCHOLIUM.

Tho' the Tube AB shou'd not be quite full of Water, as *for Example*, if it was only fill'd up to E, yet it wou'd always be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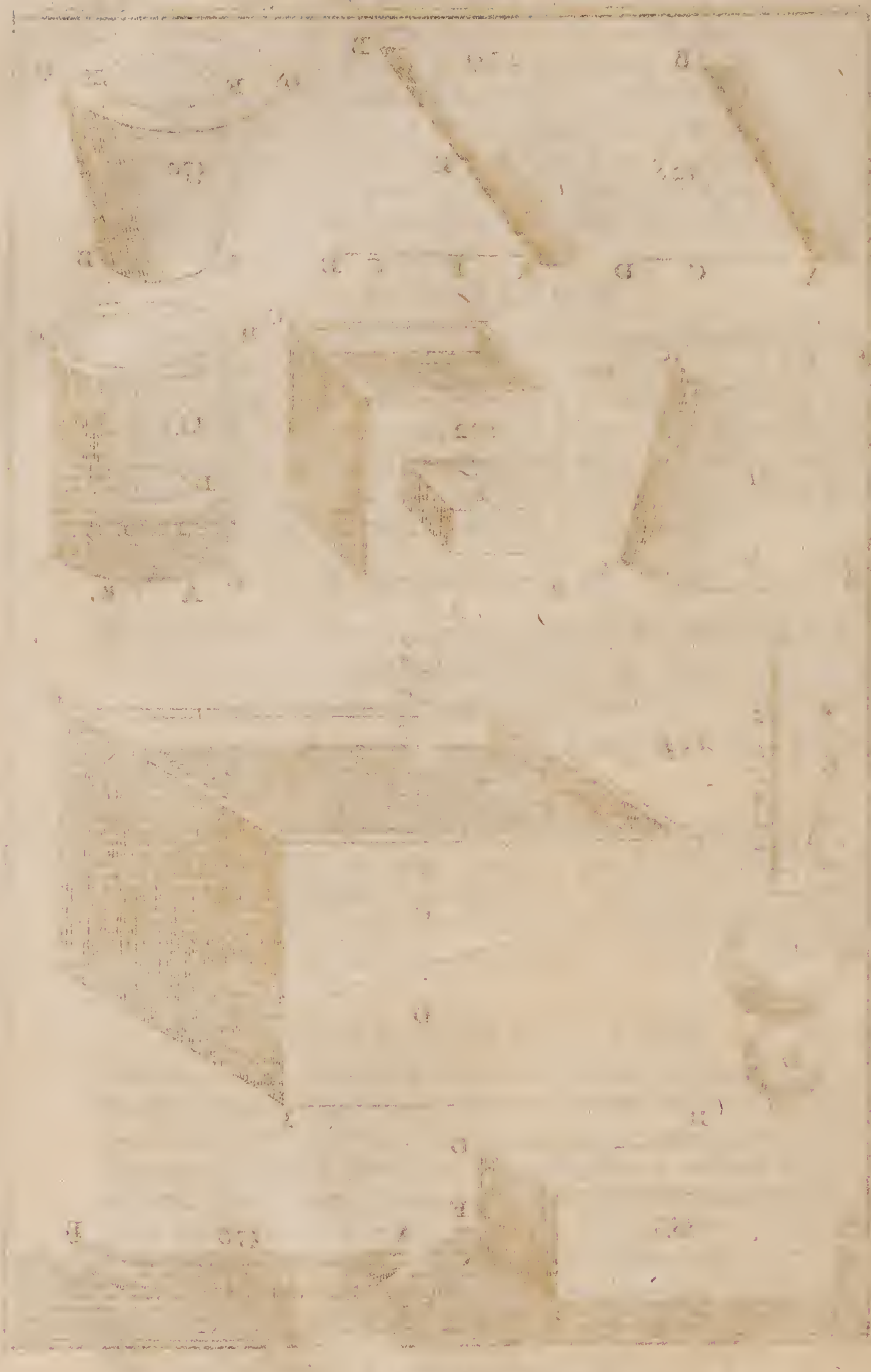


be true to say, that the Relative Gravity of the Water in its *Plate* 25.
Tube AE: will be to the Force wherewith it endeavours to *Fig.* 129.
descend thro' A the Hole at Bottom:: as the length of the Tube AB:: is to its Height BC; because the length AB: is to the Height BC:: as AE the length of the Cylinder of Water is to its Height EF, by reason of the Similar Triangles ABC, AEF, *&c.*

THEOREM VI.

If a Tube perpendicular to the Horizon, and bigger at One end than the Other, is fill'd with a heavy Liquor, that Liquor will have the same Force to go out at Bottom, as if that Hole was equal to the Hole in the Top. *

IT is evident that Water (*for Example*) contain'd in the *Fig.* 130.
Tube, or Vessel ABCD perpendicular to the Horizon, and first wider at top than at bottom, has neither more nor less Force to descend thro' AB the Hole at bottom, than if the said Hole AB shou'd be of the same Diameter with the other Hole CD; * as you may know if you imagine on the Base AB a Cylinder ABEF of the same Height with the Vessel, and perpendicular to the Horizon, for then it will be easy to judge that since Water only gravitates perpendicularly, no more of it than what is contain'd in the Cylinder ABEF, presses upon the Bottom AB, and what is contain'd in the rest of the Vessel on each side, does not at all gravitate upon the Bottom AB, but upon the Sides or inward Surface of the Tube ABCD. Wherefore, *&c.*

If the Vessel ABCD be larger at Bottom than at Top, that *Fig.* 131.
is, if the Hole AB be wider than the Hole CD, conceiving likewise upon the Base AB the Cylinder ABEF of the same Height as the Vessel, and perpendicular to the Horizon, it will be easily known that the Parts which are in the upper End of the Vessel ABCD, not only press upon those which are perpendicularly under them, but by their continual motion, press also the parts which are on every side; wherefore the parts A and B are as much press'd as the part G, and the whole Bottom AB is as much press'd as if the sides of the Tube ABCD, were the sides AF, BE, of the Cylinder ABEF.

* *Or rather, as if the Hole at Top was equal to that at Bottom.*

COROLLARY.

Plate 25. Fig. 131. It is evident from this Propoſition, That if ſeveral Veſſels of different Figure, but of the ſame Height, and perpendicular to the Horizon, be fill'd with the ſame Liquor; if they have their Bottoms of the ſame bigneſs, all thoſe Bottoms will be equally preſs'd.

THEOREM VII.

A Body whoſe Weight is equal to a Bulk of the Liquor of the bigneſs of the place that it takes up, will remain in Æquilibrio, *in a Veſſel full of the ſame Liquor.*

Fig. 132. I Say, That if the Weight of the Body A, which is immers'd in the Liquor of the Veſſel BC, be equal to the Weight of that Bulk of the ſame Liquor, (Water *for Example*) whoſe room it takes up; what ever Poſition or Place it be put into, in the Veſſel BC, it will remain in *Æquilibrio*; that is, it will neither riſe nor fall, becauſe that Body A has juſt as much Force as the Water which wou'd be in its room, ſince it is ſuppos'd as heavy as that Water, and that the ſaid Water wou'd be at reſt, if it was in the place of the Body; becauſe one Liquor does not drive away another of the ſame Specifick Gravity.

SCHOLIUM.

After the ſame manner you may know, that if the Body A was but partly immers'd in Water; ſo that a Bulk of Water equal to the immers'd Part ſhou'd be as heavy as the whole Body A; that Body A wou'd remain in *Æquilibrio*; that is, it wou'd not ſink any lower, or cauſe the Water to riſe any higher.

COROLLARY I.

This ſhews the reaſon why one cannot Fathom the Depth of ſome Seas with a Weight tied to a long Line; becauſe, tho' the Lead be Specifically heavier than Water, yet if the Line be Specifically lighter than Water, when the Depth is very great, and the Weight a ſmall one, ſuch a quantity of Line will be requir'd, that together with its Lead it may take up the Space of as much Water as is heavier than all the Rope and Lead; and in ſuch a Caſe, the Plummet will ſink no deeper, and therefore not ſhew of what depth that Sea is.

COROLLARY II.

It ſhews alſo the reaſon why you don't feel the Weight of the Bucket, when you draw Water, till after it is out of the Water; becauſe, before, it was ſuſtain'd by the Water whoſe room it took up. But for all this, you muſt not think that Water does not weigh in its Center; which wou'd be the ſame as to ſuppoſe that a Weight ceaſes to be heavy in the Scale of a Balance, becauſe we don't perceive its Gravity when an equal Weight apply'd in the other Scale keeps it in *Æquilibrio.*

COROLLARY III.

From this Propoſition it is eaſy to conclude, That a Body heavier than that Bulk of Water, whoſe room it takes up, muſt ſink wholly, and go to the Bottom. Whence it follows, that a Liquor Specifically heavier than another Liquor, will ſink, when pour'd on this laſt, and go to the loweſt place, cauſing the lighteſt Liquor to aſcend.

COROLLARY IV.

One may alſo eaſily conclude, That a Body lighter than that Bulk of the Liquor whoſe room it takes up, ought not to ſink over head into the ſaid Liquor, and therefore that a little Height of the Liquor is able to ſuſtain it. Whence it follows, That a Liquor whoſe Specifick Gravity is leſs than that of another, will remain at Top without mixing, if it be pour'd on ſoftly, and eſpecially when it is ſenſibly of a leſs Specifick Gravity than the other, as Oil in reſpect of Water, or Water in reſpect of Quick-Silver. *Plate* 25. *Fig.* 132.

Thus it is no wonder, that a Ship which had ſail'd very ſafely in the Open Sea, has Sunk and been loſt in the Mouth of a River of Freſh Water; becauſe the Sea-water is heavier than Freſh Water: For tho' the Burthen of the Ship be lighter than that Bulk of Salt-water whoſe room the Ship takes up, when it is in the Open Sea, and ſo the Veſſel is ſuſtain'd; it is heavier than that Bulk of Freſh Water whoſe room the Ship takes up in a River, and ſo the Veſſel ſinks, becauſe there is not Force enough to ſuſtain it, it being certain that the Specifick Gravity of Sea-water is much greater than that of River, Well, or Spring-water.

Neither is there any reaſon to wonder, why a Log of Wood after ſwimming a long time upon the Water, ſinks at laſt; becauſe ſuch a piece of Wood may be of the ſame, or of a greater Specifick Gravity than Water, without ſinking

Plate 25. *Fig.* 132. by reaſon of the Pores which it may have; for thoſe Pores being fill'd with Air, that Air and Wood make up One Whole lighter than that Bulk of Water whoſe room is taken up, which is the reaſon why it does not ſink: But the Water inſinuating it ſelf by degrees into the Pores, drives out the Air contain'd in them and ſupplies its place; and then the Water in the Pores, and the Wood make up One Whole heavier than the Water whoſe room it takes up, and ſo it muſt ſink of courſe.

It is alſo eaſy to underſtand why Birds Fly in the Air Specifically lighter than themſelves: And why Men Swim in Water Specifically lighter than themſelves; becauſe the Birds beat the Air with their Wings, and Men the Water with their Hands and Legs, which renders them leſs heavy, becauſe their Motion is Horizontal; beſides the Motion which they give the Liquor, cauſes that Liquor under them to preſs underneath againſt them more than it is preſs'd †.

COROLLARY V.

It follows alſo, That the ſame Body ſinks differently in Liquors of different Specifick Gravities, and deepeſt in the lighteſt Liquor. Thus we ſee that a laden Ship ſinks deeper (or draws more Water) in a River, than it does in the Sea; becauſe as we have already obſerv'd, River-Water is lighter than Sea-Water; and this ſometimes cauſes the Veſſel to ſink.

COROLLARY VI.

It follows likewiſe, That a Body weigh leſs in Water than in Air, by juſt ſo much as is the Weight of an equal Bulk of Water; that is, of as much Water as it takes up the room of. Whence it follows, That if a Balance laden with Two ſorts of Metals of different Specifick Gravities be in *Æquilibrio* in the Air, it will loſe its *Æquilibrium* in the Water; becauſe the Metals being ſuppos'd different, will not loſe of their Weight equally in Water, it being certain that that Metal whoſe Specifick Gravity is the greateſt, will loſe leſs of its Weight in Water than the other, becauſe it takes up the room of a leſs Bulk of Water.

† Reaction *is the cauſe of this: For ſince* Action *and* Reaction *are always Equal and Contrary; as much as Men, or Birds puſh the Water, or Air Backwards with their Hands, and Legs, or Wings; ſo much does the Air puſh them forwards. This holds true even when they Swim or Fly upwards; for tho' the Force of Gravity continually endeavours to preſs 'em downwards, yet if they beat the Fluid, which they are in, with a greater Force than that which Gravity is puſhing them down with, that Fluid will react with the ſame Force, and ſo cauſe them to aſcend.*

COROLLARY VII.

Laſtly, it follows, That tho' Metals, are heavier than Water, yet a Hollow Globe of Iron muſt Swim upon the Water, if ſuch a Globe with the Air contain'd in it be no heavier than an equal Bulk of Water; ſince no Heavy Body can ſink over Head unleſs it be Heavier than an equal Bulk of Water. Thus we ſee Braſs Swim upon the Water when it is hollow, as Kettles; and ſink when it is in a lump.

Nevertheleſs it happens that a common Iron or Steel-Needle, which is not wet, will Swim if it be laid gently on the Top of ſtanding Water, but this happens by reaſon of the dryneſs of the Needle to which the Water reſiſts. And as the Property of Iron when it is Free and in *Æquilibrio*, is to turn towards the Pole, Experience will ſhew that a Steel-Needle laid along upon the Surface of a ſtill Water will turn round ſeveral times; and then ſettle with one of its ends towards the North, and the other towards the South.

THEOREM VIII.

A Priſm, whoſe Specifick Gravity is leſs than that of the Water, being put into the Bottom of a Veſſel will be in Æquilibrio, *when ſo much Water is pour'd into the ſaid Veſſel, that the Height of the Water: ſhall be to the Height of the Priſm:: reciprocally as the Specifick Gravity of the Priſm: is to that of the Water.*

I Say, That if the Priſm ABCD, of a leſs Specifick Gravity than Water, be laid in the Bottom of the Veſſel EFGH, it will be in *Æquilibrio*, when Water is pour'd in to ſuch an Height, that That Height AI: may be to AD the Height of the Priſm:: reciprocally as the Specifick Gravity of the ſaid Priſm: is to the Specifick Gravity of the Water. *Plate* 25. *Fig.* 133.

DEMONSTRATION.

For ſince AI: is to AD:: as the Specifick Gravity of the Priſm ABCD is to that of the Water, *by Supp.* If in the place of the Two firſt Terms AI, AD, you put the Priſms ABKI, ABCD, which are in the ſame *Ratio*, becauſe they have equal Baſes, you will know that the Priſm ABKI: is to the Priſm ABCD:: as the Specifick Gravity of the Priſm ABCD: is to that of the Water; ſo that if the Priſm ABKI is half of the

Plate 25. Fig. 133. Prism, ABCD, the Specifick Gravity of the Prism ABCD will also be half the Specifick Gravity of the Water; and as the Prism ABIK weighs also but half of the Prism ABCD, because it has been suppos'd equal to half of it, it follows, That the Gravity of the Prism ABIK, is equal to that of the Water, and consequently, *by Prop.* 7. that the Prism ABCD is in *Æquilibrio.* *Q.E.D.*

COROLLARY.

From this Proposition it is plain, That if you wou'd cause a Prism to rise in a Vessel, by means of a Liquor Specifically Heavier than the said Prism, you must pour in so much of the Liquor, that the Height of the Liquor may to be the Height of the Prism in a *Ratio* something Greater than the Specifick Gravity of the Prism is to the Specifick Gravity of of the Liquor.

CHAPTER II.

Of the Problems.

MOST of the Hydrostatical Problems are very Pleasant and Profitable for the Uses of Human Life; we shall only give here the most Necessary, leaving those that are Pleasant and Curious for our Mathematical and Physical Recreations.

PROBLEM I.

How to find out what Proportion the Specifick Gravities of several different Liquors bear to one another.

Plate 25. Fig. 134. TAKE a long Tube of Glass, as AB, whose upper End A must be Seal'd *Hermetically*, that is, with the Glass of the Tube it self, melted at the Flame of such a Lamp as is us'd by Enamellers, and the other end B must have the Ball or Bubble C which communicates with the Tube AB, full of Air, so that the Figure ABC, represents a Bottle, whose Neck is AB, which must be divided into a certain Number of equal Parts or Degrees, which will shew how much one Liquor is heavier than another; for if the Instrument AC be put into the Liquor FG, contain'd in the Vessel HIK, adding to its under Part a little Ball, as D, with Quicksilver in it, which

which is the heavieſt of Liquors, as the Air contain'd in AC is the lighteſt of Liquors; or inſtead of Quickſilver, you may hook on a little Weight as E at the bottom, which will by its Gravity cauſe the Inſtrument to deſcend Perpendicularly, and ſink more or leſs in the Liquor, as the Liquor is lighter or heavier, the Proportion of which may be known by the number of Degrees, or equal Parts of the Tube AB, which will ſink into the Liquor. This Problem may be alſo ſolv'd by means of the following.

PROBLEM II.

How to find the Proportion between the Specifick Gravity of a Liquor and that of a Solid, ſpecifically Heavier than the ſaid Liquor.

TO find the Proportion between the ſpecifick Gravity of a Metal and that of a Liquor, weigh in the Air with a nice pair of Scales, a piece of the Metal, whoſe Weight we will ſuppoſe 10 *lib.* and having faſten'd the ſame Piece of Metal to one of the Scales of the Balance with a ſilken Thread, or Horſe-Hair, weigh it in the propos'd Liquor, in ſuch manner that it be wholly cover'd by the Liquor, without letting the Scale touch the Liquor, and if its Weight then be, *for Example*, 9 *lib.* which is One Pound leſs than it weigh'd in the Air; This difference will ſhew that a Bulk of the propos'd Liquor equal to that of the Piece of Metal weighs One Pound, and conſequently that in this Example, the ſpecifick Gravity of the Metal is Ten times greater than the ſpecifick Gravity of the Liquor.

By this Method it has been found that Gold loſes in Water about One Ninteenth Part of its Weight, Mercury or Quickſilver $\frac{1}{15}$, Lead $\frac{1}{12}$, Silver $\frac{1}{10}$, Copper $\frac{1}{9}$, Iron $\frac{1}{8}$, Tin $\frac{1}{7}$; and ſomething more, it being certain that any Body loſes of its Weight in Water in proportion to that Water whoſe room it takes up, ſo that if that Bulk of Water weighs One Pound, the Body will weigh One Pound leſs than it did in the Air, as well becauſe the Water being divided with difficulty ſupports more, as becauſe when it is forc'd out of its place, it endeavours to return into it, and preſſes proportionably the other parts of the Water which encompaſs the Body.

It is alſo by this means that it is known, that if you take equal Bulks of Metal and of Water, if the Water weigh 10 *lib.* Tin will weigh 75, Iron almoſt 81, Copper 91, Silver

104, Lead $116\frac{1}{2}$, Mercury 150, and Gold $187\frac{1}{2}$ *lib.* This Proportion will appear the better by the following Table which we have taken from Father *De Chale*'s Hydroſtaticks.

COROLLARY.

From this Problem is eaſily deduc'd the manner of knowing what Proportion the ſpecifick Gravities of Metals and Liquors bear to one another, of Liquors themſelves, and of Metals and Liquors of the ſame Species which have ſome difference; for if you know what Proportion a Liquor bears to ſuch and ſuch Metals, you will know what Proportion they bear to one another; And likewiſe if you know what Proportion a Metal bears to ſuch and ſuch Liquors, you may know what Proportion thoſe Liquors bear to one another. As if you know that the ſpecifick Gravity of freſh Water is to that of Gold, as 1 to 19, and to that of Lead, as 1 to 11, you may conclude, that the ſpecifick Gravity of Gold: is to the ſpecifick Gravity of Lead:: as 19: to 11. Likewiſe, knowing the ſpecifick Gravity of Gold to be to that of Freſh Water as 19 to 1, and to that of Mercury, as 19 to 14, one may conclude, that the ſpecifick Gravity of Water: is to the ſpecifick Gravity of Quickſilver:: as 1: to 14.

According to this Method is the following Table made, which ſhews in Numbers the Proportion of the different Weight of Metals, of Liquors, and of Stone of the ſame Bulk. Thus you ſee, that ſuppoſing a determinate Bulk of Gold to weigh 100 *lib.* the ſame Bulk of Mercury will weigh $71\frac{1}{2}$ *lib.* the ſame Bulk of or Lead $60\frac{1}{2}$ *lib.* and ſo of the reſt.

Mat-

Matters.	*Pounds.*
Fine Gold	100
Mercury	$71\frac{1}{2}$
Lead	$60\frac{1}{2}$
Silver	$54\frac{1}{2}$
Copper	$47\frac{1}{3}$
Latten.	45
Common Iron	42
Common Tin	39
Fine Pewter	$38\frac{1}{2}$
Load-Stone	26
Marble	21
Stone	14
Chryftal	$12\frac{1}{2}$
Water	$5\frac{2}{3}$
Wine	$5\frac{1}{4}$
Wax	5
Oil	$4\frac{3}{4}$

After the fame Manner is this other following Table calculated, where you may know the Weight of a Cubick Foot, and that of a Cubick Inch of different Bodies; where you muft obferve that the Pound is of Two Marks, or 16 Ounces: the Mark of 8 Ounces: the Ounce of 8 Drams: the Dram of Three Penny-Weight, or 72 Grains: the Penny-Weight 2 Mailles or 24 Grains: and the Maille 12 Grains.

Mat-

	W. of a Cubick Foot.		W. of a Cubick Inch.		
Matters.	*Pounds.*	*Ounces.*	*Ounces.*	*Drams.*	*Grains.*
Gold	1326.	4.	12.	2.	17.
Mercury	946.	10.	8.	6.	8.
Lead	802	2.	7.	3.	30.
Silver	720.	12.	6.	5.	28.
Copper	627.	12.	5.	6.	36.
Iron	558.	0.	5.	1.	24.
Tin	516.	2.	4.	6.	17.
White Marble	188.	12.	1.	6.	0.
Free Stone	139.	8.	1.	2.	24.
Water of the *Seine*	69.	12.	0.	5.	12.
Wine	68.	6.	0.	5.	5.
Wax	66.	4.	0.	4	65.
Oil	64.	0.	0.	4.	43.
Dried Oak	58.	4.	0.	4.	22.
Walnut-Tree	41.	12.	0.	3.	6.

The Weight of a Cubick Foot of any Matter once known, it is easy to know that of an Inch of the same Matter, *viz.* by dividing the known Weight of the Cubick Foot by 1728, because a Cubick Foot contains 1728 Cubick Inches. Thus knowing that a Cubick Foot of Fine Gold weighs 1326 Pounds and 4 Ounces, if you divide that Weight by 1728, the Quotient will be 12 Ounces, 2 Penny-Weight, and 17 Grains for the Weight of a Cubick Inch of Gold; and reciprocally if the Weight of a Cubick Inch of any Matter be known, you will have the Weight of a Cubick Foot of the same Matter, if you multiply that known Weight by 1728. Thus since a Cubick Inch of Lead weighs 7 Ounces, 3 Penny Weight, and 30 Grains, if this Weight be multiplied by 1728, you will have 802 Pounds, and 2 Ounces, for the Weight of a Cubick Foot of Lead.

This Table may serve to find out the Weight of a Body, whose Solidity is known; and reciprocally to know the Solidity of a Body whose Weight is known. As if you wou'd find out the Weight of a Carv'd Stone, whose Solidity is known, and is, *for Example*, of 36 Cubick Feet; multiply that Number 36 by the Weight of a Cubick Foot of Stone, which in the foregoing Table is 139 Pounds, and 8 Ounces, and the Product of the Multiplication will give 5040 Pounds for the Weight of the Stone propos'd, *&c.*

The

The foregoing Table may alſo ſerve for the Conſtruction of the following, which ſhews the Specifick Gravity of a Cylindrick Foot, and of a Cylindrick Inch of Bodies of different Matter; where you muſt obſerve that by a *Cylindrick Foot* we underſtand a Cylinder; the Diameter of whoſe Baſe is One Foot, and its Height as much; and by a *Cylindrick Inch*, a Cylinder whoſe Height is an Inch, and the Diameter of whoſe Baſe is an Inch.

The following Table has been made by means of the foregoing, by multiplying the Weight of each Body that you find in it, always by 11, and dividing the Product by 14: but if you wou'd have it more exact, you muſt Multiply by 785, and divide by 1000.

It may ſerve as the foregoing, to find out the Weight of a Cylindrick Body if you know its Solidity, or only its Height and the Diameter of its Baſe; for if the Square of the Diameter be multiply'd by the Height, and the Product by the Weight mark'd in the Table, you will have the Weight of the propos'd Cylinder, *&c.*

	Weight of a Cylindrick Foot.		Weight of a Cyl. Inch		
Matters.	*Pounds.*	*Ounces.*	*Ounces.*	*Drams.*	*Grains.*
Fine Gold	1042.	1.	7.	1.	65.
Mercury	743.	12.	5.	1.	23.
Lead	630.	4.	4.	3.	1.
Silver	566.	9.	3.	7.	35.
Copper	499.	3.	3.	3.	29.
Iron	438.	7.	3.	0.	25.
Tin	405.	8.	2.	6.	38.
White Marble	148.	5.	1.	3.	0.
Free Stone	109.	10.	1.	0.	8.
Water of the *Seine*	54.	13.	0.	3.	3.
Wine	53.	11.	0.	2.	70.
Wax	52.	1.	0.	2.	65.
Oil	50.	4.	0.	2.	51.
Dried Oak	45.	7.	0.	3.	27.
Walnut-Tree	33.	3.	0.	2.	30.

PRO-

PROBLEM III.

How to find out what Burthen a Ship can carry upon Fresh, or upon Salt Water.

Plate 23. Fig. 122. IT is evident by what has been said in *Theorem* 7. that a Ship can carry the Weight of a Bulk of Water equal to its Bulk, allowing for the Weight of the Nails and Iron Hoops that bind it; for without the Iron it wou'd not sink when full of Water, because the Timber of which it is made, is very near of the same specifick Gravity as the Water.

Therefore to solve this Problem, you must know the Capacity or Bulk of the Vessel, and likewise the specifick Gravity of the Water. It is said that a Cubick Foot of Sea-Water weighs about 73 Pounds, wherefore if the Capacity or Solidity of the Vessel be, *for Example*, of 1000 Cubick Feet, Multiplying 1000 by 73 you will have 73000 *lib.* for the Burthen of the Ship; because a Bulk of Water of 1000 Cubick Feet weighs 73000 lib.

SCHOLIUM.

At Sea a Ships Burthen is not express'd by Pounds, because their Numbers wou'd be too great; but by *Tuns*, being 2000 *lib.* or Twenty Hundred, because a Tun fill'd with Sea-Water weighs about as much. So when we say that such a Ship carries, *for Example*, 100 Tun, we mean that it carries 200000 *lib.* or 2000 Hundred Weight; because the Hundred at Sea is always 100 *lib.*

PROBLEM IV.

Knowing the Weight of a Prism, how to mark exactly how far it will sink in the Water.

Plate 25. Fig. 135. IF the Prism ABCD weighs, *for Example*, 365 Pounds, you will find how far it must sink in Water, by knowing the specifick Gravity of that Water; if it be Sea-Water, of which a Cubick Foot weighs 73 *lib.* you must by that 73, divide 365 the Weight of the Prism ABCD, and the Quotient 5 will shew, that 5 Cubick Feet of the said Water weigh also 365 Pounds. Whence it is easy to conclude, that the Prism ABCD will sink in the Water, till it has taken up the Place of 5 Cubick Feet: And therefore to know how deep it ought to sink, you must at bottom take off

a Prism

a Prifm of 5 Cubick Feet, of the fame known Bafe, becaufe that of the Prifm ABCD is known, as *for Example*, of 120 fquare Inches, by which, dividing 5 Cubick Feet, or 8640 Cubick Inches, you will have 72 Inches, or 6 Foot (Long Meafure) for the Height AE, to which ABCD the propos'd Prifm will fink into the Water, becaufe the Prifm ABCEF, which is immerg'd, is exactly of 5 Cubick Feet.

SCHOLIUM.

According to this Method, knowing the Burthen and the Bulk of a Veffel, as of the Veffel AB, one may find out how deep it will fink, and by knowing how deep it will fink, one may know its Burthen; but befides the Bulk of the Veffel, you muft alfo know the Solidity of every one of its Parts.

If, *for Example*, the Solidity from the Bottom to fuch a Height be of 450 Cubick Feet, and the Burthen of the Veffel be 32850 *lib.* which is the Weight of 450 Cubick Feet of Sea-Water, reckoning 73 *lib.* for the Weight of One Cubick Foot, you will find that it muft fink in the Sea fo deep, or a little deeper; its Burthen will be known by the Solidity of the Part which finks into the Water, which having been fuppos'd of 450 Cubick Feet, which take up the room of as much Water as weighs 32850 *lib.* thofe 32850 Pounds will confequently be the Burthen of the Veffel.

PROBLEM V.

How to know by Hydroftaticks *if a doubtful Piece of Money, Gold, or Silver, be Good or Bad.*

IF you queftion the Goodnefs of a Piece of Gold, tho' it has its due Weight, procure a Piece of good Gold which weighs as much, fo that each be *Æquilibrio* in the Air, if they be laid in the Scales of an exact Balance, Then weigh thofe Two Pieces of Gold, tying them to the Scales of the Balance with Thread, or Horfe-Hair, that they may fink into the Water without wetting the Scales: And then if thofe Two Pieces of Gold be equally Good, they will remain in *Æquilibrio* as well in the Water as in the Air, otherwife that which will weigh leaft in the Water will be Falfe; that is, mix'd with fome other Metal, more or lefs, according as it is more or lefs light in the Water, becaufe different Metals lofe differently of their Weight in Water, That lofing moft which is of a lefs fpecifick Gravity, becaufe it is fuftain'd by a greater Bulk of Water, becaufe, that it may weigh as much

as

as another Metal of a greater ſpecifick Gravity, it muſt have a greater Bulk, which will take up more room in the Water.

SCHOLIUM.

Vitruv. *l.9. c.3.* When the propos'd Piece of Gold or Silver is of a conſiderable bigneſs, (ſuch as was the Crown of Gold, which *Hiero*, King of *Syracuſa*, propos'd to *Archimedes*, to know whether the Gold-ſmith had employ'd the 18 *lib.* of pure Gold, which he had given him to make that Crown with, ſuſpecting that the Gold-ſmith had mix'd a great deal of Silver with it) it will not be neceſſary to weigh the Two pieces of Gold in Water, but it will ſuffice to immerge 'em one after another, in a Veſſel which has Water in it, it being certain that that which raiſes the Water higheſt, muſt neceſſarily be falſe, becauſe tho' it is equally heavy, it muſt be greater in Bulk, and conſequently, mix'd with a Metal of a leſs ſpecifick Gravity.

The Story ſays that this Thought came into *Archimedes*'s Head when he was in the Bath, becauſe having obſerv'd that his Body rais'd the Water in proportion of the room it took up, he gueſs'd that by that means he might eaſily find out whether there was any Silver mix'd in the Crown. For this end he procur'd a Lump of Silver, and a Lump of Gold, equal in Weight to the Crown, and he immerg'd in Water thoſe Two Bodies and the Crown, one after another, and perceiving that the Silver had driven more Water out of the Veſſel than the Crown, and the Gold; and the Crown more than the Gold: he concluded from thence that the Crown was not of pure Gold, but had Silver mixt, ſince it took up a greater Space in the Water.

To know in this Example what Quantity of Silver the Gold-ſmith had mix'd in the Crown of Gold, whoſe Weight has been ſuppos'd of 18 *lib.* one muſt exactly meaſure the different Quantity of Water which will correſpond to the Bulk of the Crown, and that of the Two Maſſes of pure Gold and of pure Silver, which I ſuppoſe equal in Weight to the Crown, and conſequently unequal in Bulk; after which one may conclude, that if the Crown takes up more room in the Water than the Maſs of Gold, it is but in proportion of the Silver which is mix'd with it, the Quantity of which may be thus known by the Rule of *Alligation.*

Let us ſuppoſe the Maſs of Gold has driven out One Pound of Water, the Maſs of Silver One Pound and a Half, and the Crown One Pound and a Third Part; in this Suppoſition the Gold which drives out One Pound muſt be mix'd with the Silver which drives out One Pound and a Half,

in ſuch manner that both together may drive out One Pound and a Third Part. For this end diſpoſe the 3 known Numbers $1, 1\frac{1}{2}, \frac{1}{3}$, as you ſee 'em here; ſo that the Number $1\frac{1}{3}$, which anſwers to the Number which we want, may be between the Two others.

1		$\frac{1}{6}$
	$1\frac{1}{3}$	
$1\frac{1}{2}$		$\frac{1}{3}$
		$\frac{1}{2}$

Then write down the difference $\frac{1}{3}$ of the Two Firſt over againſt the Third $1\frac{1}{2}$, then the difference $\frac{1}{6}$ of the Two Laſt over againſt the Firſt 1. Laſtly, add together theſe 2 differences $\frac{1}{6}$. $\frac{1}{3}$, and divide by their Sum $\frac{1}{2}$, the Number $\frac{1}{6}$ which anſwers to the Gold, and you will have $\frac{1}{3}$ for the Quantity of Gold which was in the Crown. Divide alſo by the ſame Sum $\frac{1}{2}$, the Number $\frac{1}{3}$ which anſwers to the Silver, and you will have $\frac{2}{3}$ for the Quantity of Silver that was in the Crown, whoſe Weight being of 18 lib. it will appear that in this Suppoſition there was in the Crown Six Pounds of Gold, and Twelve Pounds of Silver.

If you wou'd ſolve this Problem by *Algebra*, you muſt conſider, that ſince we have ſuppos'd the Gold to drive out One Pound of Water, the Silver One Pound and a Half, and the Crown One Pound and a Third part, it is the ſame as if a certain Meaſure of Gold ſhou'd weigh One Pound, and the ſame Meaſure of Silver One Pound and a Half; and we ſhould go about to mix theſe Two Metals together in ſuch manner, that a Meaſure of that Mixture ſhou'd weigh One Pound and One Third part. Therefore it is requir'd that we find out what Quantity of Gold, and what Quantity of Silver muſt be mix'd together to make One Meaſure of their Mixture weigh a Pound and a Third part.

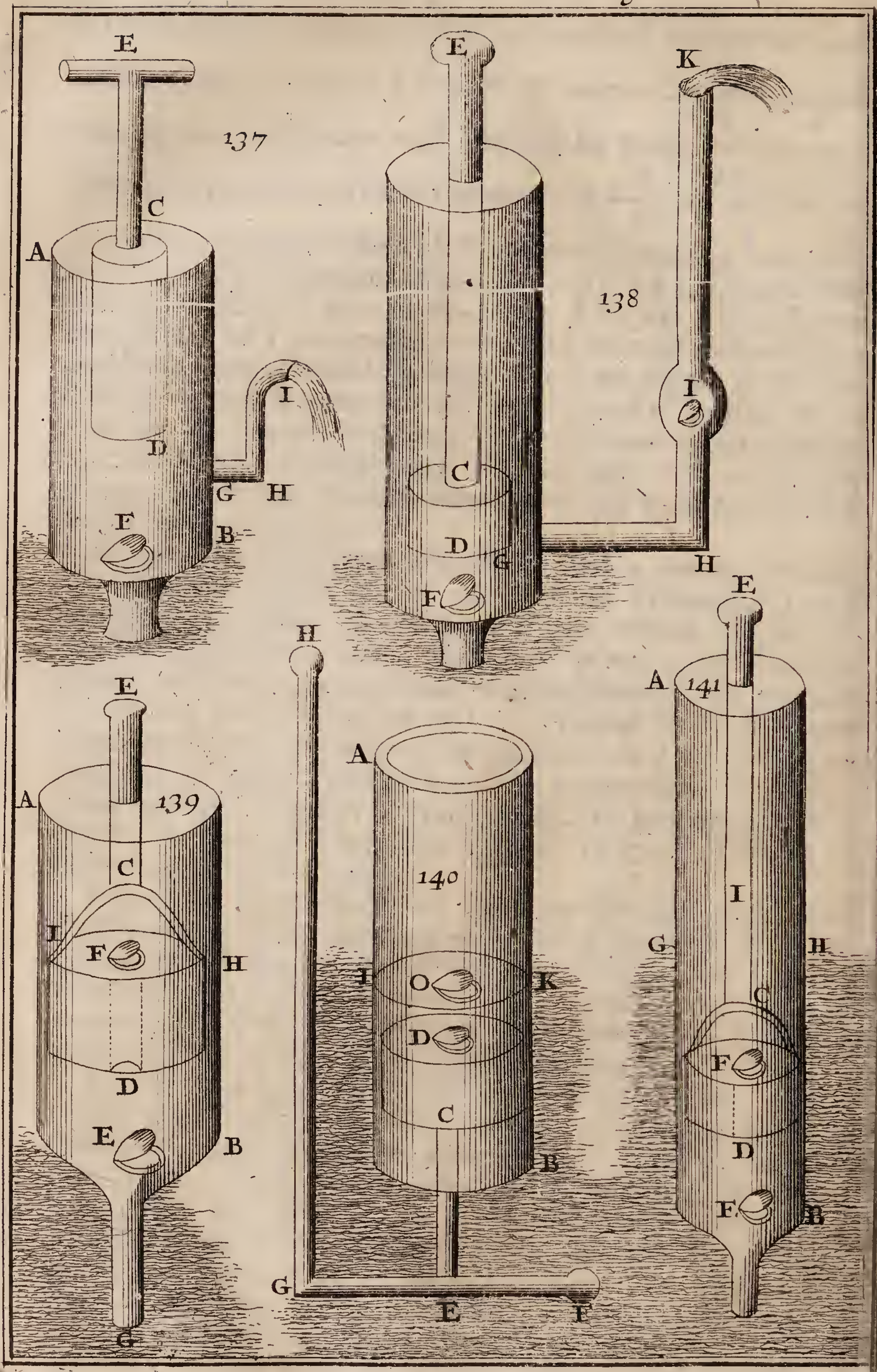
In order to this, let x ſignify the Number of Meaſures at One Pound a Meaſure, and y the Number of Meaſures at One Pound and a Half a Meaſure, and then the Meaſures at One Pound a Meaſure will be worth $1x$, and the Meaſures at One Pound and a Half a Meaſure will be worth $\frac{3}{2}y$, and the whole to-

together will be worth $1x + \frac{3}{2}y$: and as the Two Numbers of Meaſures together, or $x + y$ muſt be worth One Pound and One third, the Mixture will likewiſe be worth $\frac{4}{3}x + \frac{4}{3}y$: Therefore you will have this Equation, $1x + \frac{3}{2}y = \frac{4}{3}x + \frac{4}{3}y$, which being multiplied by 6, to avoid Fractions, you will have this other Equation, $6x + 9y = 8x + 8y$, from which taking $6x$ and $8y$, you will have this, $y = 2x$, which ſhews that inſtead of y, one may put $2x$; and becauſe we have ſuppos'd the Crown, which is worth $x + y$, to weigh 18 *lib.* we ſhall have this Equation $x + y = 18$, and if inſtead of y, you put its known Value $2x$, you will have this Equation $3x = 18$, and conſequently $x = 6$, and $y = 12$, which ſhews that in the Crown, there were Six Pounds of Gold and Twelve of Silver.

If you wou'd not have recourſe to *Algebra*, nor to the Rule of Alligation, make uſe of the Rule of Proportion; and conſider, that ſince the Maſs of Silver which weighs 18 Pounds, drives out half a Pound of Water more than the Gold, and the Crown which weighs likewiſe 18 Pounds drives out but One third part of a Pound of Water more than the Gold, by reaſon of the Silver mix'd with it; you muſt ſay, If Half a Pound Exceſs : is proportional to 18 *lib*:: what will One third of a Pound Exceſs be proportional to? and by the Rule of Three direct, you will find 12 Pounds of Silver mix'd in the Crown.

Inſtead of Two Maſſes of the ſame Weight with, and different Bulk from, the Crown; you may make uſe of Two Maſſes of the ſame Bulk and different Weight, and then it is evident that if there be Silver mix'd in the Crwn, it will weigh leſs than the Maſs of Gold in Proportion of the Silver mix'd with it, which you may find out thus.

As we have ſuppos'd the Crown to weigh 18 lib. it will weigh more than the Silver in proportion to the Gold that it contains: wherefore if the Maſs of Gold equal in Bulk to the Crown weigh, *for Example*, 24 *lib.* and that of Silver only 16 *lib.* you muſt ſay, If 8, the difference between the Weight of the Maſs of Gold and the Weight of the Maſs of Silver: anſwers to 16 *lib*. of Silver:: to how many Pounds of Silver will 6, the difference between the Weight of the Maſs of Silver and the Weight of the Crown, anſwer to? and by the Rule of Three direct, you will find 12 Pounds of Silver mix'd in the Crown, *&c.*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CHAPTER III.

Of Hydroſtatical Engines.

TO deſcribe all the Machines which have been Invented for the raiſing and drawing of Water wou'd be Endleſs; wherefore we ſhall only ſpeak of thoſe which are the moſt Uſeful, and which agree beſt with our Subject.

Of Pumps.

A *Pump* is an Engine, made like a Syringe, to draw Water out of a deep Place, and raiſe it by means of a round Piece of Wood, bound round with Tow, call'd the *Piſton*, or *Rammer*, which is mov'd backwards and forwards in a long Pipe, call'd the *Body of the Pump*, or *Barrel*.

Let AB be the Barrel, and CD the Piſton fix'd to the Rod CE, wherewith you move the Piſton in the Barrel AB, which muſt be every where cloſe and tight, except at the lower End which is in the Water, where there muſt be a ſmall Hole for the Water to enter into the Barrel, when the Piſton CD is drawn up. This Hole muſt be cover'd with a *Valve* F, which is made of Two flat pieces of Leather joyn'd together, the One of which has a Hole thro' it which the Other ſhuts up, and the cloſer theſe pieces fit together, the more perfect is the Valve. *Plate 26. Fig. 137.*

All Valves are not made the ſame way, which is the reaſon they have different Names; for when a Valve is flat, like a Board, it is call'd a *Clack*; (in French *Clapet*) and when it is round and goes tapering like a Cone, it is call'd a *Sucker* (in French *Axe*). Thoſe which are moſt in Uſe at preſent are Round and Convex, and are call'd in French *Soupapes a Queue*, when they have a Tail which comes perpendicular out of the middle of their Convexity, which Tail by its Weight draws down the convex part, to make it ſtop up cloſe the round Hole thro' which the Water paſſes, lifting up the Valve when the Piſton is rais'd.

Theſe Valves are very uſeful to ſtop the Water in a Pump, keeping it from coming back again when once it has been rais'd by means of the Piſton CD, which muſt move up and down freely in the Barrel AB, and at the ſame time exactly fill it, that the Air may not paſs between when the Piſton is drawn up; and then when the ſaid Piſton is rais'd, ſince Air can't ſucceed in the place of it, the Clack F will riſe and

Plate 26. *Fig.* 137. give way for the Water to paſs thro' the Hole which it ſtop'd before; and on the contrary, when you puſh down the Piſton CD and preſs the Water which has been rais'd, the Clack F ſhuts, and ſince the Water can't get out that way, it is forc'd out thro' the Pipe GHI, which communicates with the Body of the Pump †.

Fig. 138. Such a Pump is call'd a *Force-Pump*, becauſe it forces up the Water by preſſing it; and by means of ſuch an Engine you may raiſe the Water as high as you pleaſe, if to the Rod CE, a Power be applied as great as the Reſiſtance of the Water which is in the Pipe HI, and if there be a Clack at I to open and give paſſage to the Water, when it riſes thro' HI to enter into the Pipe IK, in which it will remain, becauſe its Weight keeps down the Clack I, which muſt riſe again and give paſſage to a freſh Quantity of Water which will riſe thro' the ſaid Pipe HI when the Piſton CD is preſs'd down. Thus by raiſing and ſinking this Piſton, the Water will continue to aſcend in the Pipe IK, until it goes out at its End K.

Fig. 139. We call *Sucking Pump*, ſuch a Pump as draws up the Water when the Piſton is rais'd, which Piſton muſt have a Hole quite thro' it from D to F, where there muſt be a Clack, that when the Water is riſen by the raiſing of the Piſton (which in ſuch a Pump is call'd the *Bucket*) it may ſtill riſe higher when the Bucket is puſh'd down; for it will preſs upon the Water under it, which will puſh up the Clack F, and run up thro' the Bucket; and this Clack will immediately ſhut again upon the raiſing of the Bucket, becauſe the Water will preſs upon it, and then open as the Piſton is ſunk to make a ſecond Quantity of Water enter into the Body of the Pump, which will at length be fill'd up to the End A, where the Water will run out.

That the Valve F may play freely, the Rod EC of the Bucket muſt be faſtned to it by means of a bended piece of Iron, as ICH, ſtrongly fix'd to the Bucket. The Tube EG, which goes into the Water, may be as long as you pleaſe, if it does not exceed 33 Foot, for then the Water wou'd not riſe; becauſe the whole Weight of the Air, which cauſes the Water to riſe, cannot raiſe it any higher than that; which *Galilæo* firſt of any one diſcover'd.

Fig. 140. Laſtly, we call ſuch a Pump as raiſes Water by puſhing it upwards, a *Lifting-Pump*; let AB be the Body of a Pump divided into Two Parts AK, BI, of which BI muſt be in the Water, as alſo the Bucket or Piſton CD, which moves upwards in this part BI, by means of the Rod FG, fix'd to the Point

† N.B. *This Pump is imperfect without a Valve ſomewhere in the Tube* GHI.

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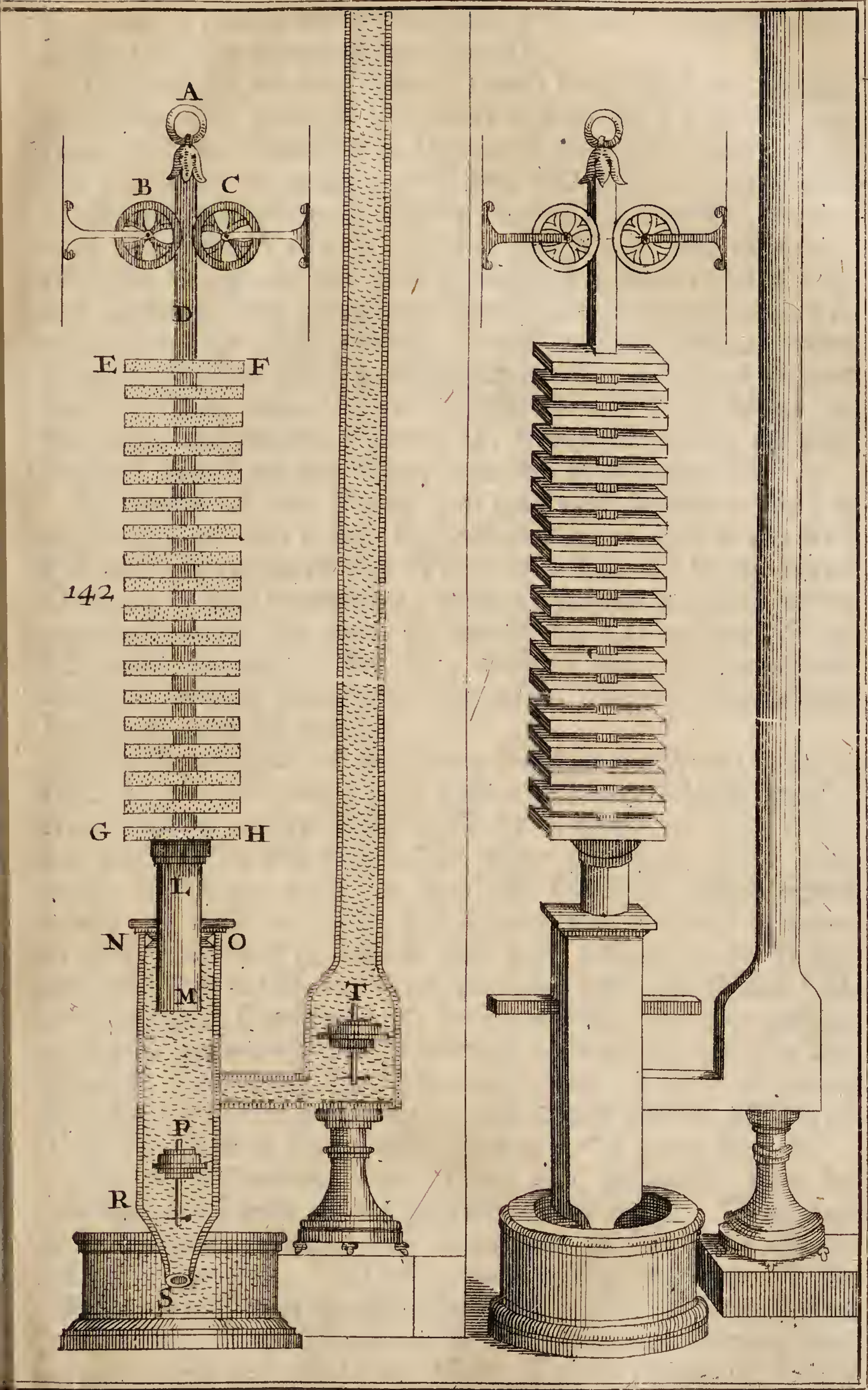
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F, round which it moves together with the Piston CD, and its Rod EC, by means of the Rod GH. *Plate* 26. *Fig.* 140.

The Rod EC of the Piston CD must be a Pipe continued in the Piston CD, quite to D, where it must have a Valve, and there must be One also at O; for if you push downwards the Rod GH, to make the Piston CD descend, the Piston pressing on the Water will Force it into the Pipe EC, which will open the Valve D, so that the Water pass above it; then the Weight of the Water will press down the Valve, and hinder it from going back the same way that it came; so when the Piston CD is rais'd, it will press the Water above it and cause it to rise (by lifting up the Clack O) and go into the part AK, where by its Weight it will press down the Clack O, and remain where it is: thus will AK be fill'd by degrees, till at last the Water runs out at the upper End A.

By means of this Pump you may raise Water as high as as you please; but it has this Inconveniency, that as the Rod FG must always be in the Water, if it happens to be out of Order, it is hard to mend it; besides since the Rod FG moves circularly about the point F, the Piston CD cannot rise or fall perpendicularly. For this reason I had rather make use of the following *Force-Pump*, which has nothing troublesome but the Length of its Rod.

Let AB the Body of the Pump stand in the Water as far as GH, *for Example*, and let the Piston CD have a Hole thro' it from D to F, where there must be a Clack to open when you push down the Piston CD, after you have rais'd it to make the Water come in thro' the Clack E, which opens when you raise the Piston and shuts when you push it down to open the Clack F, which will give passage to the Water and then shut it self as soon as you raise again the Piston CD, and the Clack E will open at the same time, and give passage to the Water which will afterwards be made to rise thro' the Clack F, by sinking the Piston as before: Thus continuing to raise and sink the Piston CD, the Barrel will be fill'd with Water, which at length will run out at the upper End A. *Fig.* 141.

Sr. *Samuel Moreland*, some Years ago, gave us a New Invention of a Pump, which he values very much. I'll explain it in his own Words, and make use of the same Figure which he gave us. 'N O R is the Profil of a Pump. P the 'Sucker which is at the Bottom of the Pump. LN the Piston 'which must be a Cylinder of Brass exactly turn'd in a Lathe, 'made to rise and fall in the Midst of the Cylinder of Water 'contain'd in the Barrel of the Pump; in such manner that it 'rubs against nothing but a small Circle of Leather, well 'prepar'd and fix'd into a little Hollow at the Top of the 'Pump on the inside over against ON; thro' this Leather the *Plate* 27. *Fig.* 142.

 'Piston

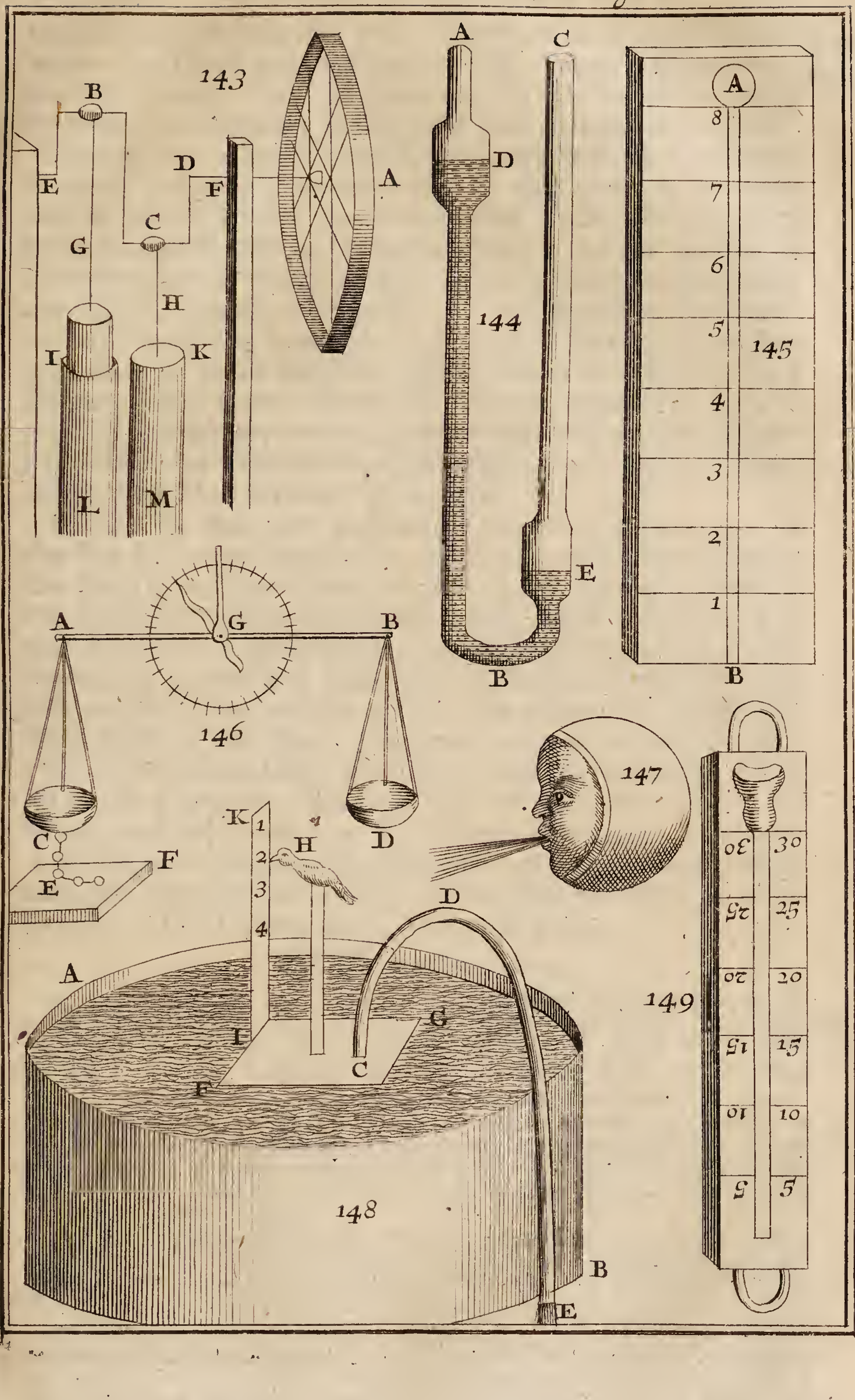
Plate 27. 'Piston goes up and down with the greatest ease imaginable
Fig. 142. 'and without any considerable Friction: to bring this to 'perfection cost me Twelve Years Study and a great deal of 'Money; and without this New Invention it wou'd have 'been impossible for me to have reduc'd the raising Water, 'to Measure and Weight, as I have done. ADL is the Rod 'of the Piston, upon which are Slipp'd on the Weights which 'are between EF and GH to counterpoise the Water, which 'is rais'd, and to keep the Piston upright between the Two 'Pullies B and C. VT is the Leaden Pipe, in which the 'Water rises after it has pass'd thro' the Sucker T, which 'hinders it from falling back into the Barrel of the Pump.

Plate 28. The Force of Rivers is commonly made use of to play this
Fig. 143. Engine by means of a Wheel as A, whose Floats dipping in the Water, are push'd by the Force of the said Water, so as to cause the Wheel to turn, which turns the bended Piece of Iron or double Crank BCD, which bearing upon the Two Fix'd Points EF, and turning upon them, successively comes nearer to, or goes farther from the Holes I, K, of the Two Barrels IL, KM, and so raises and sinks the Pistons one after another by means of their Rods BG, CH, which are fasten'd to the double Crank BCD, at the Points B, C. Instead of such a Crank, Leavers are made use of in great Engines, which by their rising and falling raise and sink the Pistons, as may be seen in the great Engine at *Marly* near *Paris*, which raises the Water of the River *Seine* up to a great Aqueduct which runs quite to *Versailles*: For want of Water, the Wind may be made use of, as in Wind-mills.

Of Barometers.

WE call *Barometer* an Engine made use of to know sensibly the different changes which happen in the Gravity of the Air, which is not the same at all times and in all places; for we know, by Experience, that the Air is heaviest when it is laden with † Vapours, and that it is lighter in a high

† *The Vapours do not increase the Gravity of the Air; for when they are in the Lower Region, as in Rainy Weather, then the Air is lightest, as it appears by the falling of the* Mercury *in the* Barometer: *and when they are in the middle Region of the Air, (that is, when the Clouds are high) they do not help the Pressure of the Air; for tho' the Air be heavier then, it is not cause the Clouds are high, but the Clouds are high because the Air is Heavy; for when the Air near the Earth is more dense than usual, then it becomes Specifically heavier than the Vapours, which therefore must ascend, and at last settle in that Region of the Air which is of the same Specifick Gravity with themselves. If the Vapours were the Cause of the encreasing of the Air's Gravity, there must be as many Vapours in the Air at a time, as are equal to Three Inches of* Mercury, *for so much we find the* Mercury *rises or falls: Now* Mercury *is about* 14 *times heavier than Water, and consequently there must be in the Air at once so*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high Place than in a low one. Barometers are made ſeveral ways; but I ſhall only explain here Mr. *Hugens*'s Barometer: because I think it very convenient, by reaſon of its being Portable, and ſhewing ſenſibly the leaſt changes in the Air. *Plate* 28. *Fig.* 144.

Let ABC be a Glaſs Tube Hermetically Seal'd at One of its Ends A, and open at its other End C, that as much Mercury may be pour'd into B, as will fill that Tube which reaches from the middle of the Cylindrick Box E, to the middle of the other Box D, which muſt be about 27 Inches diſtant from the firſt E, becauſe a Pillar of Air reaching from the Earth to the Top of the Atmoſphere, or utmoſt Surface of the Air will keep 27 or 28 Inches of Quick-Silver in *Æquilibrio* in a Perpendicular Tube: Then fill CE the remaining part of the Tube with any other Liquor which will neither Freeze in Winter, nor Diſſolve the Quick-Silver, as common Water mix'd with One Sixth of *Aqua-fortis.*

When the Mercury deſcends, One Inch, *for Example*, in the Box E, by the Gravity of the Air, it will riſe as high in the Box D, and the Water which is in the remaining part of the Tube CE, will deſcend in the Box E, and if the Capacity of that Tube be 15 times greater (*for Example*) than that of the remaining part of the Tube CE, 15 Inches of the Water of this narrow Tube will be requir'd to take up the Height of One Inch in the Box. Therefore as often as the Mercury riſes or falls One Inch, the Water will riſe or fall 15; and likewiſe when the Mercury falls or riſes One Line, the Water will fall or riſe 15; ſo that this Barometer ſhews the Alterations of the Gravity of the Air, 15 times more Senſibly than the common Barometer, and it will yet ſhew it more Senſibly, if the Boxes D, E, are made wider.

Of Thermometers.

A *Thermometer*, is a long Tube of Glaſs ſeal'd Hermetically, which has a ſmall Bubble at Top as O, and under it a long Neck as AB, which being fill'd about half *Fig.* 145.

ſo many Vapours as are equal in Weight to a Column of Water of 42 *Inches in height, and whoſe Baſis is equal to the Surface of the Earth, which is much more than falls down in Rain, during a whole Year. For a whole Year's Rain does not fill a Veſſel above* 14 *or* 15 *Inches high, as is obſerv'd in the* Hiſtory of the Royal Society at Paris.

The Reaſon then, why the Air is heavier at one time than another, ariſes from there being more Air on that part of the Earth's Surface when the Air grows heavier: And this proceeds from Winds; for Ex. *If the Wind, which is nothing but a Stream of Air, ſhould blow on any place, and the Air thus mov'd ſhould be kept in that place by Mountains or Hills; Or if Two contrary Winds ſhould blow on the ſame place, the Air will be heap'd up in the middle, and conſequently there being more Air, its Gravity will be increas'd. But if the Wind ſhould blow over a Country, the Air which is over that place, will grow leſs in Quantity, and conſequently lighter. Hence 'tis plain, that Winds are the only cauſes of the variation of the Air's Gravity.*

way with Spirit of Wine, or any other Liquor that does not Freeze in Winter (usually ting'd of some Colour to distinguish it in the Tube) serves to shew the Degrees of Heat and Cold in the outward Air. For this purpose the whole Length of the Tube is divided into Eight equal parts, and those Eight each into Eight more, to have in all 64 Degrees, to know more sensibly the Change which happens at any time in the Temper of the Air, observing to what Degree the Water rises each Hour of the Day, according as the Heat of the outward Air is encreas'd or diminish'd: For when the Air is Hot, it causes the Air in the Ball and Tube AB to be rarified, and that Air being rarified presses the Water and causes it to descend: and on the contrary when the Air is cold, it is condens'd and gives way to the Water to rise †.

Plate 28. *Fig.* 145.

With this Instrument one may compare the greatest Heat of one Summer with the greatest Heat of another, or the greatest Cold of one Winter with the greatest Cold of another, and know which of Two Rooms is the Hottest, that being Hottest where the Water will descend the lowest in the Thermometer, the least Heat being able to rarifie the Air contain'd in the Tube AB as one may find by Experience; for if the Hand be laid on the Ball A, its Heat will immediately rarify the Air and cause the Water to descend, which will creep up gently into its place again, as soon as the Hand is remov'd; which will be more easily seen if you warm the Ball by breathing upon it.

Of Hygrometers.

AN *Hygrometer* is an Instrument contriv'd to know the Degrees of the Driness and Moistness of the Air; and in some measure foretel Rain in fair Weather; for if the Air be very Moist in fair Weather it is a sign of approaching Rain. There are several kinds of Hygrometers; but I shall only explain one Sort.

Fig. 146. Make a common Balance as AB, which must be suspended by its Center of Motion G; and put into one of the Scales as D, a Weight of Lead, and in the other Scale as C, a piece of Spunge big enough to keep it in *Æquilibrio*; then it will happen that when the Weather is Moist, the Spunge growing Moist by sucking in those little Particles of Water that Swim in the Air (which it will do the more easily if it has been first dipp'd in Salt-water; for tho' the Water may be dried, yet those Saline Particles which are left behind will

† *The small end of this Thermometer must be open, and immerg'd in an open Vessel of the same Liquor with that which it contains.*

render it more ſuſceptible of the Moiſture of the Air) will become heavier than the Lead and cauſe its Scale to preponderate, and cauſe the *Examen* of the Balance, which is moveable about the fix'd Point G, to change its Poſition: On the contrary, when the Spunge is dried by the Drineſs of the Air; it will not be ſo heavy as the Lead and conſequently riſe with its Scale, and make the *Examen* alſo to turn the other way, and point to the Degrees of Drineſs mark'd upon the Circumference of the Circle deſcrib'd about the Center of Motion G. But inſtead of an *Examen* and Circle, You may faſten to the Scale C a little Chain made of ſeveral little Balls, which fall upon an Horizontal plain as EF, and will lie upon it in a greater Number when the Moiſtneſs of the Air is greater; for in ſuch a caſe, the Scale C will deſcend lower, becauſe the Spunge being Moiſter will conſequently be Heavier. *Plate* 25. *Fig.* 132.

Of Æolipiles.

AN *Æolipile* is a Hollow Globe of Braſs or any other Matter which can bear the Fire, which being half fill'd with Water thro' a ſmall Hole, and afterwards put upon live Coals, does not work till it is warm; for then the Heat rarifies the Water within it ſo much as to turn it into Wind, which ruſhes out at the ſaid Hole with an Impetuous Hiſſing, and blows ſtrong enough to cauſe a Muſical Inſtrument, as a Flageolet, to ſound.

For Ornament, ſuch an Inſtrument is made like a Head, and the Hole is made in the Mouth; it will continue blowing for the Space of an Hour or longer. It is alſo made ſometimes like a Pear with a ſlender Tail and a ſmall Hole in the End of it. When you wou'd get the Water into the *Æolipile*, Heat it very Hot and immediately throw it into cold Water, which cauſing the Air in it to be condens'd, that had been before rarified by the Heat, the Water will run in at the ſaid Hole to fill up the void Space, and ſupply the place of that Air which had been expell'd by Heat.

If inſtead of common Water, you ſhou'd fill it with rectified Spirit of Wine, and ſet Fire to the Vapour which goes out, you will ſee a continual Fire ruſh out at the Mouth of the *Æolipile* as long as the Vapour continues to go out with Violence.

Of Hour-Glaſſes.

HOur-Glaſſes (in French *Clepſydres*) are made either with Sand, or with Water; and were very valuable before the Invention of *Watches*, or *Clocks* with *Wheels*. Nevertheleſs,

Plate 28 *Fig.* 148. as those with Sand are still in use, and the Water-Clocks are very curious, we shall here say something concerning each Sort.

First, to make a Water-Clock; fill a Tub as AB with Water, and having try'd how much Water goes out of it in Twelve Hours, by means of the Syphon CDE, fix'd to the Board FG which Swims on the Water; mark in the Tub it self the Intervals of Hours, and the Board FG going down as the Water runs out at E the End of the Syphon, which must be lower than the Surface of the Water (otherwise it will not run out) will shew the Hours. You may also set upon the Board FG a little Statue, or any other Figure, as that of a Bird, which as it goes down will point to the Hours mark'd on the perpendicular Plain IK. You may also apply a String about an Horizontal Axis, whose Ends must bear and turn upon Two fix'd Points, and fasten to the End of that String a piece of Wood made, if you will, in the shape of a Ship, to float on the Water; and when the Water runs out thro' the Hole E of the Syphon CDE, (part of which may represent the Mast of that little Ship) and the said Ship descends, the Axis will turn; and if at one of its Ends there be a Dial-plate and Hand, that Hand will exactly shew the Hours, if the Hole E be of such a Diameter, that there falls out no more Water in Twelve Hours, than as much as is necessary to make the little Ship sink just so low as to give the Axis One Turn exactly; for then the End of the Hand will go quite round the Circumference of a Circle, which must be divided into Twelve equal Parts, as it is in other Dial-Plates.

Hour-Glasses with Sand are so common, that it wou'd be needless to speak of 'em more particularly: wherefore without taking any notice of such as are daily us'd, I shall speak of a New Invention of *Hour-Glasses*, which Mr. *de la Hire* of the Royal Academy of Sciences communicated to us a few Years ago, in these Words:

Fig. 149. 'Instead of one of the Glasses, of which an *Hour-Glass* is 'made, a Tube must be fix'd, of about 20 Inches in Length, 'and a Line and a half Diameter. This Tube being stopp'd 'close at that end which is not joyn'd to the Bottle, is in'stead of a Second Bottle, so that when the Sand runs out of 'the Bottle into the Tube, you may see it rise by degrees, 'and so distinctly, that you may observe how it rises, and 'see the Alterations at least every Five Seconds, and conse'quently the Minutes must be seen very distinctly, if this 'Glass runs but half an Hour.

'When all the Sand, which must run out in half an Hour, 'is run into the Tube, the Instrument must be turn'd upside 'down; and the Sand running back out of the Tube into it,

'will

'will as it goes down ſhew the Minutes and their Parts.
'To uſe this Machine conveniently, it muſt be apply'd to 'a Board, in ſuch manner that half the Bottle and half the 'Tube, be let into the Thickneſs of the Wood. Two Strings 'are faſten'd to this Inſtrument one at each End of the Wood, 'to turn it eaſily; becauſe it muſt be ſuſpended in the Air 'or ſet upright againſt ſomething or other. The diviſions of 'the Minutes are mark'd on one ſide of the Tube to ſhew 'the diſtances when it fills; and after the ſame manner on 'the other to obſerve them, when the Sand runs out of the 'Tube.

'Thoſe diviſions muſt be mark'd by means of a Pendulum, 'thus: Take a ſmall Thread and tye a Leaden Bullet at the 'End of it, to make a Pendulum. If the Length of the 'Pendulum, from the place from which the Thread hangs to 'to the Center of the Bullet, be Three Foot, and Eight Lines 'and a half, (*Paris*-Meaſure) this Pendulum will Vibrate 'Seconds; and when it has made 60 Vibrations, you muſt 'mark the Diviſions which ſhew the Minutes. All the Di-'viſions muſt be thus mark'd by help of the Pendulum as the 'Sand riſes or falls; for the Diviſions are not always equal, 'by reaſon of the inequality of the Tube, which being 'narrower in ſome places, the Sand muſt riſe faſter in ſuch 'parts of the Tube than in others, which are wider.

'You muſt obſerve, that as the Sand goes out of the Tube 'into the Bottle, it runs thro' greater Diſtances at firſt, than 'towards the laſt, which is occaſion'd by the Sands com-'ing down by jerks at firſt; but this cauſes no irregularity, 'if the Diſtances are mark'd according to the Vibrations of 'the Pendulum.

FINIS.

A
Table of the TERMS
Explain'd in the
MECHANICKS.

Body.

C.

D.

E.

Sheevers.

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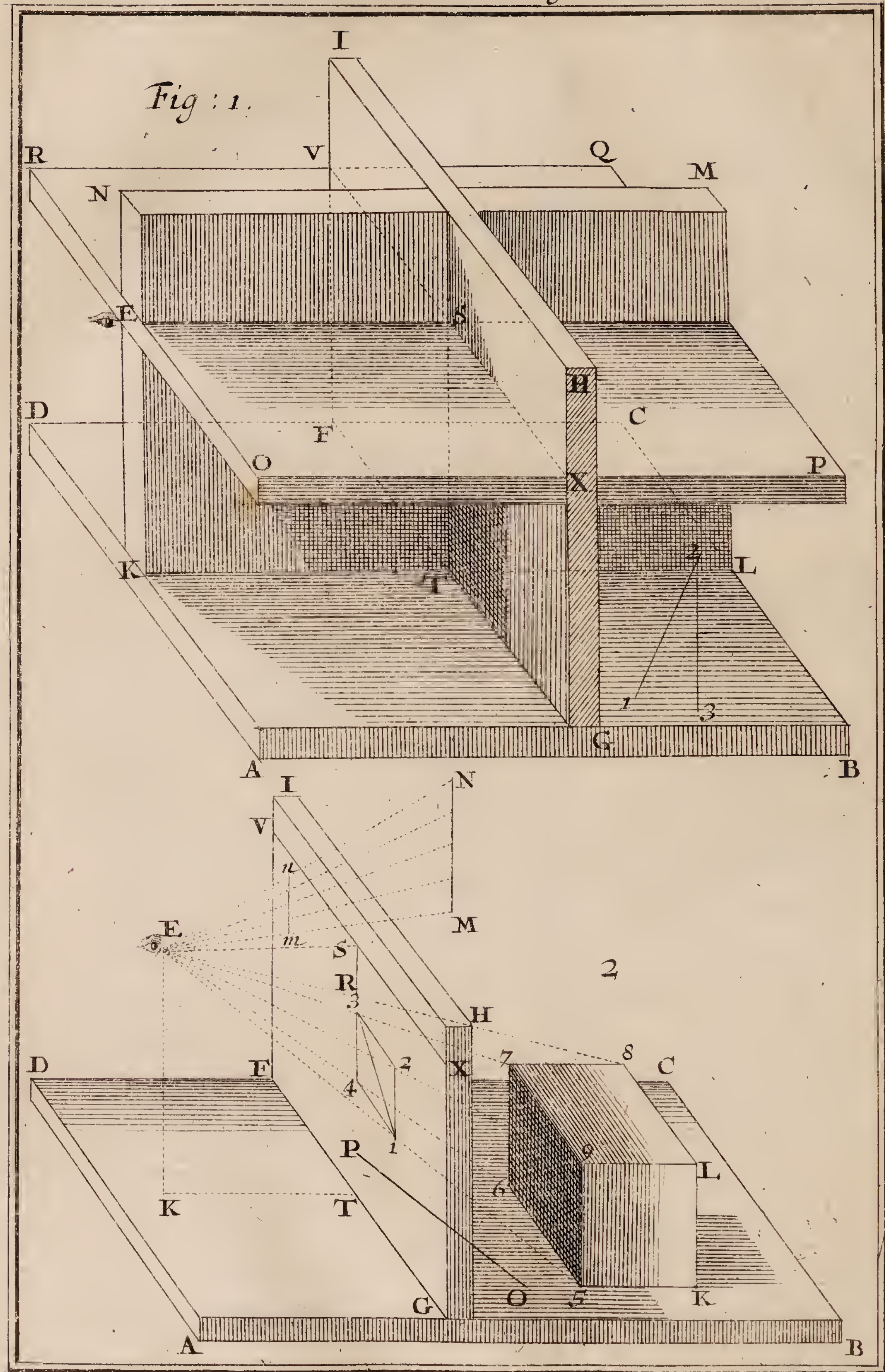
Fig: 1.
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A TREATISE OF *PERSPECTIVE.*

PERSPECTIVE is the Art of repreſenting viſible Objects in a Picture juſt as they appear to the Eye; for which reaſon this Picture ought to be ſuppos'd Tranſparent, and uſually Perpendicular to the Horizon, and likewiſe betwixt the Eye and the Object.

The chief Things to be conſider'd in Perſpective, are the Eye, the Object, the Plain of the Picture, the Geometrical Plain, the Vertical Plain, and a Fourth Plain, call'd the Horizontal Plain: and theſe are Explain'd in the following Definitions.

DEFINITIONS.

THE † *Geometrical Plain* is a plain Surface parallel to the Horizon, plac'd lower than the Eye, as ABCD, in which we imagine the Viſible Objects without any change, (except it be the reducing a great Figure into a ſmall one) and upon which the ſituation of the Object to be repreſented in Perſpective is deſcrib'd. *Plate* I. *Fig.* I.

The *Situation* of a point of an Object which is out of the Geometrical Plain, is that point of the Plain on which a line from the propos'd point falls perpendicular. Thus you will know, that the ſituation of the end 2, of the Stick 1 2, is the point 3, where the Geometrical Plain ABCD is cut by the line 2 3, which is perpendicular to it; and for this reaſon the Geometrical Plain ABCD has by ſome been alſo call'd the *Plain of Situation.*

The * *Picture* is a plain Surface, ſuppos'd as Tranſparent as Glaſs, and uſually perpendicular to the Geometrical Plain, as FGHI,

† *Call'd alſo the Baſe.* * *Called alſo the Section, Table, or Glaſs.*

Plate 1. Fig. 1. FGHI, which is always plac'd at some distance between the Eye and the Objects, to represent in it those Objects in Perspective; whence the Picture is call'd the *Perspective Plain.*

Sometimes the Picture is Inclin'd; that is, not perpendicular to the Geometrical Plain, or to the Horizon, and sometimes it has a Curve Surface, as when we wou'd paint the Surface of an Arch'd Roof, but as that is not common, we shall in the Sequel, look upon the Picture as a Plain perpendicular to the Horizon.

The *Ground-Line* is the common Section of the Geometrical Plain and of the Picture, as FG; upon this stands the Picture: Whence it also call'd the † *Base of the Picture.*

† Or *Base-Line.*

The *Horizontal Plain* is a plain Surface, which passing thro' the Eye is perpendicular to the plain of the Picture, and consequently parallel to the Horizon, as OPQR, which passes thro' the Eye suppos'd to be at E.

The *Horizontal-Line* is that Right-line in which the Horizontal Plain, and the Plain of the Picture intersect, as VX, which must necessarily be parallel to the Ground-line FG.

The *Principal Ray* is a Right-line drawn from the Eye, perpendicular to the Plain of the Picture, as ES, which therefore must of necessity run along the Horizontal Plain.

The *Visual Point*, or *Point of Sight*, which is call'd the *Principal Point*, or point of the Eye, is the point where the Picture is cut by the principal Ray, as S, which must necessarily be in the Horizontal-line VX.

The *Point of Distance* is a point in the Horizontal-line distant from the point of Sight the length of the principal Ray, as, V, or X, the lines SV, SX, being each equal to the principal Ray ES.

The *Vertical Plain* is a plain Surface, which going thro' the principal Ray, is perpendicular to the Horizon, and consequently to the Geometrical Plain, and also to the Picture, as KLMN, to which the Ground-line FG, and the Horizontal-line VX are necessarily perpendicular.

The *Line of Station* is the Right-line in which the Vertical Plain cuts the Geometrical Plain, as KL, which must necessarily be parallel to the principal Ray, and consequently perpendicular to the Picture.

The *Vertical-Line* is the Right-line, in which the Picture is cut by the Vertical Plain, as ST, which of necessity is perpendicular to the Line of Station KL, and to the principal Ray ES, because it is perpendicular to the Geometrical, and to the Horizontal Plain.

The *Height of the Eye* is a Right-line, which passing thro' the Eye, is perpendicular to the Geometrical Plain, as EK, which must necessarily be parallel and equal to the Vertical Line ST.

The

The *Accidental Point* of a Right-line is the Point where the Picture is cut, by a Right-line drawn from the Eye parallel to the Line propos'd. Thus you will know that the Accidental point of the Line 5 K, or of its parallel 9 L, is the point S. Whence it is easy to conclude, that all the Lines which are parallel to the Picture have no Accidental Point, and that all the other Lines which are parallel to one another have the same Accidental Point. You may also easily know that all the Right-lines which are perpendicular to the Picture, have the principal point S for this Accidental Point; and that all those which make an half-Right Angle with the Picture, have one of the Points of Distance for their Accidental Point. *Plate* 1. *Fig.* 2.

The *Plan* or *Ichnography* of any Object, call'd also the Situation of it, is its Orthographical Projection upon the Geometrical Plain. Thus is the Plan of a Right Cylinder, a Circle; and the Plan of a Right Cube, a Square.

We call *Orthographical Projection* of an Object, that Figure which is describ'd upon the Geometrical Plain, when from all the points of the Object, Right-lines are drawn perpendicular to the said Geometrical Plain.

But we call *Front*, or Foreright side, the Orthographical Projection of an Object, upon a plain parallel to the Picture: and *Profil*, the Orthographical Projection of an Object, upon a plain parallel to the Vertical plain.

The *Representation*, *Appearance*, or *Scenographical Appearance* of a point of any Object is a point where the Picture is cut by a Right-line drawn from the Eye to the Point of the Object propos'd. Thus one may know that the Appearance of the point M is the point *m*, and that the Appearance of the point N is the point *n*; and consequently, that the Appearance of the Line MN is *mn*.

It is plain, That if a Right-line of any Object being produc'd does not pass thro' the Eye, its Appearance in the Picture will be a Right-line, in such a place where it will be cut by a plain Surface made up of an Infinite Number of Right-lines drawn from all the points of the Line propos'd and terminated at the Eye, as so many Visual Rays, as we shall more particularly demonstrate in *Theorem*. 1.

It is also evident, That if the Surface of any Object being continued, does not pass thro' the Eye, its Appearance will be such a part of the Picture as is terminated by the Appearances of the Lines which bound the said Surface. Thus supposing that the Surface 5,6,7,9, of the Cube 5 L 8, being continued, does not pass thro' the Eye E; its Appearance will be the part 1,2,3,4, which is terminated by the Appearances 12, 23, 34, 14, of the Lines 59, 97, 76, 56, which bound the propos'd Surface 5,6,7,9.

Plate 1. Fig. 2. Lastly, it is evident that if any part of an Object touches the Picture, its Appearance will be in that place of the Picture which it touches. Thus you will know that the Appearance of the End P of the inclin'd Stick OP, which touches the Picture F G H I at the point P, is the said point P.

It follows from what has been said, that all the parts of the Objects which are lower than the Eye, or the Horizontal Plain, ought to be represented in the Picture below the Horizontal Line VX; and on the contrary, that all those which are above the Horizontal Plain, or higher than the Eye, ought to be represented in the Picture above the said Horizontal Line VX: and lastly, That all those Objects which in respect to the Eye, are on the Right-hand of the Vertical Plain, must in the Picture be represented on the Right side of the Vertical Line; and such as are on the Left, must be represented on the Left side of the said Vertical Line.

THEOREMS.

THEOREM I.

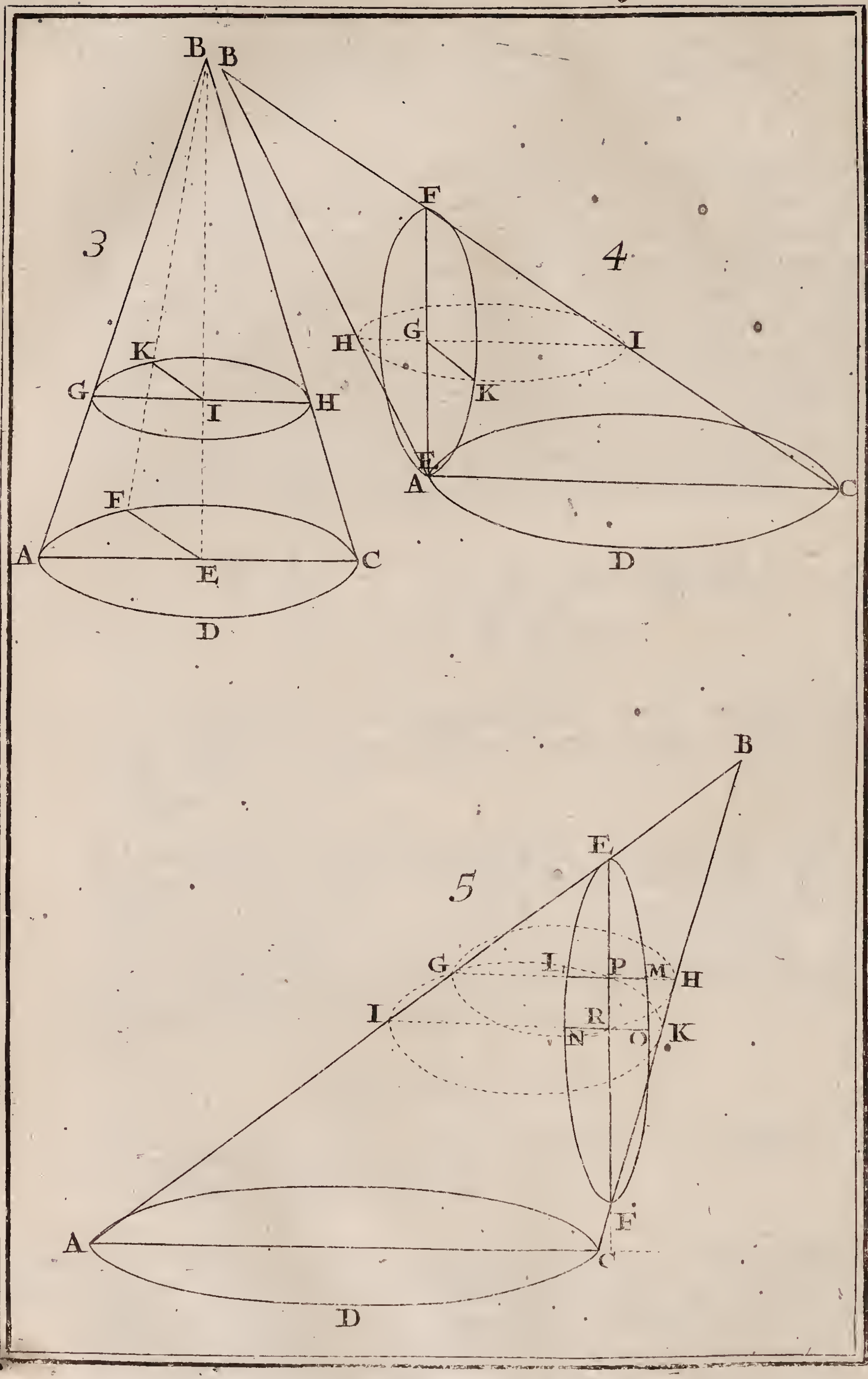
If a Right-line being produc'd, does not go thro' the Eye, its Appearance in the Picture will be a Right-line.

IF the Right-line MN being produc'd does not go thro' the Eye E, I say, that its Appearance *mn* in the Picture FGHI, is a Right-line, because its Triangular plain M E N, made up of all the Visual Rays drawn from the Eye E, thro' all the points of the Line MN, can only cut the Plain of the Picture FGHI by a Right-line, *by* 3. 11.

THEOREM II.

If a Cone be cut by a Plain parallel to its Base, the Section will be a Circle.

Plate 2. Fig. 3. THO' this Theorem be self-evident, because a Cone is made up of an Infinite Number of Circles parallel to one another, and likewise to its Base, which is also a Circle, (for which reason in our *Gnomonicks* and *Geometry* we have suppos'd it demonstrated) yet, that nothing may be wanting in this little Course of Mathematicks, I will demonstrate, that if the Cone ABCD be cut by the plain GKH parallel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allel to its Base ADC, which is a Circle, the Section GKH will also be a Circle, thus: *Plate 2. Fig. 3.*

If you draw from the *Vertex* B of the Cone thro' the Center E of the Base ACDF, which is a Circle, the *Axis* BE, and cut the Cone ABCD by a Plain which passes along its *Axis* BE, the Section will be the Triangle ABC, which for that reason is call'd the *Triangle of the Axis*; and that Section will be cut by the Plain GHK parallel to the Base ADCF in the Right-line GH, which *by* 16. 11. will be parallel to the Diameter AC, because the Two lines AC, GH are the Sections of the Two parallel Plains ADC, GHK, by the Third Plain ABC. Wherefore the Two Triangles AEB, GIB, will be Similar, as well as the Two BEC, BIH, and the *Ratio* of the Two lines AE, GI, will be the same as that of the Two CE, HI, because each of those Proportions is the same as that of the Two lines BE, BI. Whence it follows, That as the Two lines AE, CE, are equal to one another, because the point E is the Center of the Circle ADCF, the Two GI, HI, will also be equal to one another.

If thro' the point F taken, at pleasure, upon the Circumference ADC, you draw to B the *Vertex* of the Cone ABCD, the Right-line BF, which will run along the Surface of this Cone, and cut the plain GHK at the point K; and if you likewise draw the lines EF, IK, they will be parallel, *by* 16. 11. because they are the Sections of the Two parallel Plains ADC, GKH, and of the third EBF, which makes the Two Triangles BIK, BEF, be Similar, and *by* 4. 6. EF : will be to IK :: as BE : is to BI; and consequently as AE : is to GI, and as CE is to HI; whence it is easy to conclude, that as the Two AE, CE, are equal to one another, as well as the Two GI, HI; so likewise are the Three IG, IH, IK, and consequently the Section GKH is a Circle. *Q. E. D.*

THEOREM III.

If a Scalenous Cone be cut by a Plain, which being perpendicular to the Base of the Triangle of the Axis, cuts off from that Triangle (towards the Vertex) *another Triangle Similar to it in a contrary Position; the Section will be a Circle.*

I Say; That if the Scalenous Cone ABCD, be cut by a Plain perpendicular to the plain ABC, and to the Base AC of the Triangle of the Axis ABC, in such manner that the Triangle BEF (terminated by EF the Section of the cutting Plain and Triangle of the Axis) be Similar to the said Triangle *Plate 2. Fig. 4.*

Plate 2. *Fig.* 4. ABC in a contrary Position, which is call'd a *Subcontrary Section*; that is, if the Angle BEF be equal to the Angle ACB, and the Angle BFE to the Angle BAC, the Section EKF of the Cone and cutting Plain is a Circle,

DEMONSTRATION.

If thro'the point G, taken, at pleasure, upon EF the common Section of the Plain IKF, and of the Triangle of the Axis ABC, you draw the line HI parallel to AC the Diameter of ADC Base of the Cone, and that a Plain be made to pass along HI, being parallel to the said Base ADC; the Section HKI of that plain HKI, and of the Cone will be a Circle, whose Diameter is HI, *by Theor.* 2. and because the Plain EKF, as well as the Plain HKI, is perpendicular to the Plain of ABC the Triangle of the Axis, their common Section GK will likewise be perpendicular to the said Triangle ABC, *by* 19. 11. and consequently to the Two lines HI, EF: and because each of the Triangles BEF, BHI, is Similar to ABC the Triangle of the Axis, they will be Similar to one another, and the Angle F will be equal to the Angle H, and the Angle E to the Angle I, which makes the Triangles EGH, IGF Similar, and it will be known *by* 4.6. that the Four lines GH : GE : : GF : GI; are Proportional, and *by* 16. 6. that the Rectangle of the Two lines GE, GF, is equal to that of the Two GH, GI, that is, *by* 35. 3; to the Square of the line GK; whence it is easy to conclude, that the Section EKF, is a Circle. *Q. E. D.*

THEOREM IV.

If a Cone be cut by a Plain, which being perpendicular to the Base of the Triangle of the Axis, cuts off from the Triangle towards the Vertex *another Triangle Dissimilar, the Section will be an Ellipsis.*

Plate 2. *Fig.* 5. I Say, That if the Cone ABCD, whose Basis is the Circle ADC, and the Triangle of the Axis ABC, be cut by a Plain, which being perpendicular to the Base, as well as the Plain of the Triangle of the Axis ABC, and cutting the Two Sides AB AC of the Triangle at the points E, F, takes off from the said Triangle ABC, the little Dissimilar Triangle BEF, whose Base EF is the common Section of the cutting Plain, and of the Triangle of the Axis ABC; the Section EN,

FH

FH of that cutting Plain, and of the Cone is an *Ellipsis*; (*viz.*) a plain Figure terminated by a Curve-line, in which the Squares of the Ordinates to one Diameter, as to the Diameter EF, are proportional to the Rectangles under the correspondent parts of the said Diameter. *Plate* 2. *Fig.* 5.

PREPARATION.

If by thought you cut the Cone ABCD, by a Plain, (which going between the ends E, F, of the line EF which is call'd the *Diameter of the Section*, be parallel to the Base ADC of the Cone ABC,) you will have by that Section the Circle GLHM, whose Diameter GH being the common Section of the cutting Plain, and of the Triangle of the Axis ABC, will be parallel to AC the Diameter of the Base ADC.

Then let the Cone ABCD be cut by another Plain, (which going between the said ends EF of the Diameter of the Section EF, must also be parallel to the Base ADC of the Cone ABCD,) to have by that Second Section, the Circle INKO, whose Diameter IK being the common Section of that Second cutting Plain, and of the Triangle of the Axis ABC, will be parallel to AC the Base of the said Triangle ABC, and consequently to the Diameter GH.

Lastly, Draw thro' the opposite points L, M, where the Section ENFH is cut by the Circle GLHM, the Line LM, which will be divided at Right-angles, and into Two equal parts by the Diameter of the Section EF, at the point P, where the Two Diameters EF, GH intersect. Likewise draw thro' the Two opposite points N, O, where the said Section ENFH is cut by the Circle INKO, the Right-line NO, which will be also cut at Right-angles, and into Two equal parts by the Diameter of the Section EF, at the point R, where the Two Diameters EF, IK, intersect. Whence it follows, That the Two Lines LM, NO, are Ordinates to the Diameter EF, and that the said Diameter EF is an Axis.

DEMONSTRATION.

This Preparation being made, you will have in the similar Triangles GPE, IRE, this Analogy, GP: IR: : EP: ER; and in the Two similar Triangles HPF, KRF, you will have this, HP: KR: : FP: FR; and if of the Homologous Terms of these Two Analogies, Rectangles be made, as you see here, you will have a Third Analogy thus:

Plate 2. *Fig.* 5.

GP:	IR::	EP:	ER.
HP:	KR::	FP:	FR.
GPHP:	IRKR::	EPFP:	ERFR.

GPHP: IRKR:: EPFP: ERFR, in which, if instead of the Two First Terms, that is, the Rectangle of the lines GP,HP, and the Rectangle of the lines IR, KR, you put the Two Squares PL, RN, which are equal to 'em, by the Nature of the Circle, you will know that the Square PL: is to the Square RN:: as the Rectangle under the lines EP,FP, is to the Rectangle under the lines ER,FR, and consequently, that the Section ENFH is an Ellipsis. *Q.E.D.*

THEOREM V.

If a Circle be parallel to the Picture, its Appearance in the Picture will also be a Circle.

IF you imagine Rays to come from all the Points of the propos'd Circle and be terminated in the Eye, they will form a Cone, whose *Vertex* will be the Eye, and Basis the Circle: And as that Cone is cut by a Plain parallel to its Base (*viz.* by the Picture) it follows, *by Theorem* 2. that the Section or Appearance is a Circle. *Q.E.D.*

THEOREM VI.

If a Circle is not parallel to the Picture, and its Plain being continued does not pass thro' the Eye, its Appearance on the Picture will be either an Ellipsis, or a Circle.

IF you imagine Rays to come from all the points of the propos'd Circle and be terminated in the Eye, they will, as before, form a Cone, which will be cut obliquely by the plain of the Picture; and consequently, the Section can only be an Ellipsis, *by Theorem* 4. unless the Section of the Cone be *subcontrary*, in which case it will be a Circle, *by Theorem*.

THEO-

THEOREM VII.

If a Right-line be parallel to the Picture, the Appearance of it in the Picture will be parallel to the said Right-line.

IF the line 6,7, be parallel to the Picture FGHI, I say that its Appearance 3, 4, is parallel to it: For if you suppose a Plain along the propos'd Line 6, 7, to be parallel to the Picture, as the plain 5, 6, 7, 9, the Sections of these Two parallel plains FGHI, 5 6 7 9, by the Third Triangular Plain 6E7, namely, 6, 7, and 3, 4, will be parallel, *by* 16. 11. *Plate* I. *Fig.* 2.

COROLLARY.

From this Proposition it follows, That if the Line propos'd be parallel to the Ground-line FG, as 5, 6, its Appearance 1, 4, will also be parallel to the Ground-line FG: And that if the propos'd Line be parallel to the Vertical Plain, or perpendicular to the Horizon, as 5, 9, its Appearance 1, 2, will be perpendicular to the Ground-line FG. And lastly, That if the Line propos'd be Inclin'd to the Horizon, as 5, 7, its Appearance 1, 3, will be likewise Inclin'd; so that if it be produc'd so far as to meet the Ground-line FG, it will make with it an Angle equal to that, which the Line propos'd makes with the Geometrical Plain.

THEOREM VIII.

If a Right-line being produc'd meets the Picture, its Appearance in the Picture being produc'd, will go thro' its Accidental Point.

IF the line 7, 8, being produc'd, meets the Picture FGHI, I say, That its Appearance 3 R in the Picture will be part of the line S 3, which is drawn thro' the Appearance 3 of the point 7, and thro' the accidental point S, terminated in the Picture by the Ray ER parallel to the propos'd line 7, 8; that is to say, That if in the line 7, 8, you take as many points as you please, as 8, and that from thence you draw as many Rays towards the Eye E, as E8, the Ray E8 will pass thro' some point of the line S3, as R. *Fig.* 2.

DE-

DEMONSTRATION.

Plate 1. Fig. 2. For the Plain, which goes thro' the Two parallel Lines ES, 78, cuts that of the Picture by the line S3, and because the points E8, are taken in Two parallel Lines, the line E8 drawn from one point to the other, is of necessity in their Plain, *by* 7. 11. wherefore when it passes into the Picture, it must be in the common Section S3. *Q. E. D.*

COROLLARY.

It is evident from this Theorem, That the Appearance of a line perpendicular to the Picture, such as is here the line propos'd 7, 8, is a Right-line, which, being produc'd, goes thro' the principal Point S, and that the appearance of a Horizontal Line which makes half a Right-angle with the Picture, or an Angle of 45 Degrees, goes thro' the point of Distance on that side.

THEOREM IX.

If Two Right-lines, which are parallel to the Picture, and equal to one another, proceed from the same Point; their Appearances in the Picture will likewise be equal to one another.

Plate 3. Fig. 6. IF from the point L, proceed Two equal Right-lines LM, LN, parallel to the Picture FGHI, I say, That their Appearances 12, 13, are also equal to one another; as you may know by drawing from the Eye E, the Rays EL,EM.

DEMONSTRATION.

For the line 12 is parallel to the line LM, and the line 13 to the Line LN, *by Theorem* 6, which causes the Two Triangles ELN, E 13 to be Similar, as also the Two ELM, E 12; whence it is easy to conclude, *by* 4. 6. That as EL: is to E1:: so is LM: to 12; and LN: to 13, and consequently, that the Four LM: LN:: 12: 13, are proportional: And because the Two First LM,LN, are suppos'd equal; the Two Last 12, 13, must of necessity be also equal. *Q.E.D.*

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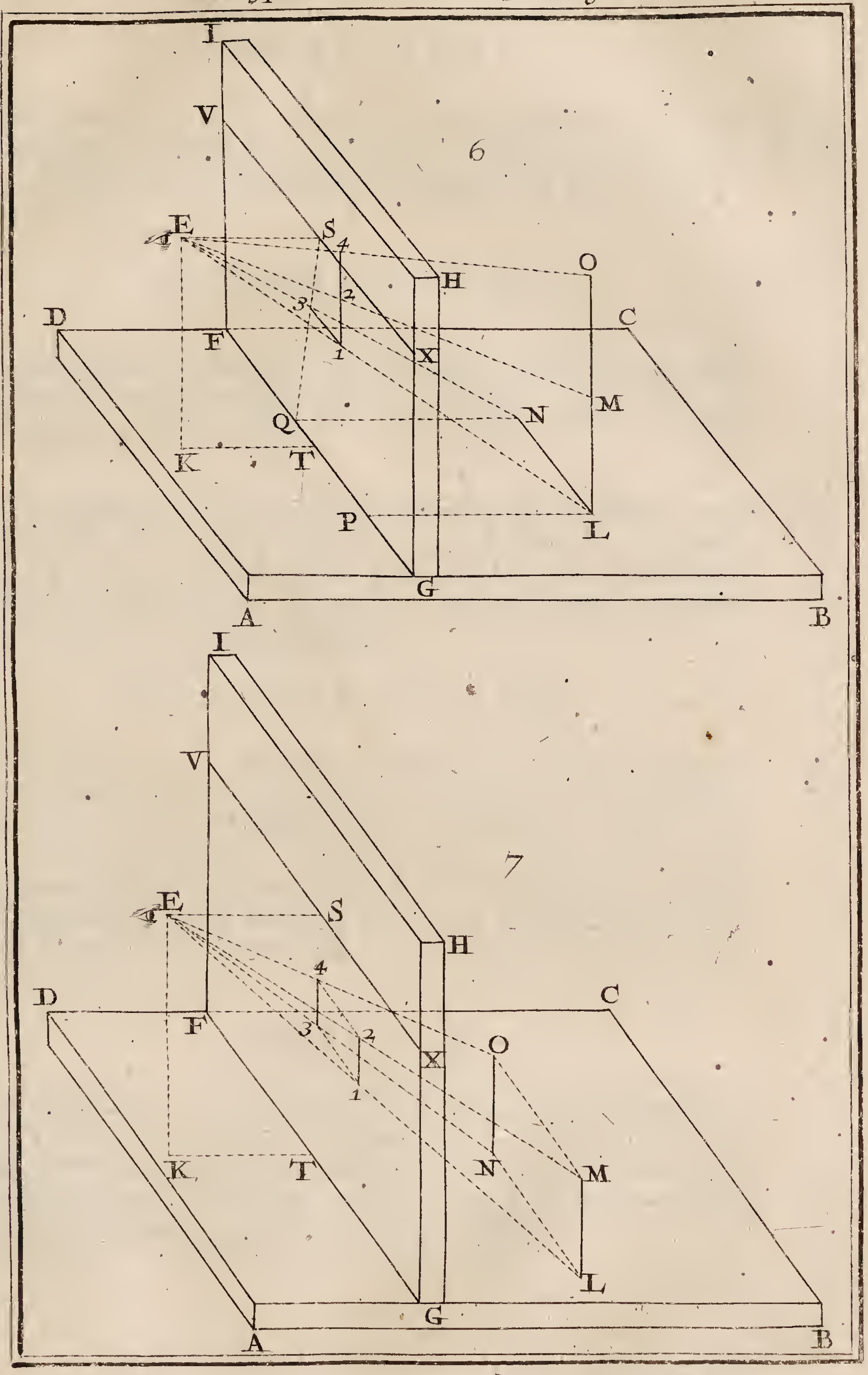
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COROLLARY.

It follows, *by* 10, 11. That ſince the Two lines LM, LN, are parallel to the Two 12, 13, the Angle L of the Two lines LM, LN, is alſo equal to the Angle 1 of their Appearances 12, 13. *Plate* 3. *Fig.* 6.

THEOREM X.

If a Right-line parallel to the Picture be divided into equal Parts, the Appearances of thoſe Parts in the Picture will be Equal.

IF the Right-line LO be parallel to the Picture FGHI, and divided, *for Example*, into Two equal parts at the point M; I ſay, That the appearances 12, 24, of the equal parts LM, NO, are alſo equal; as may be known, if from the Eye E, you draw the Rays EL, EM, EO. *Fig.* 6.

DEMONSTRATION.

For the line 14 is parallel to the line LO, *by Theorem* 7: which cauſes the Two Triangles ELM, E 12, to be Equiangular, as alſo the Two EMO, E 24, whence we conclude, *by* 4. 6. That EM: is to E 2:: as LM: is to 12, and as MO: is to 24, and conſequently, that the Four Lines LM: MO:: 12: 24 are proportional: And becauſe the Two Firſt LM, MO are ſuppos'd equal, the Two Laſt 12, 24, muſt alſo be equal. *Q. E. D.*

SCHOLIUM.

If the line LO was produc'd to O, in ſuch manner that the part added to it ſhou'd be equal to LM, or to MO; One might demonſtrate after the ſame manner, That the Appearance of that new Line added will be equal to the part LM, or to the appearance 24 of the other part MO.

THEO-

THEOREM XI.

If Two Right-lines Equal to one another, and Parallel to the Picture, be equally Distant from the Picture; their Appearances in the Picture will be Equal.

Plate 3. Fig. 7. THE lines LM, NO, are suppos'd equal, and parallel to one another, and to the Picture FGHI, and also equally distant from the Picture, so that the line LN, or MO, which joins their Extremities, is parallel to the Ground-line FG, and consequently to the Horizontal-line VX. I say, That in such a case, the Appearances 12, 34, of the Two equal lines LM, NO, are also equal.

DEMONSTRATION.

For since, *by Theorem 6.* the Appearances of the lines LM, NO, which are parallel to the Picture FGHI, *viz.* 12, 34, are parallel to one another, as well as the Two 13, 24, which are the Appearances of the lines LN,MO, parallel to one another and to the Picture; the Figure 1,2,4,3, will be a Paralellogram, whose Two opposite sides 12, 34, are, *by* 34. 1. equal to one another. *Q. E. D.*

THEOREM XII.

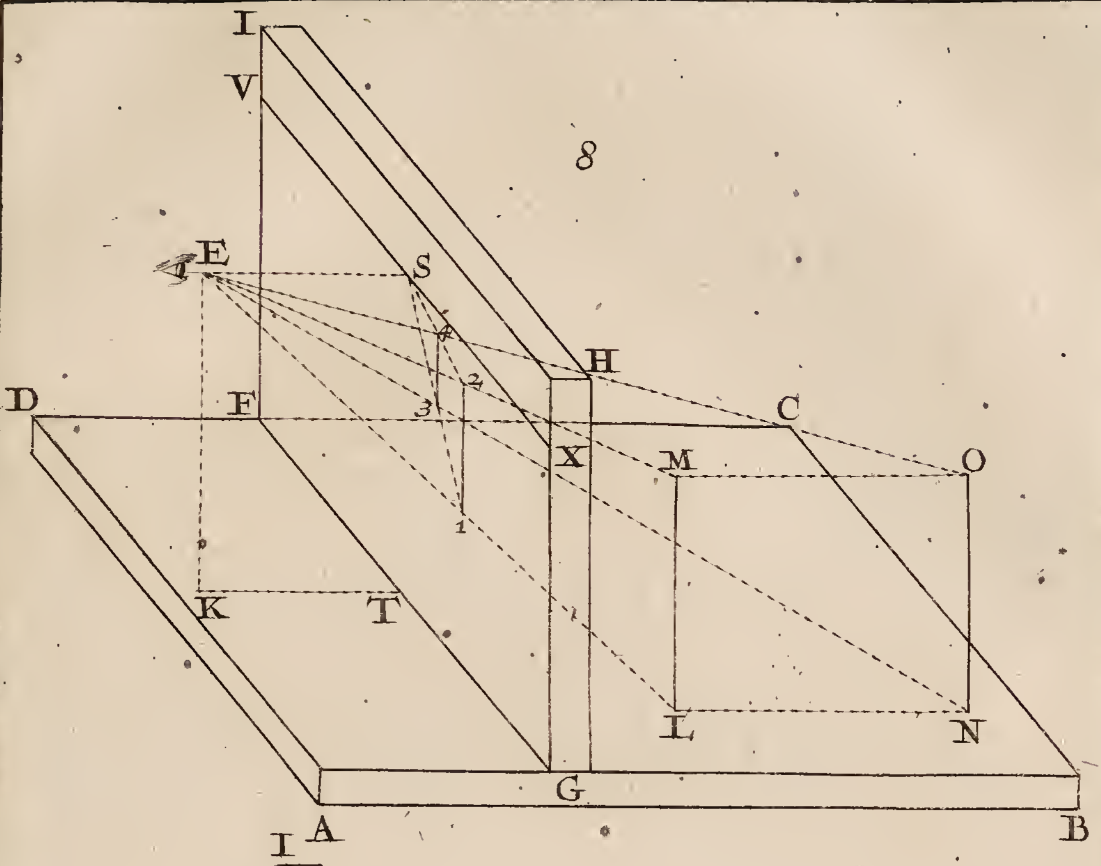
If from as many Points as you will of a Right-line, (which being produc'd meets to the Picture) be drawn as many equal Right-lines, parallel to one another, and to the Picture, their Appearances in the Picture will be terminated by Right-lines, which being produc'd, will go thro' the Accidental Point of that Right-line.

Plate 4. Fig. 8. FROM the Two points L,N, of the Right-line LN, whose accidental point is S, draw the Two equal Lines LM, NO, parallel to one another, and to the Picture FGHI; then I say, That the Appearances 12, 34, of those Two Lines LM, NO, must be terminated by the lines 13, 24, which being produc'd, will fall into the accidental point S.

DEMONSTRATION.

For since the lines LM, NO, are parallel, and equal to one another, the lines LN,MO, which join their Extremities will

also



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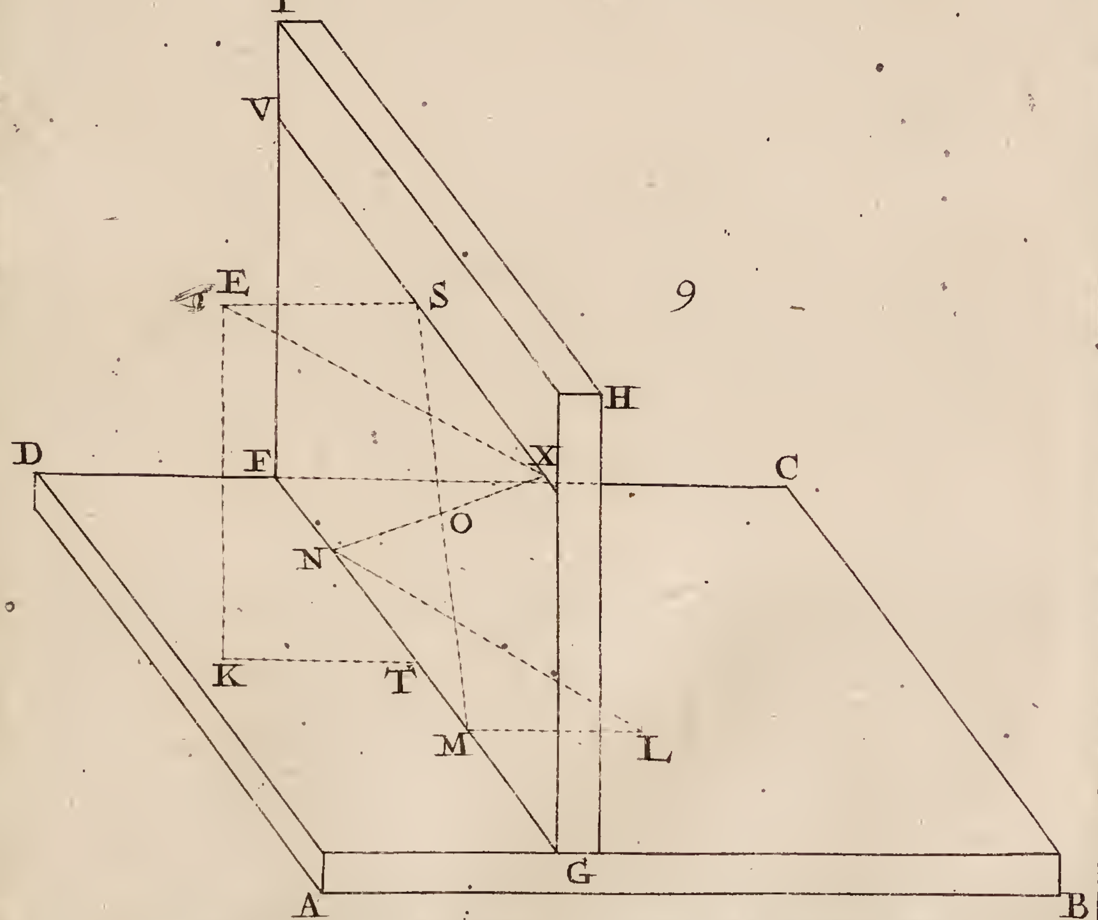

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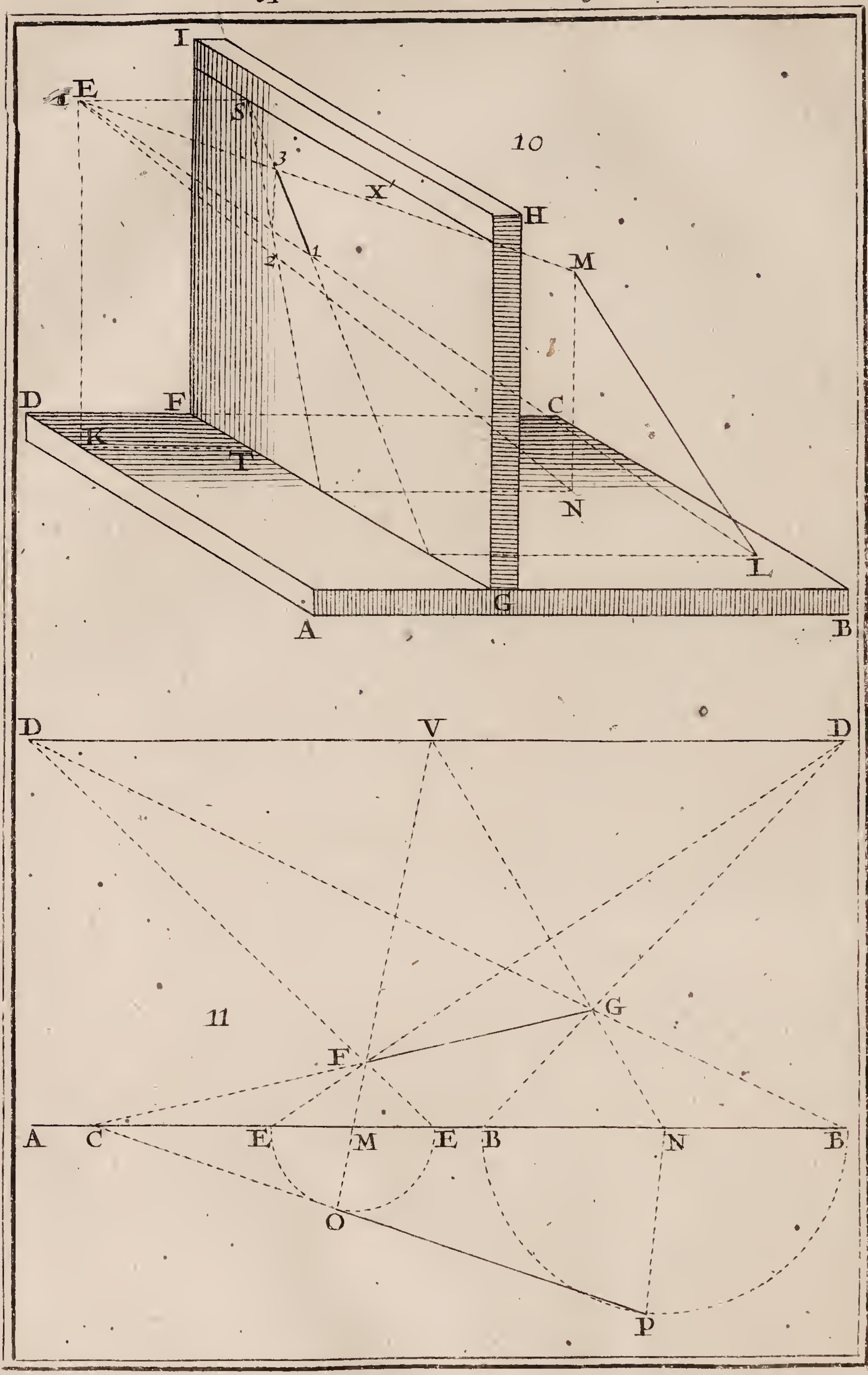
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alſo be equal and parallel, *by* 33. 1. and one of thoſe Two lines, *viz.* LN being ſuppos'd parallel to the Ray ES, the other line MO will be alſo parallel to the ſaid Ray ES, and the point S will be the Accidental Point of the Two Lines LN, MO, where their Appearances 13, 24, muſt concur, *by Theorem* 7.

PROBLEMS.

PROBLEM I.

A Point being given in the Geometrical Plain, how to find its Appearance in the Picture.

LET L be the point given in the Geometrical Plain, whoſe Appearance is to be found in the Picture FGHI, whoſe point of Sight is S, in reſpect of the Eye at E, and the Horizontal Line VX; mark upon that Horizontal Line VX, the Two parts SV,SX, each equal to the principal Ray ES, or to the diſtance from the Eye to the Picture, to have at V and X the Two points of Diſtance, by means of which, the Appearance of L the point propos'd will be thus found: *Plate* 4. *Fig.* 9.

Draw from that point L, the line LM perpendicular to the Ground-line FG; and from the point M, where that Perpendicular cuts the Horizontal-line, draw to the principal Point S, the Right-line MS. Set off the Length of the perpendicular LM, from M along the Ground-line FG, either on the Right, or on the left, *as for Example,* to N, and draw thro' that point N, and thro' X the oppoſite point of Diſtance, the line XN, which will upon the line SM give the Appearance of the propos'd point L, at O.

DEMONSTRATION.

For if you join EX,LN, you will eaſily know that they are parallel to one another, becauſe the Angles which they make with the Picture are Half-right, by reaſon of the Rectangular Iſoſceles Triangles ESK,LMN; wherefore X the point of Diſtance will be the Accidental Point of the Line LN, and, *by Theorem* 8. the Appearance of the point L will be in ſome point of the line XN, and as it is alſo in the line SM, becauſe LM is perpendicular to the Picture, the point O their common Section muſt be the Repreſentation of L the Point propos'd. *Q. E. I. & D.*

SCHO-

SCHOLIUM.

Plate 4. *Fig.* 9. It is plain that O is the Appearance of the point L but in respect to the point E, where we have suppos'd the Eye, and where consequently it must be plac'd when the Picture is to be look'd at from such a place, that O may exactly Represent the point L: For if the Eye be any where else besides E, either the point of the Sight will be chang'd, or the Distance betwixt the Eye and the Picture, and then the Points of Distance V, X, will no longer be the same: And the Representation of the point L will no longer be in O.

One may by help of this Problem represent in the Picture any Figure that is suppos'd to be in the Geometrical Plain: For if that Figure be made up of several Right-lines, the Appearance of each in particular may be found, by finding out the Appearances of the Two points which terminate it: And if it have some Curve Lines; their Appearances may be found, if by One Line you join several points in the Picture, which will be the Appearances of as many other points taken at pleasure on the Curves of the Geometrical Plain.

PROBLEM II.

A Point being given in the Geometrical Plain from whence proceeds a Right-line perpendicular to the Horizon, whose Length is given; how to find the Appearance of such a Line in the Picture.

Plate 3. *Fig.* 6. THE point L is given in the Geometrical Plain, and there stands upon it an upright Line, whose length LM is given. It is requir'd to find the Appearance of that line LM in the Picture FGHI, whose Principal Point is S, and the Two points of Distance V, X.

Having thro' the point L, drawn the line LN parallel to the Ground-line FG, and equal to the propos'd LM, draw from the point L,N, the lines LP, NQ, perpendicular to the Ground-line FG, and *by Problem* 1. find the Appearances 1, 3, of the Two points LN; that is, the Appearance 13 of the line LN. Then raise from the point I, the line 12, perpendicular to the Ground-line FG, and equal to the line 13; and that perpendicular 12 will be the Appearance of LM, the Line propos'd.

DEMONSTRATION.

For if the line LM be perpendicular to the Geometrical Plain ABCD, its Appearance in the Picture will be perpendicular to the Ground-line FG, *by Theorem* 7. and it will pass thro' the point 1, which is the Appearance of the point L: And

And becauſe LM is perpendicular and equal to LN, which proceeds from the point L, and is parallel to the Ground-line FG, the Appearances of theſe Two equal lines LM,LN, ought to be equal, *by Theorem* 9. wherefore the Line 12, which goes from the Point 1, having been drawn perpendicular to the Ground-line FG, and equal to 13, which is the Appearance of the Line LN, will be the Appearance of the Line LM. *Q. E. I & D.* *Plate* 3. *Fig.* 6.

SCHOLIUM.

In the Practice you need not draw the line LN, only after you have drawn from the point L, LP perpendicular to the Ground-line FG, you muſt take upon that Ground-line PG the part PQ equal to the propos'd line LM, and draw from the principal Point S, thro' the point Q, the Right-line SQ, which will terminate at the point 3, the line 13 parallel to the Ground-line FG; and that line 13 will be the length of 12 the perpendicular requir'd.

One may by means of this Problem repreſent in the Picture any Priſm of known Height, whoſe Plan is given in the Geometrical Plain, by deſcribing the Appearance of that Plan in the Picture *by Probl.* 1. and raiſing from the Points of that Appearance Lines perpendicular to the Ground-line, and equal in Height to the propos'd Priſm, as we have juſt taught.

PROBLEM III.

A Point being given in the Geometrical Plain, from whence an Inclin'd Right-line of known length proceeds; how to find in the Picture the Appearance of that Inclin'd Line.

SUppoſe the Inclin'd Line LM, of given Length and Poſition, to be drawn from L a point given in the Geometrical Plain ABCD. To find its Appearance in the Picture FGHI, whoſe point of Sight is S, and one of the points of Diſtance X; draw from the upper End M, the line MN perpendicular to the Geometrical Plain ABCD, to have at N upon that Geometrical Plain the Situation of the end M; and having *by Probl.* 1. found out the Appearances 1, 2, of the Two points LN, which are upon the Geometrical Plain ABCD, find out *by Probl.* 2. the Appearance 2, 3, of the Perpendicular MN, and draw the Right-line 1, 3, which will be the Appearance of the Inclin'd Line LM, becauſe the point L is repreſented by the point 1, and the point M by the point 3. *Plate* 5. *Fig.* 10.

SCHO-

SCHOLIUM.

One may also by means of this Problem represent in the *Plate* 5. Picture a Body Inclin'd and cut Sloping, whose Ichnography *Fig.* 10. you have on the Geometrical Plain, the Height of all its parts being known; namely, by finding out *by Probl.* 1. the Appearance of the Plan of the Body, and afterwards finding out the Appearance of all the Inclin'd Lines which terminate that Inclin'd Body, as has been taught.

Perspective Practical, which we shall teach after these Problems, will shew you better how to put the Three foregoing Problems in practice, which will suffice for the common Operations of *Perspective*: But to solve several Difficulties which may occur, we shall here add the following Problems.

PROBLEM IV.

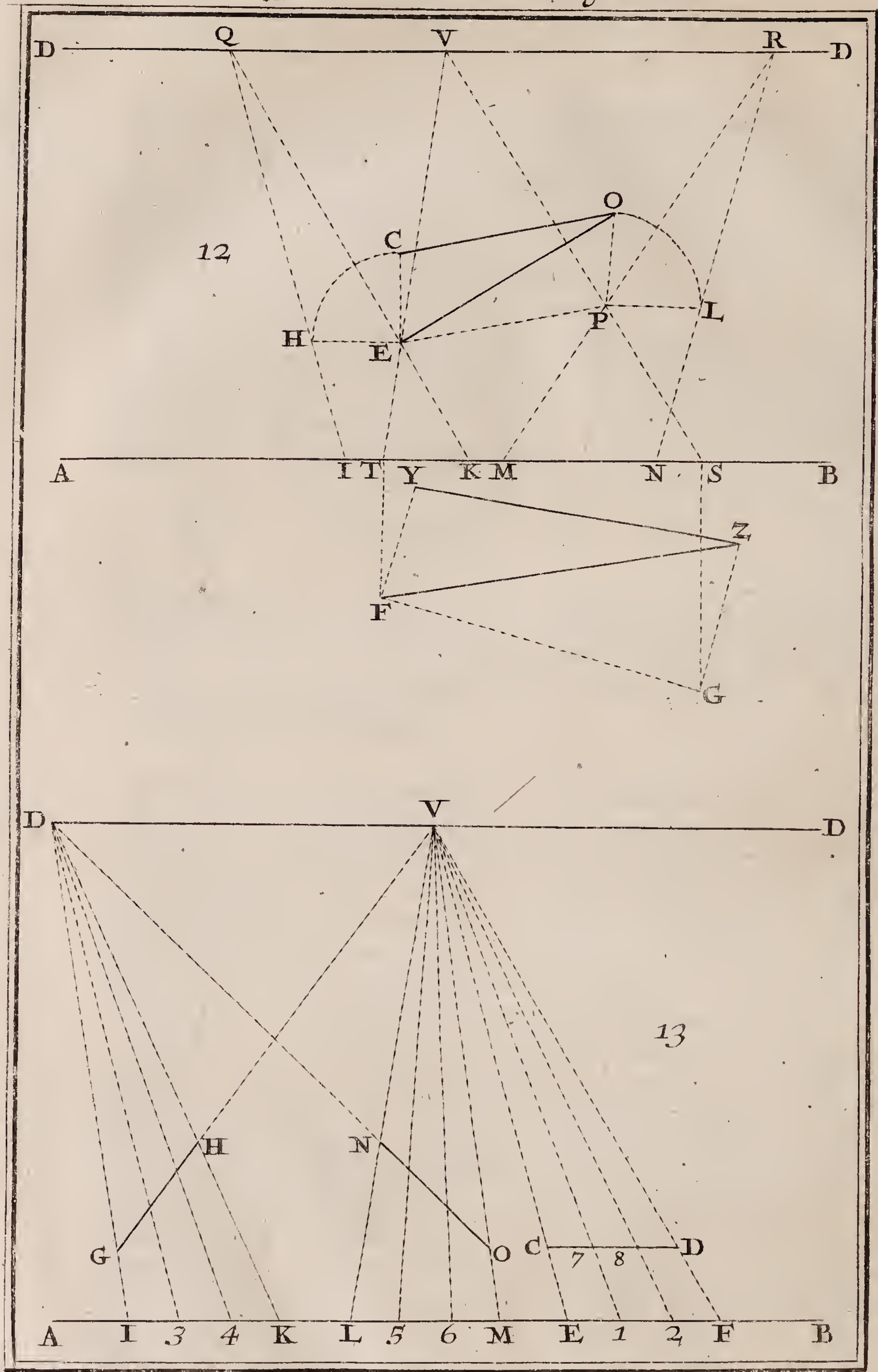
The Appearance of a Right-line of the Geometrical Plain being given in the Picture; how to find out the Length and Position of that Right-line in the Geometrical Plain.

Fig. 11. THE Line AB represents the Ground-line, and its Parallel DD the Horizontal Line, upon which are mark'd the Point of Sight V, and D, D, the Two points of Distance, equally distant from V the principal Point. We shall always mark these things by the same Letters, that we may not be oblig'd to repeat 'em every time. The Rest which is below the Ground-line AB must be taken for the Geometrical Plain, which must be suppos'd behind the Picture.

The line FG is the Appearance of a Line of the Geometrical Plain, and you must find upon the Geometrical Plain the length and position of that Line which is represented in the Picture by the line FG. Draw thro' F, G, the Two ends of the propos'd line FG, to One of the points of Distance D, the Right-lines DB, DE; and the principal Point V the Right-lines VM, VN; and thro' the Points M, N, of the Ground-line AB, draw to the said Ground-line the Perpendiculars MO, NP, in such manner that MO may be equal to ME, and NP to NB, and draw the Right-line OP, which will be the Line requir'd.

DEMONSTRATION.

For it is plain *by Probl.* 1. that the point F is the Appearance of the point O, and the point G the Appearance of the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the point P, and consequently that the line FG is the Appearance of the line OP. Thus we have found upon the Geometrical Plain the Length and Position of the line OP, whose Appearance FG was given in the Picture. *Q. E. I. & D.* *Plate 9. Fig. 11.*

SCHOLIUM.

You may do it without having any regard to the principal Point V, if you have the points of Distance as here; namely, if you draw thro' those Two points of Distance D, D, and thro' the Ends F, G, of FG the line propos'd, the Right-lines DE, DB; and if you divide the distance EE into Two equal parts at the point M, and the distance BB at the point N, to make an End of the rest as before.

It is evident that when FG the line propos'd is not parallel to the Horizontal-line AB, being produc'd it will meet the said Horizontal-line in a point, as C, thro' which also the line OP will pass when produc'd, whose Appearance is the line FG; and this may afford us a shorter Method in the Practice.

If FG the Line propos'd was a Curve, in which case it wou'd also represent a Curve, you might after the same Manner find that Curve upon the Geometrical Plain; namely, by finding several of its points upon the Geometrical Plain, after the same manner as you found out the point O, whose Appearance is F, and the point P, whose Appearance is G.

If FG the Line propos'd shou'd tend to the principal Point V, in which case the Two points M, N, wou'd coincide, it wou'd represent a Line perpendicular to the Picture, *by Theor.* 8. and then it wou'd suffice to find out one of its Ends upon the Geometrical Plain, to draw from it a Perpendicular to the Ground-line AB, which being equal to the distance of the points EB, terminated by the Two Rays which go from the same point of Distance D, will be the Line requir'd.

PROBLEM V.

The Appearance and the Situation of a Right-line rais'd above the Geometrical Plain been given in the Picture, how to find the Length and Height of that Line above the said Geometrical Plain.

THE Right-line CO and the Situation EP of a Right-line rais'd above the Horizon being given in the Picture, the length of the line CO, and the Height of the Two Ends C, O, that is, the Height of the Two Perpendiculars CE, OP, are requir'd. *Plate 6. Fig. 12.*

Plate 6. Fig. 12. Draw firſt from V the point of Sight, thro' the points, E P, the Right-lines VE, VP, which being produc'd will give upon the Ground-line AB, the points T, S, by means of which, and by the help of the foregoing Problem, you will find out the poſition and length FG of the Situation EP.

Then draw thro' E the point of Situation, EH parallel to the Ground-line AB and equal to the Perpendicular CE; and from the point Q taken at pleaſure upon the Horizontal line DD, draw thro' the points E, H, the Right-lines QI, QK, which will give upon the Ground-line AB, the Height IK of the point C.

Likewiſe draw thro' P the other point of Situation PL, parallel to the Ground-line AB, and equal to the perpendicular OP; and from the point R taken at pleaſure upon the Horizontal Line DD, draw thro' the points P, L, the Lines RM, RN, which will give upon the Ground-Line AB, the Height MN of the point G.

Laſtly, draw from the point F, FY perpendicular to FG and equal to IK the Height which is found: And likewiſe from the point G, the line GZ perpendicular to the ſaid Line FG, and equal to MN the Height which is alſo found; then joyn YZ which will repreſent the length of the line propos'd CO.

SCHOLIUM.

If CO the Line propos'd be a Curve, you muſt find out the Height of ſeveral of its points, as we have already found that of the points C, O; then find upon the Line FG, which may be a Right or a Curve line, the Poſition of the ſaid points, to draw from thoſe New points of Situation Perpendiculars to the line FG, which muſt be equal to the Heights which you have found out, and which correſpond to the ſaid points; and if you joyn the Ends of all thoſe Perpendiculars by a Curve, this new Curve will be that which is repreſented in the Picture by the Curve propos'd.

Whenever the Heights IK, MN are equal to one another, that will ſhew that CO the Right-line propos'd is Horizontal, that is, parallel to the Geometrical Plain, and then it will not be neceſſary to draw the Two Perpendiculars FY, GZ, to find out the Length of the line CO; becauſe in ſuch a caſe that length will be equal to the Line FG, becauſe of the Two equal and parallel lines FY, GZ, *&c.*

By means of this Problem you may eaſily find on the Geometrical Plain, the Length and Poſition of an Inclin'd Line, whoſe Appearance and Situation you have in the Picture. As if the Inclin'd Line EO, and its Situation EP, be given in the Picture, you muſt only find on the Geometrical Plain the point F, whoſe Repreſentation is E; and the line FG, whoſe Appearance

ance is EP; with the Height MN, whose Appearance is OP; and raise from the point G the perpendicular GZ equal to the known Height MN upon FG, in order to joyn the Right-line FZ, which will give the Length and Position of the Inclin'd line EO. Plate 6. Fig. 12.

PROBLEM VI.

** How to divide into Parts equal in Representation the Appearance given in the Picture of a Right-line Situated upon the Geometrical Plain.*

THERE may be several cases because the Line propos'd in the Picture may be parallel to the Ground-line, or concur with the point of Sight, or with One of the Two points of Distance, or with any other point of the Horizontal-line: But all these Cases will be resolv'd the same way, by means of a point which we shall take indifferently upon the Horizontal-line, as you will see. Fig. 13.

To divide GH, the Line propos'd, which tends to the Principal Point V, *for Example*, into Three parts which are equal in Representation, draw thro' its Two ends G, H, from the point D, taken at pleasure upon the Horizontal-line DD, the Right-lines DG, DH, and produce them till they meet the Ground-line AB, in Two points, as I, K. Then divide the part IK into Three equal parts at the points 3,4, thro' which drawing to the said point D, Right-lines, they will divide the propos'd Right-line GH into Three parts equal in Representation.

DEMONSTRATION.

The Demonstration of this Operation will be evident to him that considers the point D as the Accidental point of Four parallel Lines which are represented by lines which proceed from that Accidental point D, and divide into Three equal parts the line of the Geometrical Plain, whose Appearance is the propos'd line GH.

Likewise to divide into Three parts equal in Representation, the line NO, which tends to the point of Distance D, draw thro' their ends N, O, from the point V taken at pleasure upon the Horizontal-line DD, the Right-lines VL, VM, and having divided the part LM of the Ground-line AB, into Three equal parts at the points 5, 6, you must draw thro'

* *That is, to divide a Line in the Picture in such manner that it shall represent a Line on the Geometrical Plain which is divided into equal parts.*

Plate 6. *Fig.* 13. thoſe points 5, 6, to the ſaid point V, Right-lines, which divide NO the line propos'd into Three parts equal in Repreſentation.

The Operation muſt be the ſame for any other Line; but when it happens to be parallel to the Ground-line, as CD, it will ſuffice to divide it into Three equal parts at the points 7, 8, which will be the ſame thing as if the part EF was divided into Three equal parts at the points 1, 2, becauſe that line CD repreſenting a line parallel to the Picture, *by Theor.* 7. its Diviſions muſt *by Theor.* 10. be equal to one another.

SCHOLIUM.

This Problem may alſo be ſolv'd in general by finding upon the Geometrical Plain *by Probl.* 1. that line whoſe Appearance is the line propos'd in the Picture, and dividing into parts really equal, that line of the Geometrical Plain, which we ſhall hence-forward call the *Geometrical Line*, and which is alſo call'd the *Objective Line*, this Term generally ſignifying every line which being out of the Picture belongs to any Object. Then draw from the points of Diviſions of that Geometrical Line, Perpendiculars to the Ground-line, which will be cut by thoſe Perpendiculars in points, from which drawing Right-lines to the point of Sight, thoſe Right-lines will divide the Line propos'd into parts equal in Repreſentation. This may alſo be done more generally *by Probl.* 8.

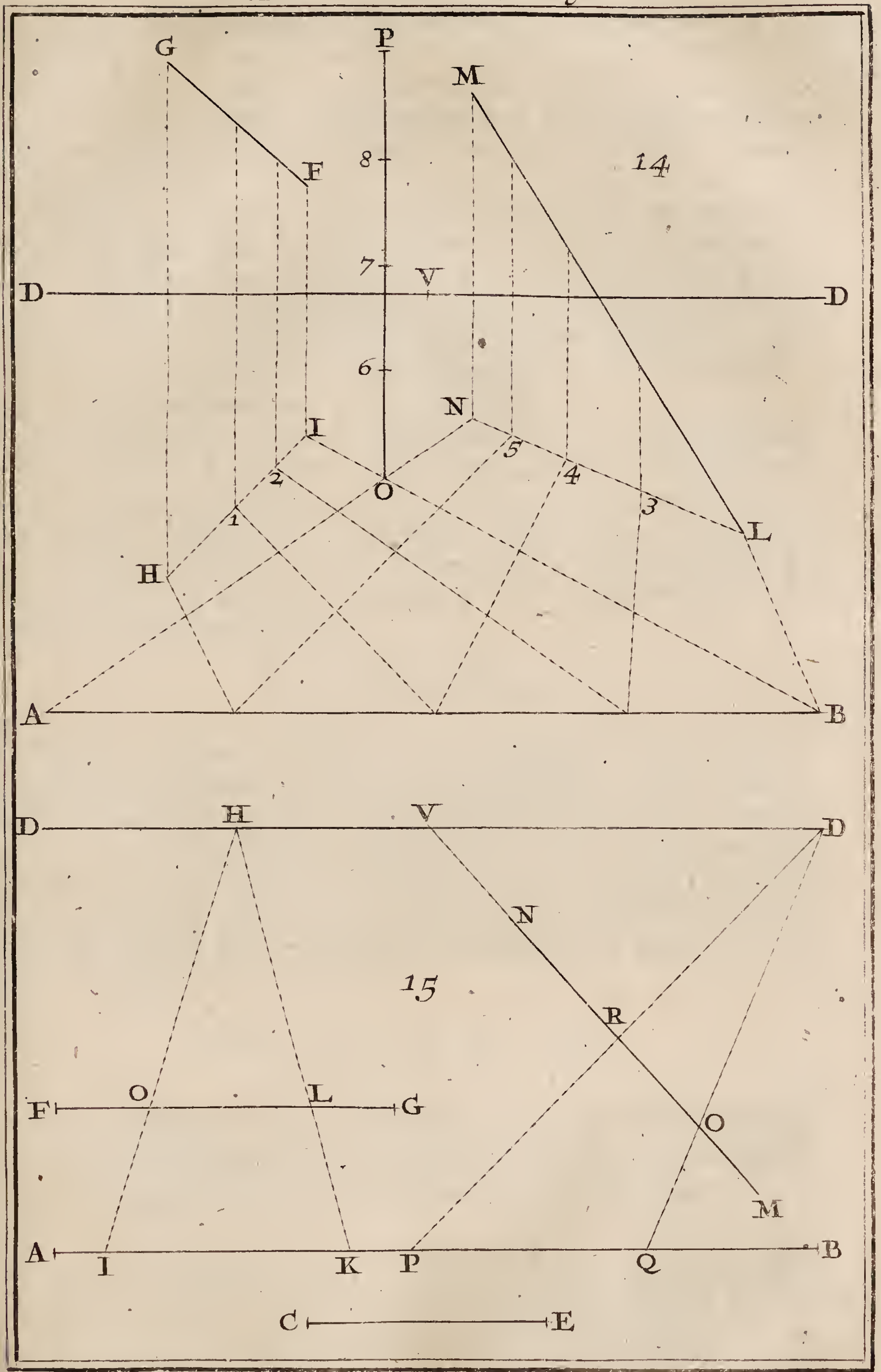
PROBLEM VII.

How to divide into Parts equal in Repreſentation the Appearance given in the Picture of an Objective Line rais'd upon the Geometrical Plain.

THERE may be ſeveral different caſes becauſe the line propos'd in the Picture may repreſent a line which is wholly out of the Geometrical Plain, or a line of which One end touches the Geometrical Plain, the other end being in the Air; and in thoſe Two Caſes that Line may be either Inclin'd or Perpendicular to the Horizon.

All theſe Caſes may be ſolv'd by means of the Situation of the line propos'd, except that in which the ſaid line is perpendicular to the Ground-line AB, as OP, becauſe its Situation being but One point, it cannot be made uſe of to divide the line propos'd into parts equal in Repreſentation; but it will be eaſy to ſolve this laſt Caſe as you will find when you have ſolv'd the firſt, thus:

Firſt, to divide the line FG, which repreſents an Objective line rais'd upon the Geometrical Plain, (*for Example*) into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into Three parts equal in Reprefentation, divide *by Probl. 6.* the Situation HI into Three parts equal in Reprefentation at the points 1, 2, and draw from each of thofe points 1, 2, Lines perpendicular to the Ground-line AB, which will divide the propos'd Line FG into Three parts equal in Reprefentation. *Plate 7. Fig. 14.*

Likewife to divide (*for Example*) into Four parts equal in Reprefentation the Inclin'd Line LM, whofe Situation is LN; let that Situation *by Probl. 6.* be divided into Four parts equal in Reprefentation, and from the Points of Divifion 3, 4, 5, draw as many Perpendiculars to the Ground-line AB, which will divide the propos'd LM into Four parts equal in Reprefentation, as was requir'd.

Becaufe the line OP is perpendicular to the Ground-line AB, it is known *by Theor.* 7. that it reprefents an Objective Line parallel to the Picture, and *by Theor.* 10. that its Divifions are equal: Wherefore to divide it, (*for Example*) into Four parts equal in Reprefentation, you muft divide it into Four parts really equal at the points 6, 7, 8, which will be the points of Divifion requir'd.

SCHOLIUM.

Becaufe the line FG tends to the principal Point V, its Situation HI tends alfo to the Point of Sight V, and in fuch a Cafe to divide it into parts equal in Reprefentation, you may make ufe of the point of Diftance D, which is its Dividing Center: But as the Situation LN of the Inclin'd Line LM does not tend to the principal Point V, its Dividing Center will be in fome other point of the Horizontal Line DD. It may be found by the means of

PROBLEM VIII.

How, from a Point given upon the Appearance given in the Picture of a Geometrical Line, to cut off a part equal in Reprefentation to a given Line.

HERE may alfo happen feveral different Cafes; becaufe the line given in the Picture may be parallel to the Ground-line AB, or may tend to the principal Point V, or to One of the points of Diftance D, or to fome other point of the Horizontal Line DD. *Fig. 15.*

All thefe Cafes may be folv'd the fame way; *viz.* by a point which we fhall mark upon the Horizontal Line, and call the *Dividing Center*, which will be found by different

Plate 7. *Fig.* 15. means; according to the Poſition of the line given in the Picture, as you will ſee.

Firſt, if the Line propos'd in the Picture be parallel to the Ground-line AB, as FG, its Dividing Center may be taken in what point you will of the Horizontal Line DD, as in H; Wherefore, if you muſt from O towards G cut off a part equal in Repreſentation to the given line CE, draw from the Dividing Center H, thro' the point O, the Ray HO, and produce it, till it meets the Ground-line AB in ſome point as I. Then make IK equal to CE, and draw the Ray HK, which will upon the given Line FG, determine the part OL equal in Repreſentation to the given Line CE.

Secondly, if the Line given in the Picture tends to the principal Point V, as MN, its Dividing Center will be which you will of the Two points of Diſtance D. If then the point O be given upon the ſaid Line; from it to cut off upon the line in the Picture a part equal in Repreſentation to the given Line CE; you muſt from the point D draw thro' O the Ray DQ, and having made QP equal to CE, draw the Ray DP, which will cut off from the given Line, MN, the part OR equal in Repreſentation to the given Line CE.

Plate 8. *Fig.* 16. But if the Line given in the Picture tends to any other point of the Horizontal Line DD, *for Example*, to the point of Diſtance D, as FG, its Dividing Center will be found if you draw thro' the principal Point V, the line VL perpendicular to the Horizontal Line DD, and equal to VD the the Diſtance from the Eye to the Picture, and ſet off the Diſtance DV from D towards the Right or towards the Left Hand upon the Horizontal Line DD towards M, which will be the Dividing Center of FG the Line propos'd. If then the line CE and the point O be given, you muſt draw the Ray MOH, and having made HA equal to CE, the Ray MA will upon the propos'd Line FG determine the part OK, equal in Repreſentation to the given Line CE.

Likewiſe to find the Dividing Center of the Line PQ, which tends to the point R of the Horizontal Line DD, ſet off RL the Diſtance from L to the Eye, from the ſaid point R on the Right or on the Left upon the Horizontal Line DD to N, which will be the Dividing Center of the Line PQ. If then the Line CE be given, and alſo the point O upon PQ the Line propos'd, and the Ray NOS be drawn, to make SB equal to CE, by drawing the Ray NB; you will upon the Line propos'd PQ have the part OI equal in Repreſentation to the given Line CE.

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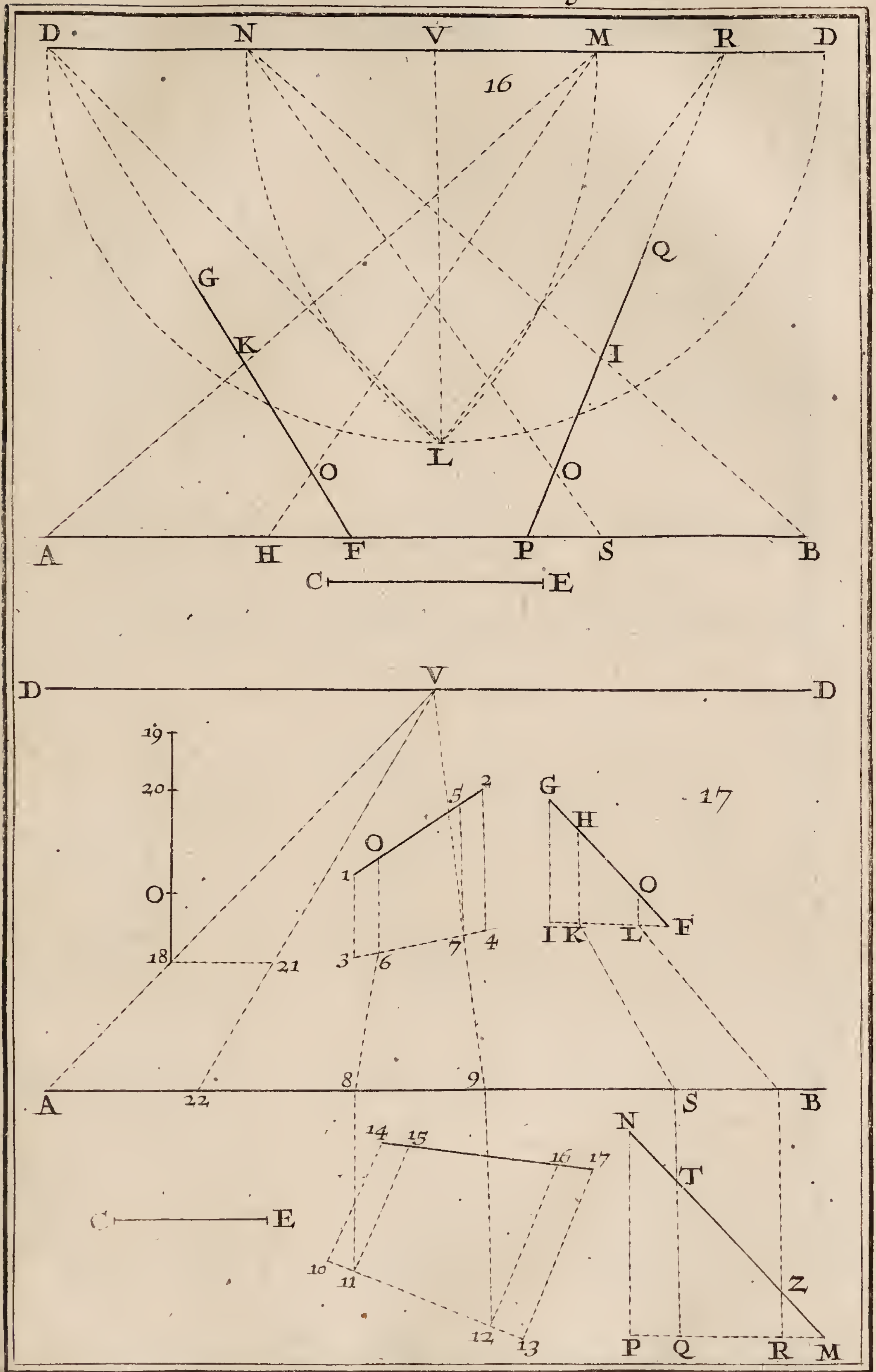
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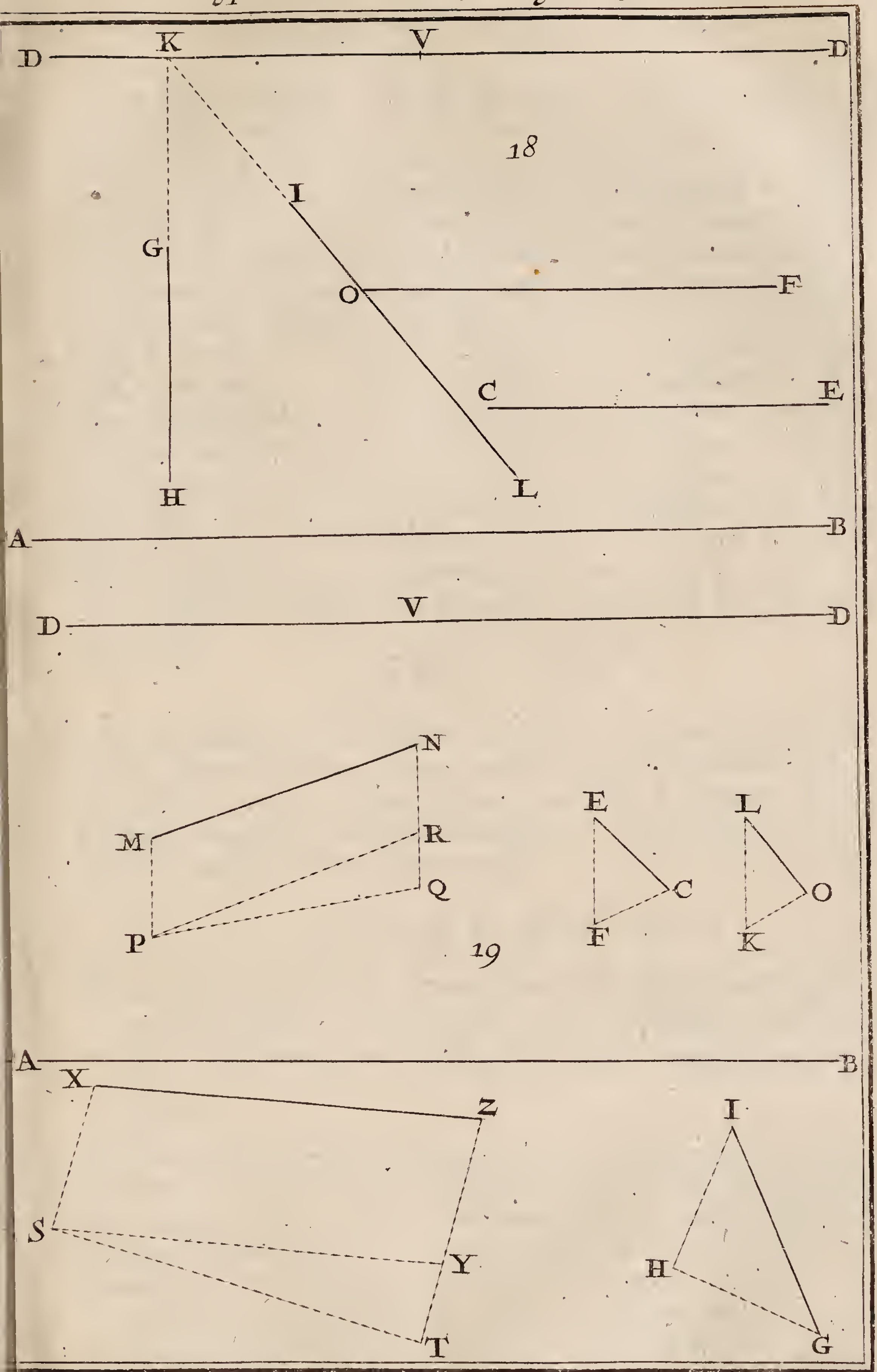
Perspective Plate 9. Page 23.



DEMONSTRATION.

The Demonstration of this Operation will be evident, if you consider that the point R is the Accidental Point of that Geometrical Line, whose Appearance is PQ, *by Theor.* 8. and that the point L represents the Eye, looking upon LV as the principal Ray, so that the line LR will be that which determines in the Picture the Accidental Point R, and which will consequently be parallel to the Geometrical Line represented by PQ in the Picture; For if a Plain be made to pass along those Two parallel Lines, and that together with their Plain they be made to turn Horizontally, (the Geometrical Line about the point P, and its Parallel LR about the point R,) till that Plain falls in with the Plain of the Picture, in which case the point L will come to N, and the Geometrical Line will coincide with the part PB of the Ground-line AB, That point N upon the Horizontal Line DD will have the same Effect upon the plain of the Picture as the point L in the Air, and therefore it will be the Dividing Center of the propos'd Line PQ. *Plate* 8. *Fig.* 16.

SCHOLIUM.

From what has been said it is easy to imagine, that to such a Line given in the Picture, a line may be added equal in Representation to a given Line, and that by means of this Problem one may solve the foregoing; and this may also be solv'd by means of the following.

PROBLEM IX.

How, from a Point given upon the Appearance given in the Picture of a Right-line rais'd above the Geometrical Plain, to cut off a Part equal to a given Line.

SEveral Cases may also happen here, because the Line given in the Picture, may represent a Line Inclin'd upon the Geometrical Plain, or perpendicular to the Geometrical Plain, or wholly above the Geometrical Plain, which may be Parallel, Inclin'd, or Perpendicular to the said Geometrical Plain.

All these Cases may be solv'd the same way, *viz*; by means of the Situation of the Line propos'd in the Picture, except when that line is perpendicular to the Ground-line; because in such a case its Situation being but One point, it can't be made use of as if it was a Right-line; but it will be easy to solve this Case, after we have solv'd the others thus:

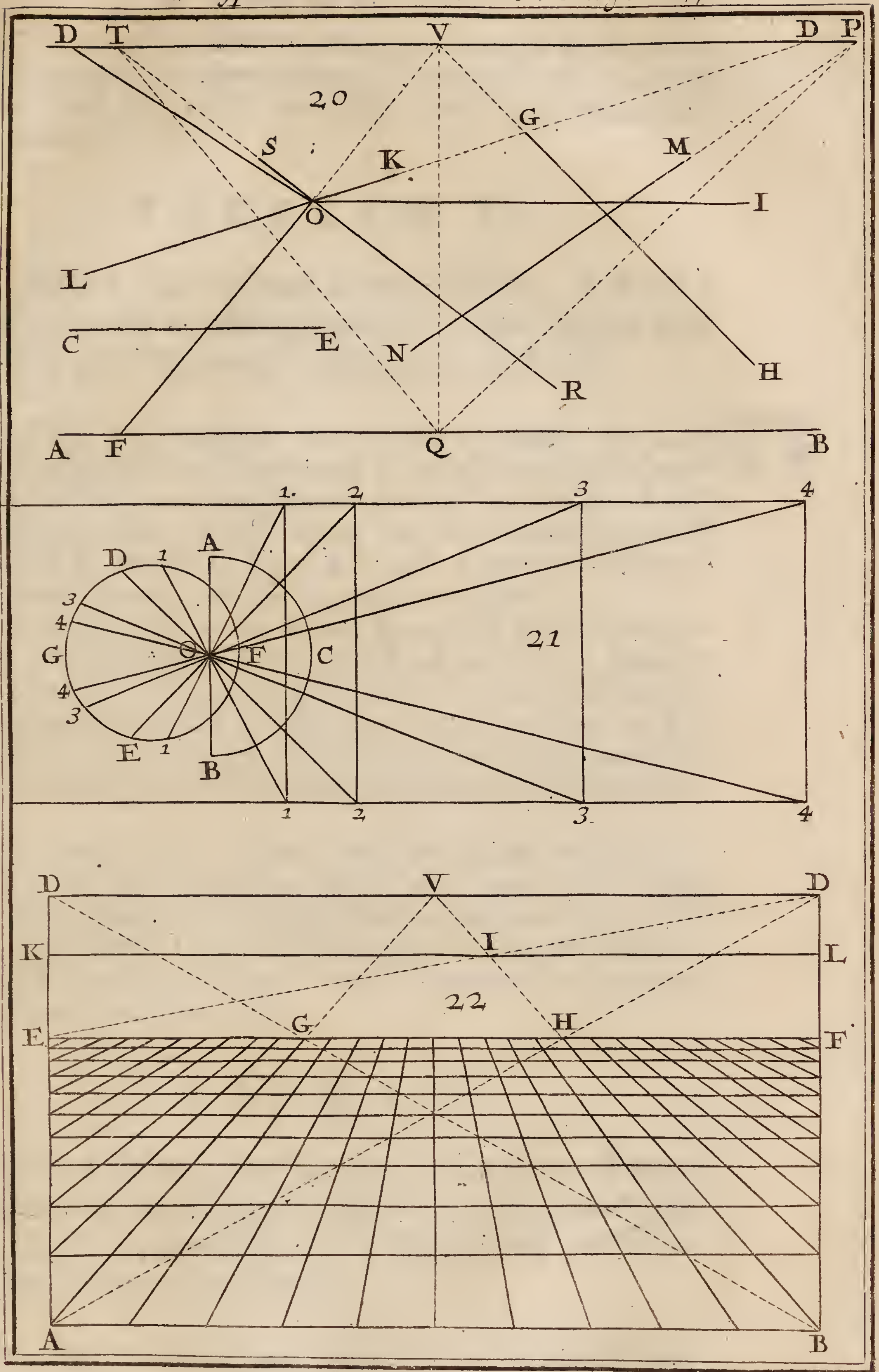
Plate 8. *Fig.* 17. Let the line FG, whose Situation is IF represent a line Inclin'd upon the Geometrical Plain; and let it be requir'd to cut off from the given Point O towards G, such a part of it as may be equal in Representation to the given Line CE.

Having *by Prob.* 5. found the line PM, whose Representation is the Situation IF; and the line MN, whose Appearance is the Line propos'd FG; draw from O the given Point, the line OL perpendicular to the Ground-line AB, and from the principal Point V, thro' the point L, draw the line LB, which here meets the Ground-line AB at B; thro' which point you must draw to the said line AB, the perpendicular BR, which will upon PM give the point R, whose Appearance is L. From that point R upon PM raise the Perpendicular RZ, which upon MN will give the point Z, whose Appearance is O. Make ZT equal to the given Line CE, and draw from the point T, TQ perpendicular to PM; and from the point Q, QS perpendicular to the Ground-line AB. Lastly, from the principal Point V, thro' the point S, draw the Ray SK, and from the point K, KH perpendicular to the Ground-line AB, which will upon the propos'd Line FG, determine the part OH equal in Representation to the line ZT, or to the given Line CE.

Likewise if the point O be given upon the given Appearance 1, 2, of a line rais'd upon the Geometrical Plain, whose Situation is 3. 4, to cut off from it a part equal in Representation to the given Line CE; draw from that point O, O 6 perpendicular to the Ground-line AB; and having *by Probl.* 5. found the line 10, 13, whose Appearance is the Situation 3, 4, and the line 14, 17, whose Appearance is the Line propos'd 1, 2; draw from the principal Point V, thro' the point 6, the Right-line 6, 8, and thro' the point 8, the line 8, 11, perpendicular to the Ground-line AB, and likewise thro' the point 11, the line 11, 15 perpendicular to the Geometrical Situation 10, 13, which will upon the line 14, 17, give the point 15, from which upon the line 14, 17, must be taken the part 15, 16, equal to the given line CE, to draw from the point 16 to the Line 10, 13, the Perpendicular 16, 12, and from the point 12 to the Ground-line AB, the Perpendicular 12, 9; then draw from the point 9 to the principal Point V, the Ray 7, V, and from the point 7, raise 7, 5, perpendicular to the Ground-line AB, and you will upon the Line propos'd 1, 2, have the part O 5 equal in Representation to the given line CE.

If the point O be given upon a line perpendicular to the given Line AB, as 18, 19, whose point of Situation is 18, draw from that point 18, to the Ground-line AB, the parallel 18, 21, terminated by Two Rays, drawn from the point V taken at pleasure upon the Horizontal-line DD, thro' the

points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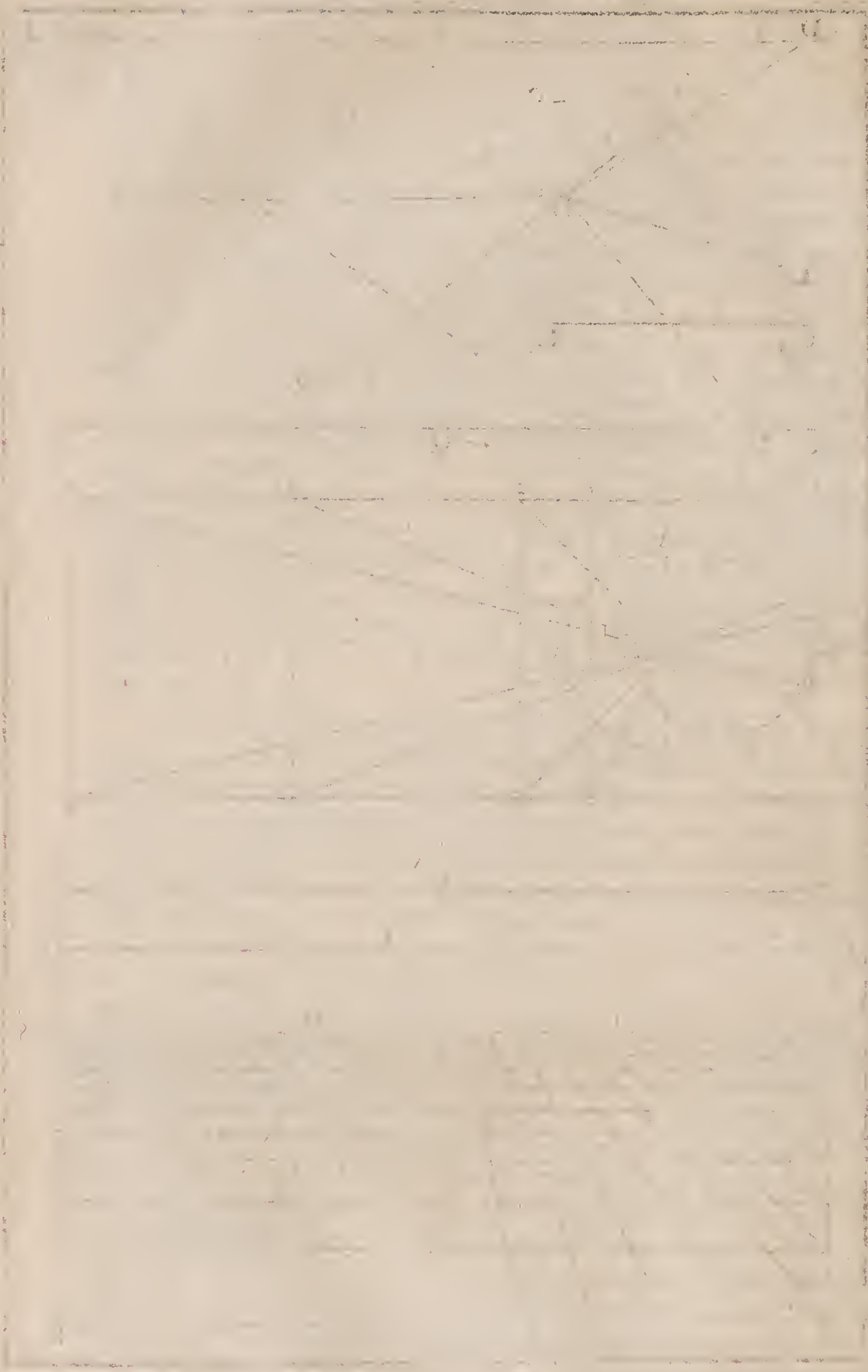


points A, 22, (diſtant from one another upon the Ground-line AB the length of the given Line CE) and make O, 20 equal to 18, 21; and the part O, 20 will be equal in Repreſentation to the given Line CE.

PROBLEM X.

How, from a Point given in the Picture, to draw a Line parallel in Repreſentation to the Appearance given in the Picture of a Geometrical Line.

THERE may happen Two Caſes; becauſe the Line propos'd in the Picture may be parallel to the Ground-line AB, as CE; or it may when produc'd, meet the Horizontal Line, as GH, which meets the Horizontal Line DD at the point K, which *by Theor.* 8. is the Accidental Point of the Geometrical-line, whoſe Appearance is GH. *Plate* 9. *Fig.* 18.

Firſt to draw to the given Line CE parallel to the Ground-line AB, thro' the given Point O, a Line parallel in Appearance; draw thro' the ſaid point O, O F parallel to the Ground-line AB, which *by Theor.* 4. will be parallel in Repreſentation to the given CE; that is, it will repreſent a Geometrical Line parallel to that which is repreſented by the line CE.

In the Second place, to draw thro' the ſaid given Point O, a Line parallel in Repreſentation to the given Line GH, which being produc'd meets the Horizontal-line DD at the point K, draw thro' that point K and thro' the given point O, the line IL, which *by Theor.* 5. will be parallel in Repreſentation to the given Line GH.

PROBLEM XI.

How, from a Point given in the Picture, to draw a Line parallel in Repreſentation to the Appearance given in the ſaid Picture of a Right-line rais'd above the Geometrical Plain.

SEveral different Caſes may happen concerning the Line given in the Picture, becauſe it may repreſent a Right-line One of the Ends of which touches the Geometrical Plain, or a Line which is wholly above the Geometrical Plain; and in thoſe Two Caſes the Line propos'd may repreſent a Line which is inclin'd, or One which is parallel, or One which is perpen-

Plate 9. *Fig.* 19. perpendicular to the Geometrical Plain, in which caſe the Line propos'd is perpendicular to the Ground-line; and in ſuch a Caſe the Solution is particular, as you will ſee.

Firſt, to draw thro' the given Point O, a Line in the Picture parallel in Repreſentation to the Appearance given CE of a Right-line Inclin'd to the Geometrical Plain, and touching the ſaid Geometrical Plain in a point whoſe Appear-is C; draw *by Probl.* 10. thro' the given Point O, to the Situation CF, the parallel OK, equal in Repreſentation to the ſaid Situation CF, or to the line GH, whoſe Repreſentation is CF, which may be done *by Probl.* 8. for I ſuppoſe that the Figure GHI, whoſe Appearance is CEF, has been found upon the Geometrical Plain *by Probl.* 3. Then draw from the point K, to the Ground-line AB, the Perpendicular KL, equal in Repreſentation to the Perpendicular EF, or to the Perpendicular HI, whoſe Repreſentation is EF, which may be done *by Probl.* 2. and draw OL, which will be parallel in Repreſentation to CE the Line propos'd.

If the Line propos'd be wholly rais'd above the Geometrical Plain, as MN, whoſe Situation is PQ: having *by Probl.* 3. found the Figure STZX, upon the Geometrical Plain, which Figure is repreſented in the Picture by the Figure PQNM; draw thro' the Point S of the Geometrical Plain, SY parallel to XZ, and having *by Probl.* 2. made QR equal in Repreſentation to the Line TY, draw the Right-line PR, which will repreſent a Line Inclin'd to the Geometrical Plain and Parallel to the propos'd MN. Wherefore if, as we have taught in the firſt Caſe, thro' the point given in the Picture a Line be drawn parallel in Repreſentation to the Line PR, that Parallel will alſo be parallel in Repreſentation to the Line propos'd MN.

It is evident *by Theor.* 4. That if the Line given in the Picture be parallel to the Ground-line AB, its Parallel in Repreſentation will alſo be parallel to the ſaid Ground-line AB; and if the Line given in the Picture be perpendicular to the Ground-line AB, in ſuch manner that it repreſents a Line perpendicular to the Horizon, its parallel in Repreſentation will alſo be perpendicular to the ſaid Ground-line AB.

COROLLARY.

This Problem gives us a Method to find the Accidental Point of a Line propos'd in the Picture: For if thro' a Point taken at pleaſure in the Picture, a Line be drawn parallel in Repreſentation to the Line propos'd, in the Picture the point of Section of thoſe Two Lines parallel in Repreſentation, will be the Accidental Point of the Line propos'd.

PRO-

PROBLEM XII.

How, from a Point given in the Picture, to draw a Line perpendicular in Representation to a Right-line given in the said Picture.

THERE may happen Two principal Cases, because the Line given in the Picture may represent a Geometrical Line, or a Line rais'd above the Geometrical Plain, but as this Second case is of no great use, we shall only speak of the First, which may also have several different cases, because the Line given in the Picture may be parallel to the Ground-line, or meet with the Point of Sight, or One of the Points of Distance, or any other point of the Horizontal Line.

First, if the Line given in the Picture be parallel to the Ground-line AB, as CE, and O be the Point given; draw from that point O, thro' the principal Point V, the line OF, which will be perpendicular in Representation to the given Line CE. *Plate* 10. *Fig.* 20.

If the Line given in the Picture tends to the Principal Point V, as GH; draw thro' the point O, to the Ground-line AB, the Parallel OI, which will be perpendicular in Representation to GH the Line propos'd.

If the Line given in the Picture tends to one of the Points of Distance D, as KL; draw to the other Point of Distance D, thro' O the given Point, the line DO, which will be perpendicular in Representation to the propos'd Line KL; because Each of these lines makes half a Right-angle with the Picture; and therefore, these Two lines meet so as to make a Right-angle.

Lastly, if the Line given in the Picture meets the Hozontal Line DD in any other point, as MN, which meets it at the point P; draw from the Principal Point V, the Right-line VQ perpendicular to the Horizontal Line DD, and equal to VD the Distance of the Eye from the Picture, and having joyn'd QP, draw to it thro' the point Q the perpendicular QT, which will, upon the Horizontal Line DD, give the point T, from which you must draw thro' O the given Point, the line RS, which will be perpendicular in Representation to the propos'd Line MN. We shall not give the Demonstration of these little Operations; because whoever understands the foregoing Problems may easily find it out.

Perspective Practical.

PERSPECTIVE *Practical*, is that which by short Methods, that is, by Rules which are short and easy, teaches to Represent in Perspective in the Picture whatever you please: it is divided into *Lineal Perspective*, which is the Diminution of the Lines in the Plain of the Picture where they represent others distant from the Picture: And into *Aerial Perspective*, which is the Diminution of Teints and Colours, and belongs to Painters only, wherefore we shall say no more of it here.

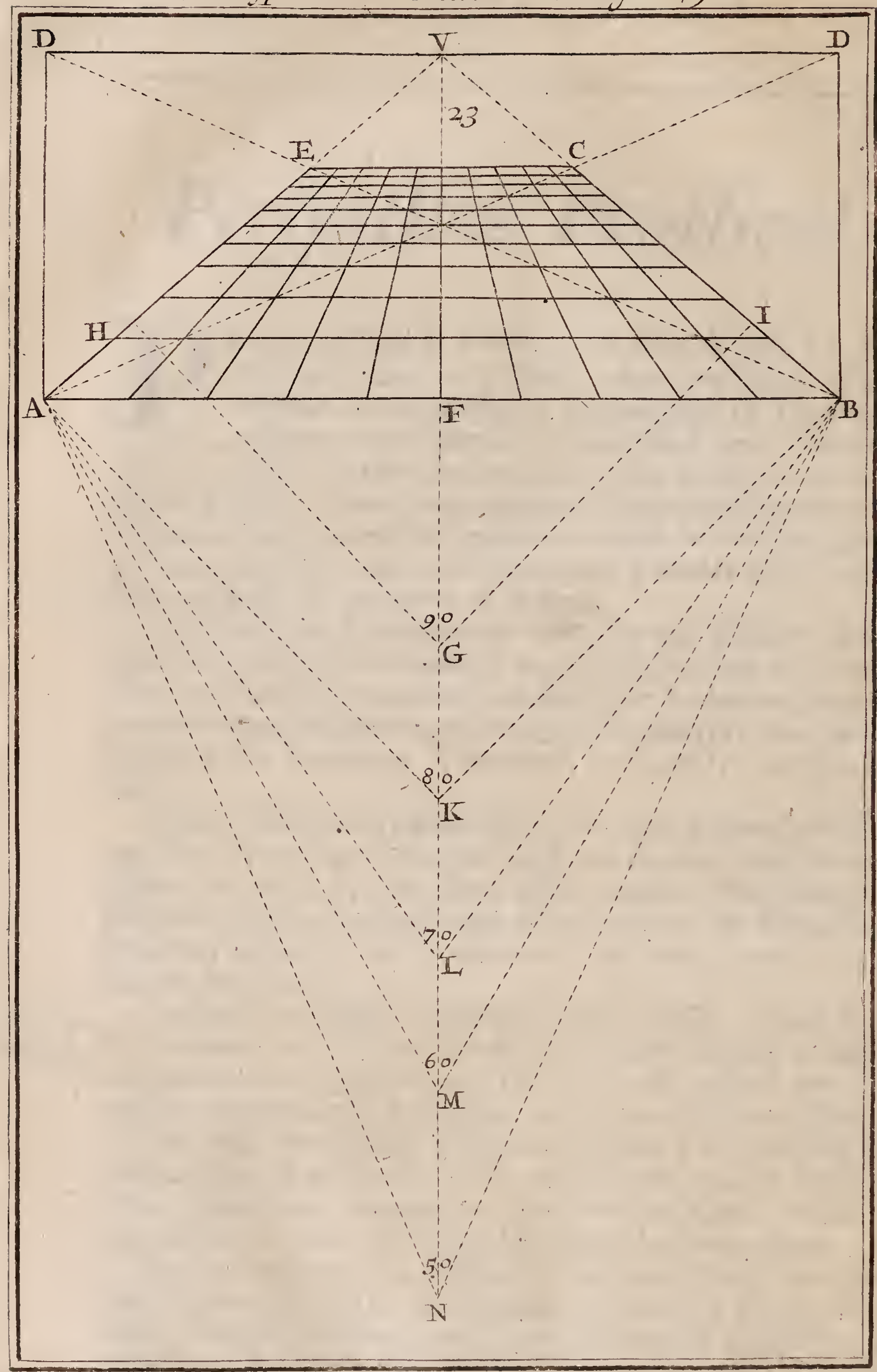
The foregoing Problems are only for the Theory of Perspective consider'd in General, and the following will be for Practical Perspective, where we shall give no Demonstrations; because whoever knows any thing of Geometry, and has understood the foregoing Theorems, will easily Comprehend them.

Before we begin any Operation, we must know about how near the Points of Distance ought to be from the Point of Sight; or what is the same thing, under what Angle the Picture ought to be seen, that whatever is to be Represented in it may appear in just Proportion, and taken in easily by the Eye at one View.

Plate 10. *Fig.* 21. To find that Angle, consider the Eye DGE, whose Pupil is towards F, and *Retina* towards G, whose Extent at most is no greater than the distance from D to E, which are Two points diametrically Opposite. It is certain that That Eye can see only the Objects which are contain'd within the Compass of the Semi-circle ACB, and which can at the same time paint their Images on the *Retina* DGE, which for that reason we have not extended beyond a Semi-circle.

This being suppos'd, if the Eye sees the Object 2,2, under the Right-angle 2O2, its Representation will fill the *Retina* DGE; but its ends 2,2, will not be seen so distinctly, because their Rays O2, O2, will fall upon the ends D,E, of the *Retina*, and the Eye will endure some pain if it endeavours to see distinctly the whole Object 2,2.

The same Eye cou'd not see distinctly the Extremities of the Object 1,1, because the Rays O1, O1, wou'd not fall upon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upon the *Retina* DGE. It wou'd ſee very conveniently the Object 3, 3, becauſe it wou'd ſee it under the Angle 3O3 less than a Right, which wou'd not put it to ſo much pain. It wou'd yet ſee more eaſily the Object 4, 4, for the ſame reaſon. But if the Object was very diſtant, the Viſual Angle wou'd be too little, and the Repreſentation of ſuch an Object wou'd not be diſtinct enough, by reaſon of the Confuſion of Viſual Rays. *Plate* 10. *Fig.* 21.

To know immediately under what Angle an Object ſhou'd be ſeen, as *for Example*, a Square, which takes in all that you wou'd Repreſent in Perſpective: Let us ſuppoſe the Plain ABDD to be the Plain of the Picture, whoſe Horizontal-line is DD, Point of Sight V, Points of Diſtance DD, and Ground-line AB, parallel to the Horizontal-line DD, which is diſtant from it the whole Height of the Eye above the Geometrical Plain. Let us alſo ſuppoſe AB to be the Length of the ſide of the Square, which we wou'd Repreſent in Perſpective, whoſe Appearance in the Picture is call'd the *Perſpective Square*, as ABCE. *Plate* 11. *Fig.* 23.

This being ſuppos'd, if the Eye of the Spectator was at G, he cou'd not ſee the Two ends A,B, of the Perſpective Square, becauſe the Eye cou'd ſee at moſt but under the Angle HGI, which is a Right. But if it were at K, it cou'd ſee the ends AB, becauſe the Angle AKB is a Right. It wou'd ſee the ends AB better at L, and better yet at M, where the Angle AMB is of 60 Degrees: And from ſuch a diſtance the Objects may be ſeen in Perfection, becauſe the Repreſentation which from that diſtance is made in the Eye, is neither too great, nor too little; becauſe the Viſual Angle AMB is neither too great, nor too little. The Angle ANB it alſo very well.

Therefore to have the Points of Diſtance, take the diſtance FK, or rather the diſtance FL, or FM, or elſe the diſtance FN, and ſet it off on either ſide upon the Horizontal-line DD, from the principal Point V to the point DD, which will be the Points of Diſtance. For if that diſtance VD ſhou'd be leſs than the diſtance FK, the Perſpective Square ABCE wou'd appear Irregular, becauſe it wou'd be ſeen under an Obtuſe, and conſequently too great an Angle.

Thus you may look upon this as a general Maxim, *viz.* that the diſtance betwixt the Points of Sight and the Points of Diſtance muſt be, at leaſt, equal to FA or FB; that is, to the Space betwixt the end of the Vertical-line VF, and One of the Corners of the Perſpective: And it will not be amiſs to let that diſtance be ſomething greater, and as in this Caſe, the the Plain of the Picture might not be wide enough to receive Two Points of Diſtance, you muſt mark only One,

and

Plate 11. Fig. 25. and inſtead of placing the principal Point fore-right, it may be plac'd on One ſide.

To begin by what is eaſieſt, *viz.* the Repreſentations of Points, Lines, and Plans, you muſt divide into Two parts the Sheet of Paper which you wou'd Work upon, by the Line AB, which will be your Ground-line, and which we ſhall always mark with the ſame Letters AB; the upper part of your Paper ABDD, muſt be taken for the Plain of the Picture, and the lower part for the Geometrical Plain. The principal Point V, and the points of Diſtance, ſhall alſo be always mark'd by the ſame Letters as we have ſaid elſewhere.

OPERATION I.

How to find in the Picture the Appearance of a Point given in the Geometrical Plain.

Plate 12. Fig. 24. TO find the Appearance of the point C, which is given in the Geometrical Plain, let fall from that point C upon the Ground-line AB, the Perpendicular CE, and from the point E, draw to the principal Point V, the Line VE. Take upon the Ground-line AB, from E the part EF, or EA, equal to the Perpendicular CE; and draw from A to the oppoſite Point of Diſtance D, or from F to the oppoſite Point of Diſtance D, a Line which will upon the Line VE give at G the Appearance of the Point given C.

SCHOLIUM.

As this Problem is the Foundation of all the following, I ſhall here teach ſome other ways of ſolving it, which may be very uſeful in ſome caſes; and I ſhall alſo reſolve Two Difficulties which may happen in the Practice.

You may then in the Firſt place, find out the Appearance G of the given Point C, without making uſe of the principal Point V, when you have in the Picture the Two Points of Diſtance D, D, *viz.* in the Interſection of the Two Rays DA, DF. But as uſually we have but One Point of Diſtance upon the Horizontal-line DD, this Second Method is more curious than uſeful; wherefore I ſhall add a Third, which is ſhorter and more convenient.

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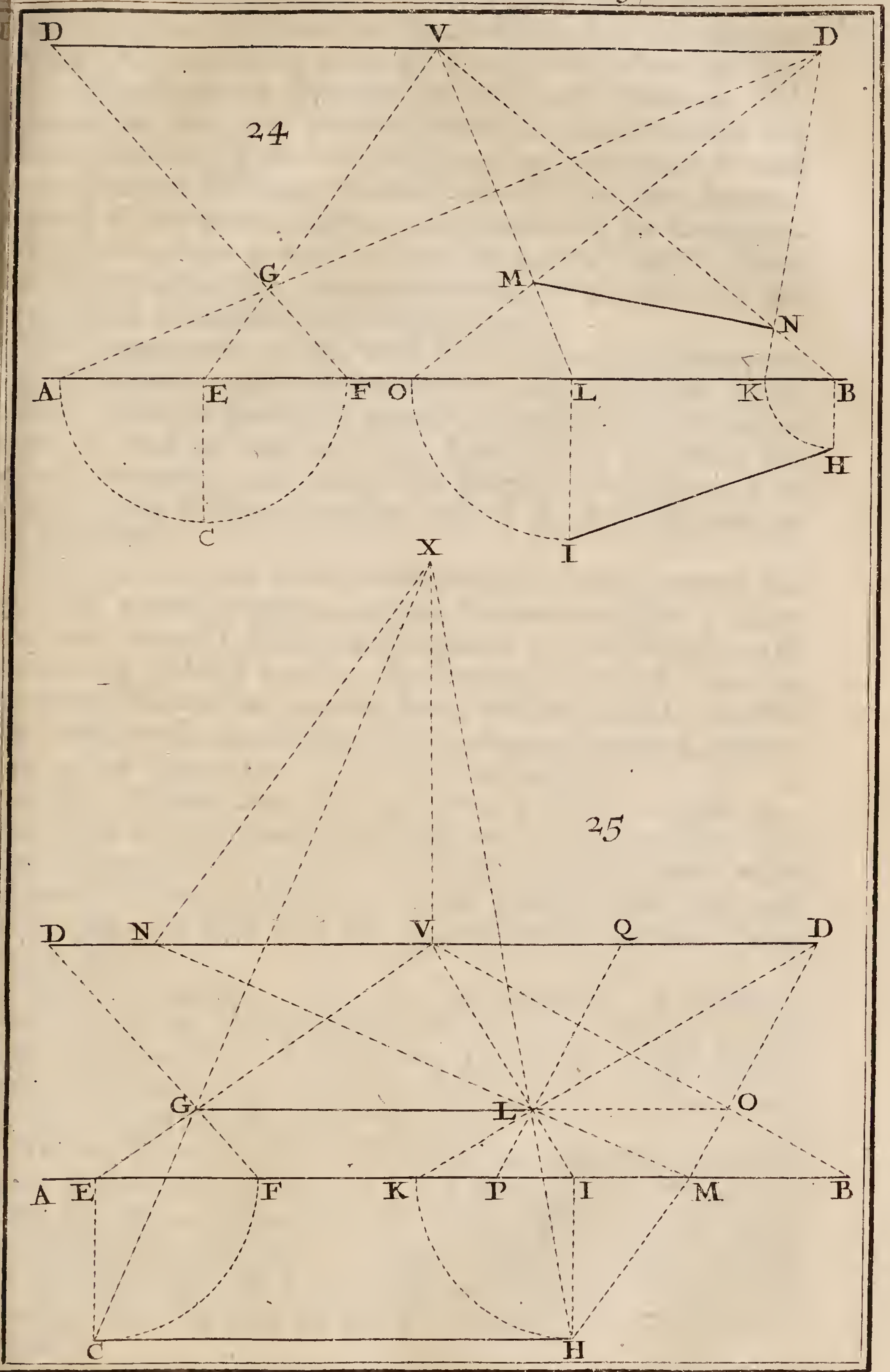
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Therefore to find the Appearance of the point C which is given upon the Geometrical Plain, let fall as before, from that Point given C, upon the Ground-line AB, the Perpendicular CE, and having made EF equal to CE, draw the Ray DE. Then from the principal Point V, raiſe upon the Horizontal-line DD, the Perpendicular VX, equal to VD the diſtance from the Eye to the Picture, and the point X will be the Point of Diſtance, to find in the Picture the Appearances of all the points that you will ſuppoſe in the Geometrical Plain: As here it will ſerve for the given Point C; for if you draw the Ray CX, you will have upon the Ray DF, the Appearance of the propos'd Point C, at G. Plate 12. Fig. 25.

This Appearance might alſo have been found upon the Ray EV; but as it is hard for the Two Rays EV, CX, to Interſect, when the point C is almoſt over againſt the principal Point V; and as they will not Interſect at all, when the point C is exactly over againſt the Point of Sight; it will be better in the Practice to make uſe of the Ray DF, than of the Ray EV.

Thus, to find out by this Method the Appearance of the point H, which is given upon the Geometrical Plain; draw from that point H, HI perpendicular to the Ground-line AB; and having made IK equal to the Perpendicular HI, draw to the point K from the oppoſite Point of Diſtance D, the Ray DK, which will cut the Ray XH, in ſome point, as L, which will be the Appearance of H the Point propos'd.

Inſtead of drawing from the given Point H, a Line perpendicular to the Ground-line AB, you may draw to it any other oblique Line, as HM, to which you muſt draw thro' the point X, the parallel XN; for by joining MN you will have upon the Ray XH, the Appearance L of the Point propos'd H.

The Firſt Method, where One of the Points of Diſtance mark'd upon the Horizontal-line DD is made uſe of, is the moſt uſual and eaſy; wherefore we ſhall in the Sequel always make uſe of it, and to render it more Familiar, we ſhall here repeat it again for the given Point H.

Having, from the point H, drawn the Line HI perpendicular to the Ground-line AB; and having drawn from the point I to the principal Point V, the Ray VI, make IK equal to the Perpendicular IH; and draw thro' the point K, and the oppoſite Point of Diſtance D, the Ray DK, which will upon the Ray VI, give the point L which is the Appearance requir'd.

If it happens that when the length of the Perpendicular HI is ſet off upon the Ground-line AB, the point K falls out of the breadth of the Picture, it muſt be ſet off on the other

Plate 12. Fig. 25. other side, if there be another Point of Distance: But if there be but one, which is more usual, draw thro' the point B taken at pleasure upon the Ground-line AB, and thro' the principal Point V, the Ray VB, and having made MB equal to the Perpendicular HI, draw the Ray DM, which upon the Ray VB will give the Point O, thro' which you must draw to the Ground-line AB, the Parallel OL, which upon the Ray VI will give the Point requir'd L.

That every thing that you design to represent in Perspective may appear in just Proportion, we are some time oblig'd to have the Eye at a great distance from the Picture, which may hinder us from marking the Point of Distance D upon the Horizontal-line DD; in such a Case you must only mark half of the distance of the Eye from the Picture upon the Horizontul-line DD, from the principal Point V to Q, which will represent one of the Points of Distance: And then you must only set off half of the Perpendicular HI upon the Ground-line AB, from I to P, and drawing the Ray QP you will upon the Ray VI, have the point L as before, for the Appearance of H, the Point propos'd.

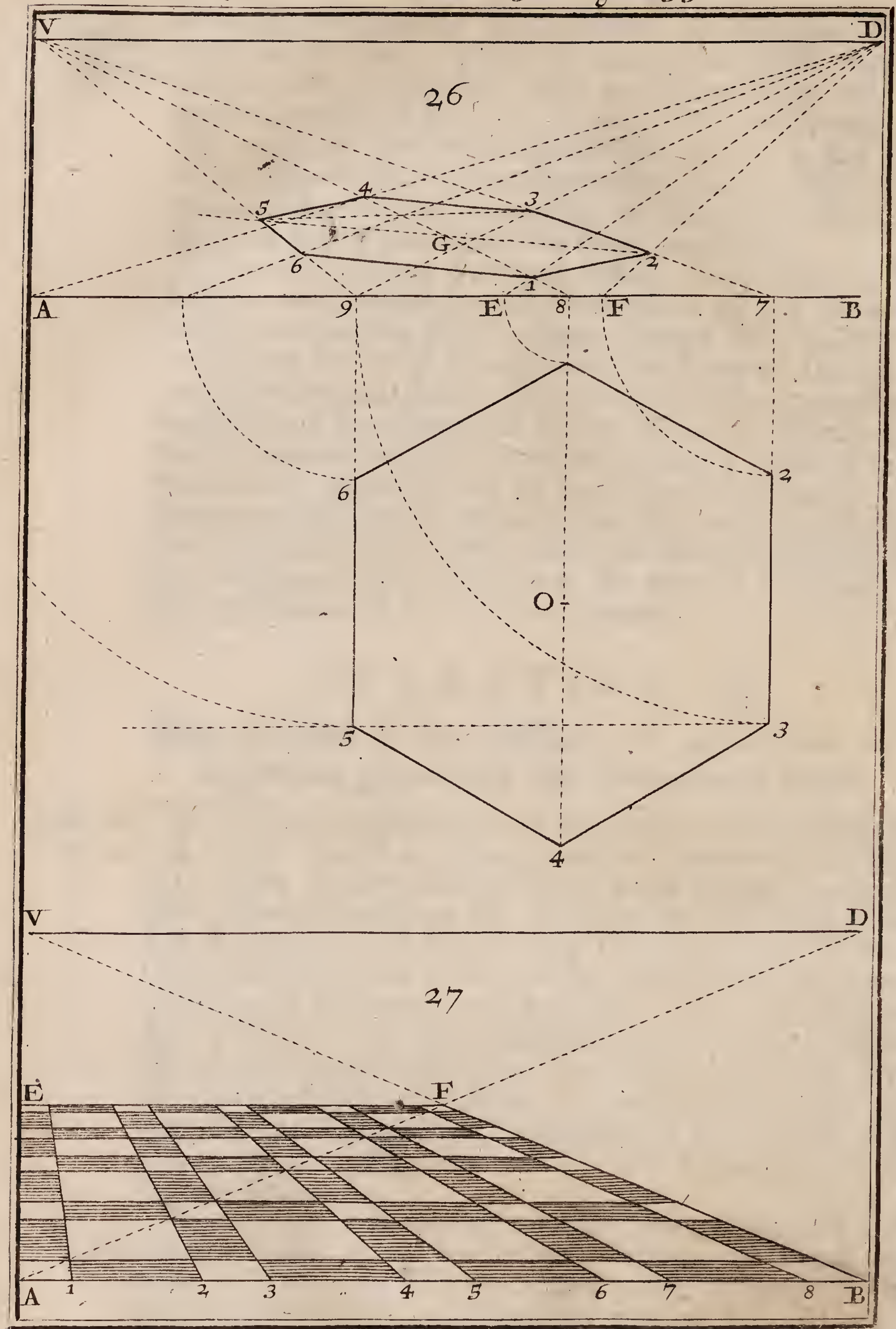
OPERATION II.

How to find in the Picture the Appearance of a Right-line given upon the Geometrical Plain.

Plate 12. Fig. 24. TO find in the Picture the Appearance of the Right-line HI, which is given upon the Geometrical Plain; draw from its Two Ends H,I, the Lines HB, IL, perpendicular to the Ground-line AB, and draw thro' the Two points L,B, to the principal Point V, the Two Rays VL,VB. Make LO equal to LI, and draw from the point O thro' the opposite Point of Distance D, the Ray DO, which will upon the Ray VL give at M the Appearance of the point I. Make BK likewise equal to BH; and draw from the point K, thro' the opposite Point of Distance D, the Ray DK which will upon the Ray VB, give the Appearance of the point H at N. Last of all, join MN, which will be the Appearance of HI the Line propos'd.

[Fi]g. 25. Likewise to find the Appearance of the Right-line CH, which is given upon the Geometrical Plain; having drawn from its Two ends CH, the lines CE, HI, perpendicular to the Ground-line AB; and from the principal Point V, thro' the Two points E,I, (where the Two Perpendiculars cut the Ground-line) the Rays VE, VI; make EF equal to EC, and IK equal to IH, and draw thro' the point F, to the opposite Point of Distance D, the Ray DF; which will upon the Ray

VE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VE give the Appearance of the point C at G, and likewiſe draw from the point K to the oppoſite Point of Diſtance D, the Ray DK, which will upon the Ray VI give the Appearance of the point H at L, wherefore if you join the Right-line GL, you will have the Appearance of the propos'd Line CH, *&c.* *Plate* 12. *Fig.* 25.

SCHOLIUM.

The Theory and Practice will give you ſeveral ſhort Methods, it being certain, *by Theorem* 7. That if the propos'd Line CH, be parallel to the Ground-line AB, its Appearance in the Picture, (*viz.*) GL, will alſo be parallel to the ſaid Ground-line: and, *by Theorem* 8. That when the Line given upon the Geometrical Plain is perpendicular to the Ground-line, its Appearance in the Picture being continued, will paſs thro' the Point of Sight, and thro' the Point of Diſtance, when it makes half a Right-Angle with the Ground-line.

When the Line given on the Geometrical Plain is not a Right-line, you muſt from ſeveral of its points draw Perpendiculars to the Ground-line, by means of which, and what has been ſaid, you will find in the Picture the Appearances of all thoſe points, which being join'd by a Curve Line, that Curve will be the Appearance of the Line propos'd.

OPERATION III.

How to find in the Picture the Appearance of a plain Figure given on the Geometrical Plain.

TO find in the Picture ABDV, the Appearance of the Regular Hexagon 1,2,3,4,5,6, which is given in the Geometrical Plain, draw from all its Angles as many Lines perpendicular to the Ground-line AB; and thro' the points 7,8,9, where they cut the ſaid Ground-line AB, draw to the Principal Point V the Lines V7, V8, V9, by means of which, and by help of what has been ſaid, you will find the Appearances of the Lines which bound the propos'd Hexagon. As *for Example*, to find the Appearance of the Line 1, 2, you muſt ſet off the Length of the Perpendicular 8, 1, upon the Ground-line AB from 8 to E, and the Length of the Perpendicular 7, 2, upon the ſame Ground-line AB, from 7 to F, and draw from the oppoſite Point of Diſtance D, the Rays DE, DF, *&c.* *Plate* 13. *Fig.* 26.

SCHOLIUM.

One may here make uſe of a ſhort Method, to find the
Plate 13. Appearance of the Point 5, whoſe Perpendicular 9,5, cannot
Fig, 26. be ſet off upon the Ground-line AB, from the point 9 towards the part oppoſite to the Point of Diſtance; becauſe the Plain of the Picture is not large enough: For as the Two points 5, 3, are in a line parallel to the Ground-line AB, having found in the Picture the Appearance of the point 3 upon the line V7, you need only draw thro' the Appearance 3, to the Ground-line AB, a Parallel, which will upon the Ray V9 give the Appearance of the other point 5. You may, if you will, do the ſame thing, with the Two points 2,6, which are alſo equally diſtant from the Ground-line AB.

There is alſo another way to find the Appearance of the point 5, not only by the ſhort Methods which have been taught in *Operation* 1. but alſo in this manner. Becauſe the propos'd Polygon 1, 2, 3, 4, 5, 6, is Regular, if upon the line V8, you find the Appearance G of its Center O, and from that point G, which is call'd the *Apparent Center*, you draw to the point 2, of the Appearance of the point 2, diametrically oppos'd to the point 5, a Right-line, that Right-line being produc'd, will, upon the Ray V9, give the Appearance of the point 5, which is requir'd. Thus will the Ray V8 give upon the Ray DA the Appearance of the point 4, diametrically oppos'd to the point 1, and ſo on.

OPERATION IV.

How to repreſent in Perſpective a Floor of equal Squares ſeen foreright, without a Geometrical Plain.

DIvide the Ground-line AB into as many equal parts as
Plate 22. you pleaſe, each of which ſhall repreſent the ſide of
Fig. 10. One Square. Draw thro' each Point of Diviſion to the principal Point V, as many Right-lines or Rays, the Two laſt of which will be VA, VB, and thro' the point A where the Diviſion begins, draw to the oppoſite Point of Diſtance D, the Ray AD, which will cut the foregoing Rays in Points, thro' which you muſt draw to the Ground-line AB, as many Parallels, whoſe laſt will be EF, which will have its diviſions equal between the Two points G, H, which may be continued from G to E, and from H to F, to draw thro' thoſe new Points of Diviſion, from the principal Point V, other Rays which will give upon the Two lines DA, DB, (which

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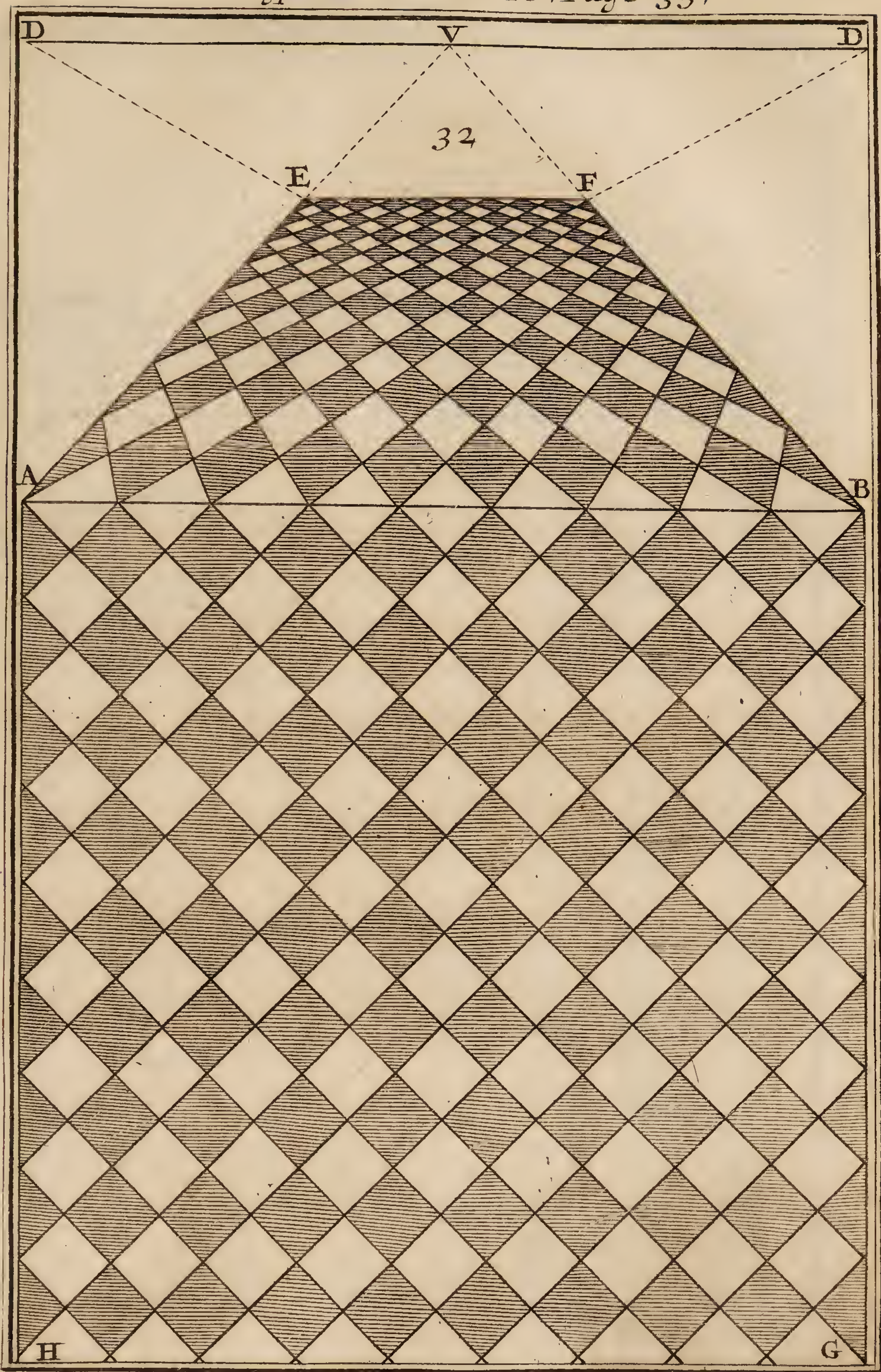
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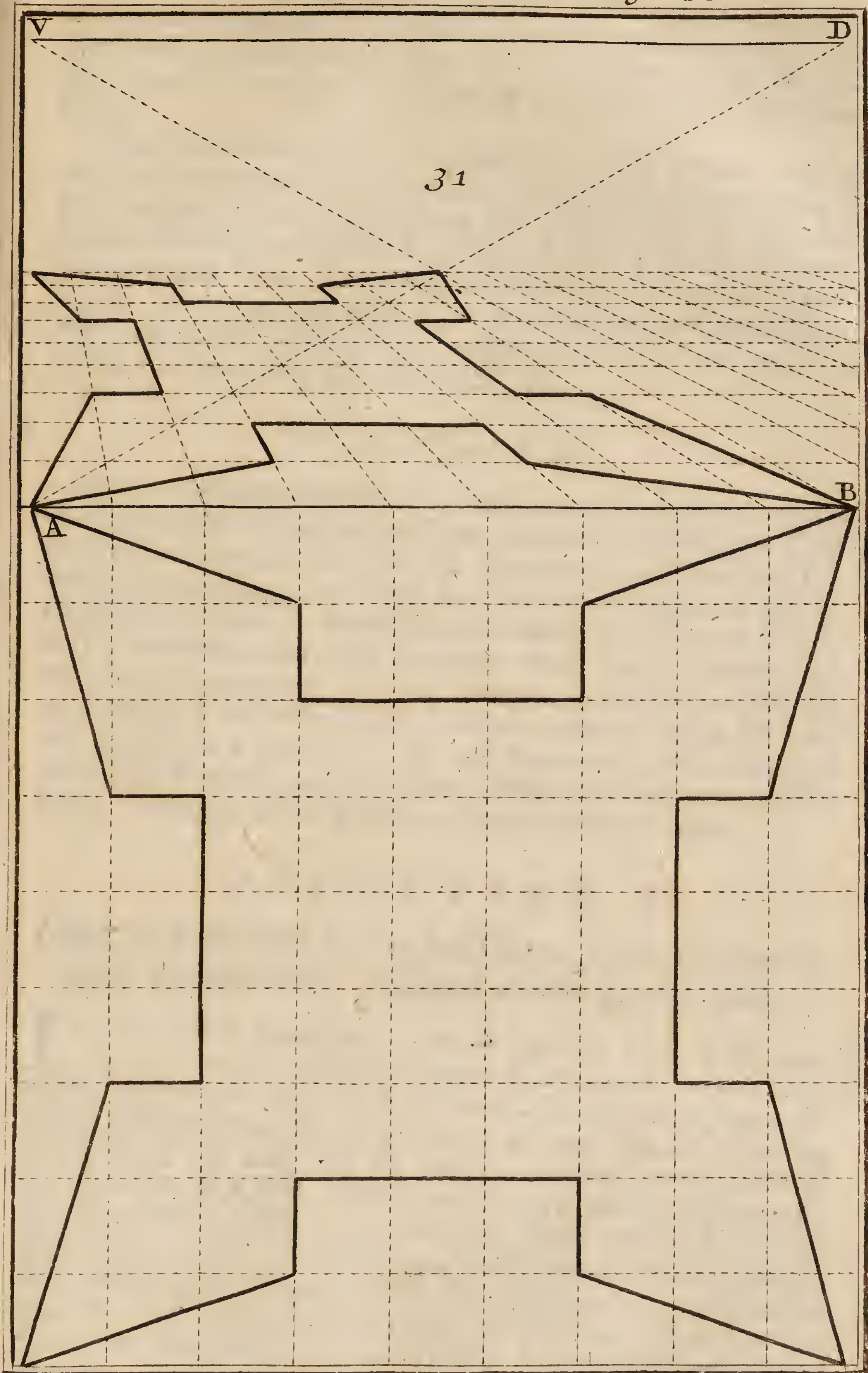
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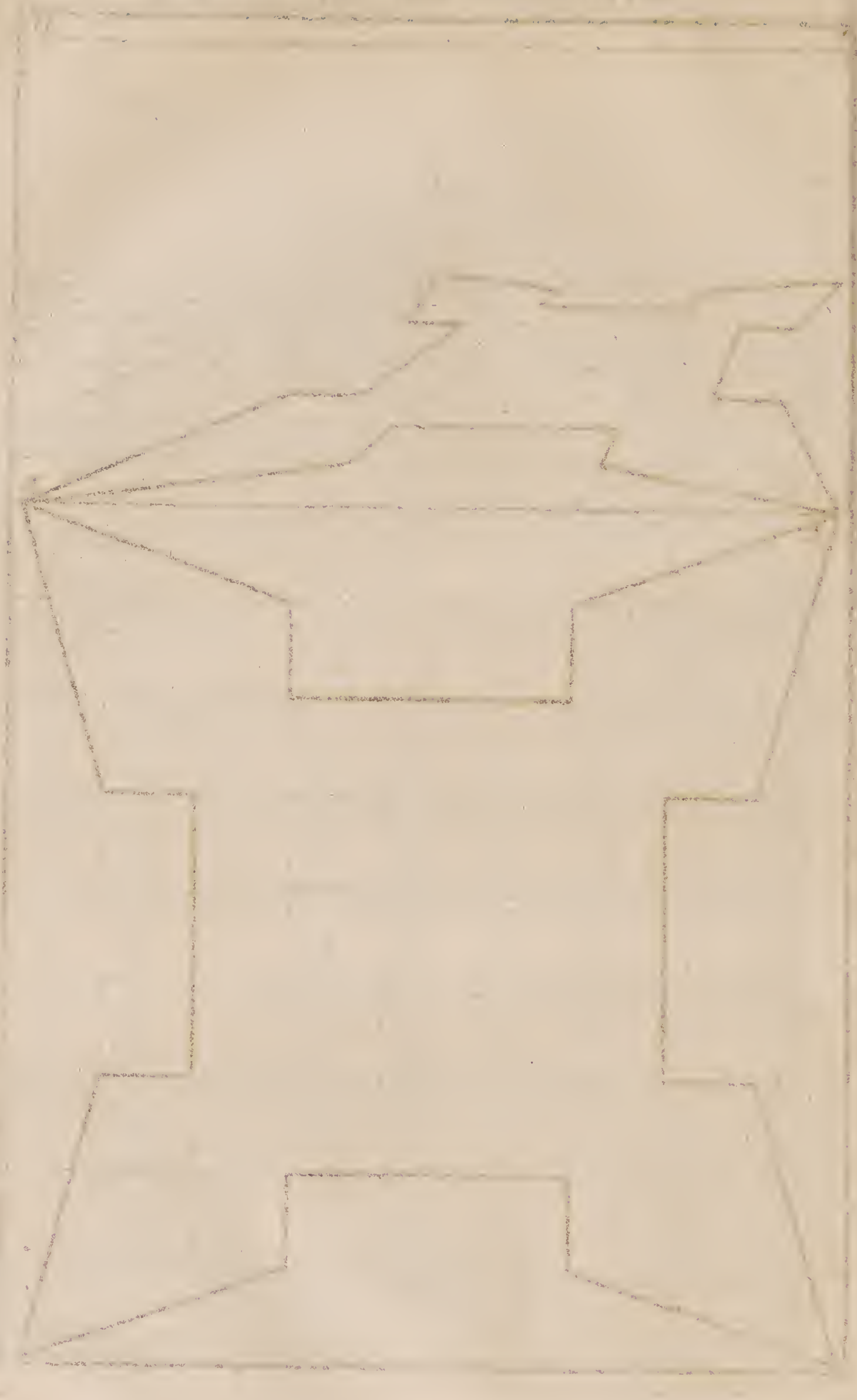


go thro' the Two Points of Diſtance D) the ſame points thro' which the foregoing Parallels to the Ground-lines paſs, and thro' which conſequently Rays may be drawn, to the principal Point V, without lengthning the Diviſion of the line GH. *Plate* 10. *Fig.* 22.

Thus you will have in Perſpective all the Squares which can be comprehended in the ſpace ABFE; and if you wou'd have more of 'em, draw from the point E to the oppoſite Point of Diſtance D, the Ray DE, which will cut all thoſe that proceed from the principal Point V, in points, thro' which, as before, you muſt draw Parallels to the Ground-line AB, whoſe laſt will be KL, which paſſes thro' the point 1, where the Rays VB, DE, Interſect, *&c.*

SCHOLIUM.

If you deſcribe Squares in the Geometrical-plain, whoſe ſides be equal to the parts of the Ground-line AB, the little Perſpective Squares will be the Appearances of thoſe in the Geometrical-plain, and they may be very uſeful to put in Perſpective, One or more Figures, made up of ſeveral Lines, as, *for Example*, a Fortified Polygon, which being deſcrib'd on the Geometrical-plain with the Squares, it will not be difficult to deſcribe it in Perſpective, with the Squares of the Picture, drawing ſuch a part of it, in each Square of the Picture, as is deſcrib'd in the correſpondant Square of the Geometrical-Plain. A ſight of the Figure will make all this clear. *Plate* 15. *Fig.* 31.

OPERATION V.

How to repreſent in Perſpective a Floor of Squares ſeen Corner-wiſe, without a Geometrical-plain.

If you wou'd repreſent all thoſe Squares ſeen with their Angles towards you in a Square Floor which is ſeen fore-right, and whoſe Side AB is determin'd upon the Ground-line; firſt deſcribe the Appearance of that Square, drawing from the Two ends AB of the ſide, to the principal Point V, the Rays VA, VB, and to the Two Points of Diſtance D, the Rays DFA, DEB, which will cut the former VA, VB, in Two points, as F, E, which you muſt join by the Right-line EF, which will be parallel to the Ground-line; this ſhews that by help of only One Point of Diſtance, you may deſcribe the Perſpective Square ABFE, which you muſt divide into Squares ſeen from their Angles, thus: *Plate* 16. *Fig.* 32.

Having, as in *Operation* 4. divided the part AB of the Ground-line, or the ſide of the Perſpective Square into equal

parts, draw thro' the Points of Division to each Point of
Plate 16. Distance D, as many Right-lines as there are Points of Divi-
Fig. 32. sion, which for their common Intersections will form the Appearances of the Squares seen Corner-wise, with which Squares it will be easy to fill the Perspective Square ABFE, because all the Rays which proceed from the Two Points of Distance D, divide equally in the same points the side EF parallel to AB, which is also divided equally and in the same Points, by the said Rays which goes from the Two Points of Distance D.

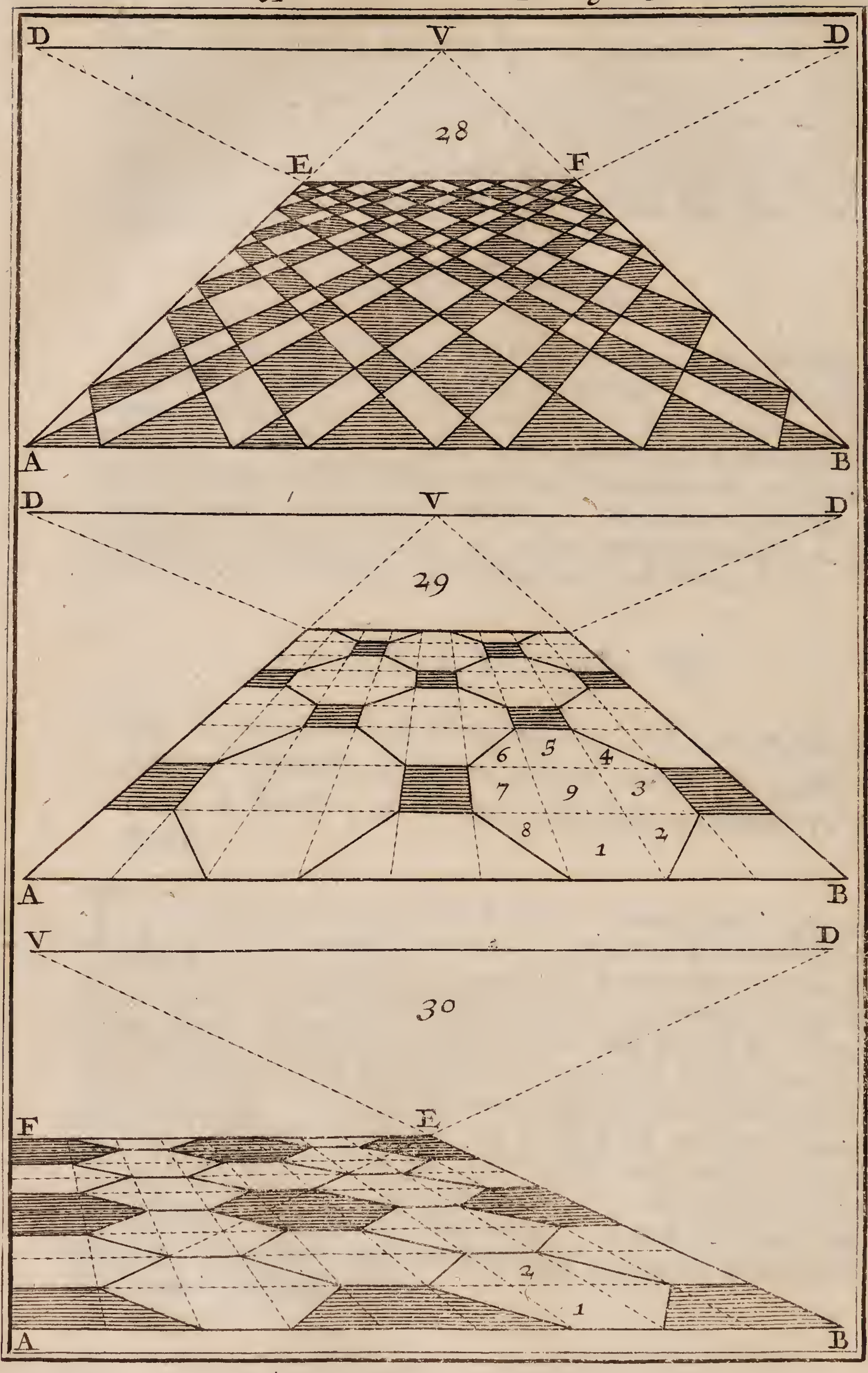
SCHOLIUM.

If upon the said side AB a Square be drawn in the Geometrical Plain, as ABGH, whose Representation is the Perspective Square ABEF, and that Square on the Geometrical Plain ABGH be divided into other little Squares by Lines parallel to the Diagonals AG, BH, you may make use of that Square thus divided, as has been taught in *Operation* 4. to put in Perspective several Things at once, whose Draught is describ'd upon the Square of the Geometrical Plain.

OPERATION VI.

How to represent in Perspective a Floor made of several Squares seen foreright, and encompass'd with a Border, or Frame, without a Geometrical Plain.

DIvide the Ground-line AB into as many Parts, alterna-
Plate 13. tively Equal and Unequal, as you wou'd have Squares
Fig. 27. and Borders at the points 1,2,3,4,5,6,7,8, and draw from all those points to the principal Point V, as many Rays or Right-lines; whose last will be VB, and first VA. Then draw from the point A to the opposite Point of Distance D, the Ray DA, which will cut those that go from the Point of Sight, in Points thro' which you must draw as many Parallels to the Ground-line AB, whose last will be EF, which terminates the Perspective Square ABFE, and you will have the Representation of the Floor requir'd; and you may continue it, if you draw another Ray thro' the point E, and thro' the point of Distance D, &c.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OPERATION VII.

How to repreſent in Perſpective a Floor made of Equal Squares ſeen Corner-wiſe, and encompaſs'd with a Border, without a Geometrical Plain.

DIvide the Ground-line AB into Parts alternatively Equaland Unequal, as you did before; and draw thro' the Points of Diviſion to the Points of Diſtance DD, as many Lines, which you will terminate in the Perſpective Square as before, which will be deſcrib'd as taught. *Plate 14. Fig. 28.*

OPERATION VIII.

How to repreſent in Perſpective a Floor of Octogons mix'd with Little Squares, without a Geometrical Plain.

DIvide the Ground-line AB into equal parts, as if you wou'd only draw a Floor of equal Squares ſeen foreright, and deſcribe ſuch a One Actually, as has been taught in *Operation* 4. Then take at pleaſure the Square 9, and the Eight others which are round about it, *viz.*, 1,2,3,4, 5,6,7,8, and in the Squares 2.4,6,8, draw the Diagonals which muſt tend to the Points of Diſtance D,D, and you will have an Octogon; and the others are done the ſame way. *Fig. 29.*

SCHOLIUM.

It is plain, that betwixt the Octogons ſo deſcrib'd, there will be ſuch Little Squares as are requir'd, and that the ſaid Octogons are not Regular, becauſe Four of their ſides are equal to the ſides of the Little Squares, and the other Four are equal to the Diagonals of the ſaid Squares.

OPERATION IX.

How to represent in Perspective a Floor made of Hexagons, without a Geometrical Plain.

Plate 14. Fig. 30. FIRST draw a Floor ABEF, upon the Ground-line AB, as has been taught in the 4th *Operation*. Then take at pleasure Two contiguous Squares as 1,2, and draw the Diagonals of the Two other Squares, which are on the Right and on the Left, and you will have an Hexagon, which will be a Model for the rest, because they are all describ'd the same way.

SCHOLIUM.

It is also plain that those Hexagons thus describ'd, do not represent Regular Hexagons, because their Sides are not equal in Representation, the longest being the Diagonals of those Squares which are made upon the shortest, as in the foregoing Octogons.

OPERATION X.

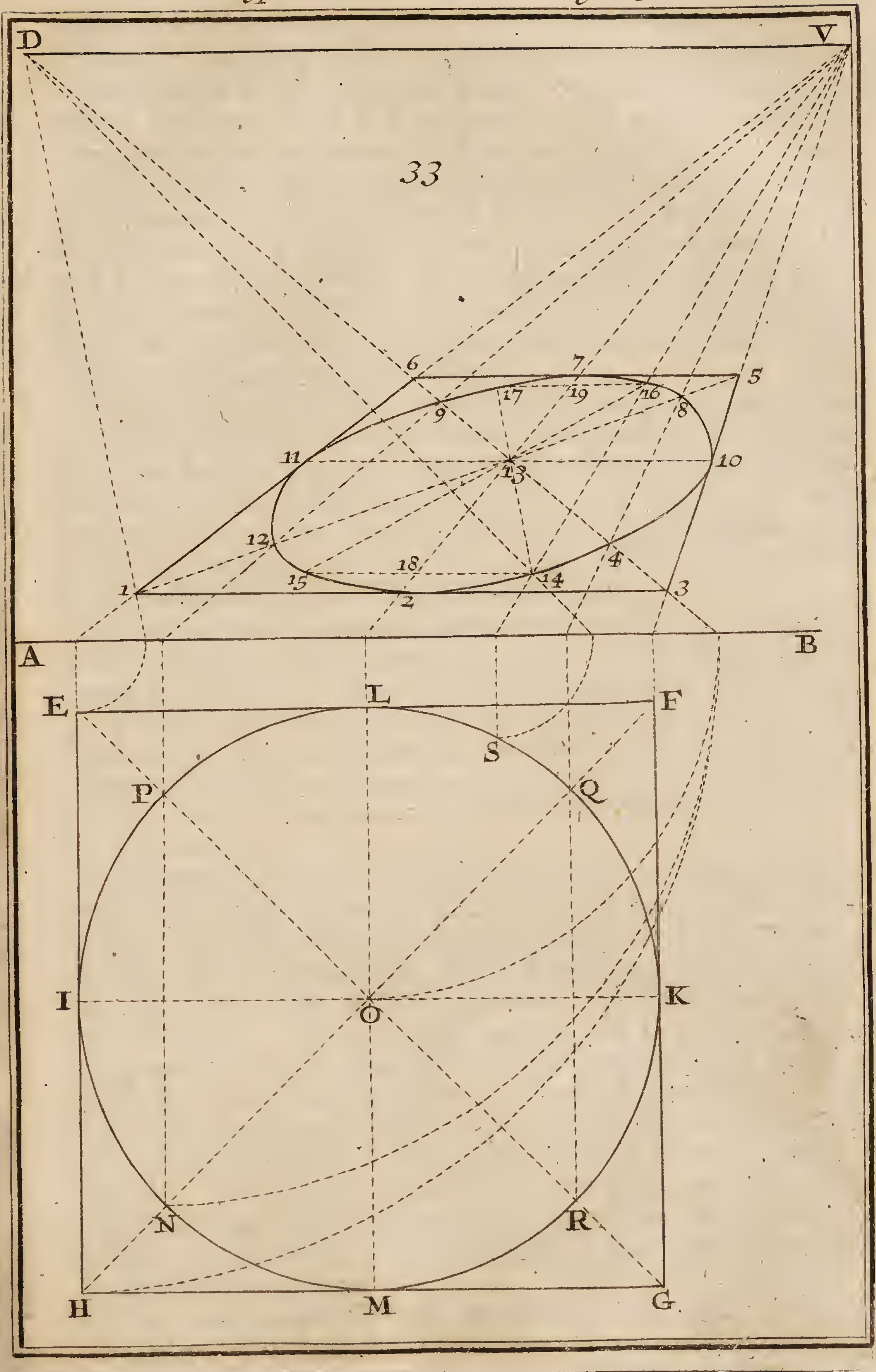
How to find in the Picture, the Appearance of a Circle given in the Geometrical Plain.

Plate 17. Fig. 33. TO find the Appearance of the Circle ILKM, which is given in the Geometrical Plain, describe about that Circle the Square EFGH, One side of which as EF or GH, must be parallel to the Ground-line AB, and the other Side EH or FG must consequently be perpendicular to the said Ground-line AB. Draw the Two Diagonals EG, FH, which intersecting at O, the Center of the Circle, will give upon IKLM the Circumference of it, the Four points P, Q, R, N.

Lastly, find in the Picture ABVD, the Appearance of the Square EFGH, which is the Perspective Square 1, 3, 5, 6, with all its Divisions, and thro' the points 2,4,10,8,7,9,11,12, which are the Appearances of the points L,Q,K,R,M,N,I,P, of the Circumference of the Circle given, draw a Curve which will determine the Appearance of IKLM, the Circle propos'd.

SCHOLIUM.

To describe more easily the Curve, which represents the Circumference of the propos'd Circle ILKM: it will not be amiss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amiſs to find ſome other points between thoſe whoſe Appearances are ſomething diſtant: As here, the Appearances 2, 4, of the points L, Q, being at ſome diſtance from one another, you muſt find the Appearance 14 of the point S, taken at pleaſure between the Two points L, Q. *Plate* 17. *Fig.* 33.

It is not abſolutely neceſſary to have the whole Circle upon the Geometrical Plain, in order to mark its Appearance in the Picture, for it is ſufficient to have a Fourth part of it, as LK, becauſe by means of the Appearances of the points of that Quarter LK of the Circle, you may by a ſhorter Way find the Appearances of the Correſpondant Points, of the Three other Quarters KM, MI, IL.

Thus having found the Appearance 14 of the point S, and the Appearance 13 of the Center O, you muſt thro' the point 14 draw the line 14, 15, parallel to the Ground-line AB, and the line 18, 15, muſt be made equal to the line 18, 14; and the point 15 will be the Appearance of the Correſpondant Point of the Quarter LI, that is, of a Point as far diſtant from the Ground-line AB as the point 14.

If from 13 the Apparent Center you draw to 15, the point which you have found, the line 13, 15, and produce it till it meets the line V 14, you will have the point 16 for the Appearance of the Correſpondant Point of the part KM: And if you draw thro' the point 16 the line 16, 17, parallel to the Ground-line AB, and make 19, 17, equal to 19, 16, the point 17 will be the Appearance of the Correſpondant Point of the Quarter MI.

That point 17 might alſo be found, by drawing thro' the point 14, and the Apparent Center 13, the line 13, 17, which is call'd the *Apparent Diameter*, becauſe it is the Appearance of One of the Diameters of the Circle propos'd, namely, of that which goes thro' the point S, whoſe Appearance is the point 14 For the ſame reaſon the line 11, 10, is an Apparent Diameter, becauſe it is the Appearance of the Diameter IK, which is parallel to the Ground-line AB: and the line 2, 7, is an Apparent Diameter, becauſe it is the Appearance of the Diameter LM perpendicular to the ſaid Ground-line AB. For this reaſon alſo the line 4, 6, is an Apparent Diameter, becauſe it is the Appearance of the Diameter QN, which makes half a Right-angle with the Ground-line AB: And ſo of the others.

The Appearance of the propos'd Circle ILKM happens here to be an Ellipſis, but it may be a Circle, when the Circle given touches the Ground-line AB, at the point L of the Vertical Line LV, and the diſtance between the Eye and the Picture be a Determinate length, which we ſhall find thus:

Plate 18. Fig. 34. Having produc'd the Side of the circumscrib'd Square EFGH, till it meets at Right-angles the Horizontal-line DD in a point as S, draw the Right-line SL, and cut off from it the part SC equal to the part SV, and the remaining part LC will be the distance VD, which must be between the Eye and and the Picture that the propos'd Circle ILKM may be also represented in the Picture by a Circle, which will like the Ellipsis touch the Two sides E6, F5 at the Two Ends 11,10, of the Apparent Diameter 11, 10.

If VD the distance from the Eye to the Picture was given, and you wou'd also find the Height of the Eye, or the distance VL from the Horizontal-line DD to the Ground-line AB, that the Appearance of ILKM the Circle given may be a true Circle; the Rectangular Triangle SVL shews that the Square of the line EL, or the Square of the *Radius* of the given Circle IKLM must be taken from the Square of the line LS, or from the Height requir'd.

The given Circle IKLM may also be represented in Perspective by a true Circle; tho' it does not touch the Plain of the Picture, so that its Center O be seen foreright; that is, if it be in the line LM, which being perpendicular to the Ground-line AB passes thro' the End of the Vertical Line VL: and we only made it touch the Picture to determine more easily the distance from the Eye to the Picture, and also to have an easier Calculation when we found out this distance by the new *Analysis*, thus:

Plate 19. Fig. 35. Let ABCD be the Geometrical Plain, containing the given Circle IKLM, which touches the Picture AGHB at the point L, in such manner that the Diameter LM of that Circle be perpendicular to the Ground-line AB. Let the Eye be at E rais'd above the Geometrical Plain by the Height EF, which I suppose known, and distant from the Picture the length of the Principal Ray EV, which can only be One determinated distance, when the Representation L, 11, 7, 10, of the Circle IKLM is a true Circle; that is, when the Angle EL7 is equal to the Angle EML, and consequently that the Triangle E7L is Similar to the Triangle ELM, which we have call'd *Subcontrary Section*.

Let a be put for the Height of the Eye EF, or for the length of the Vertical Line VL, b for the Diameter LM of the given Circle IKLM, and x for the Distance EV, or FL from the Eye to the Picture; and you will have $b + x$ for the line FM, and $\frac{ab}{b+x}$ for the Diameter L7, because of the Two Similar Triangles MFE, ML7. If you add together the Two Squares EF, FL, you will have $aa + xx$ for the

Square

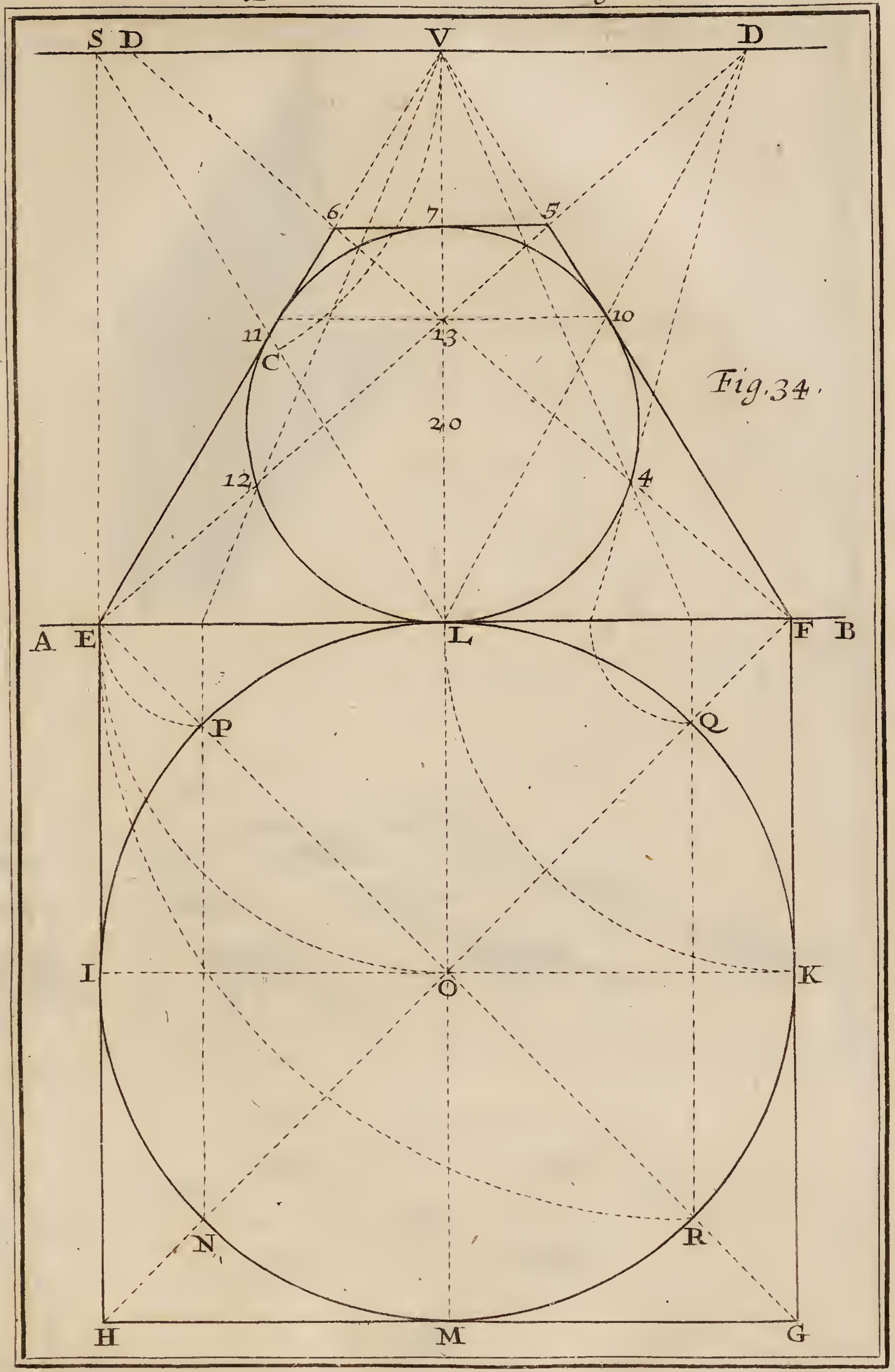


Fig. 34.

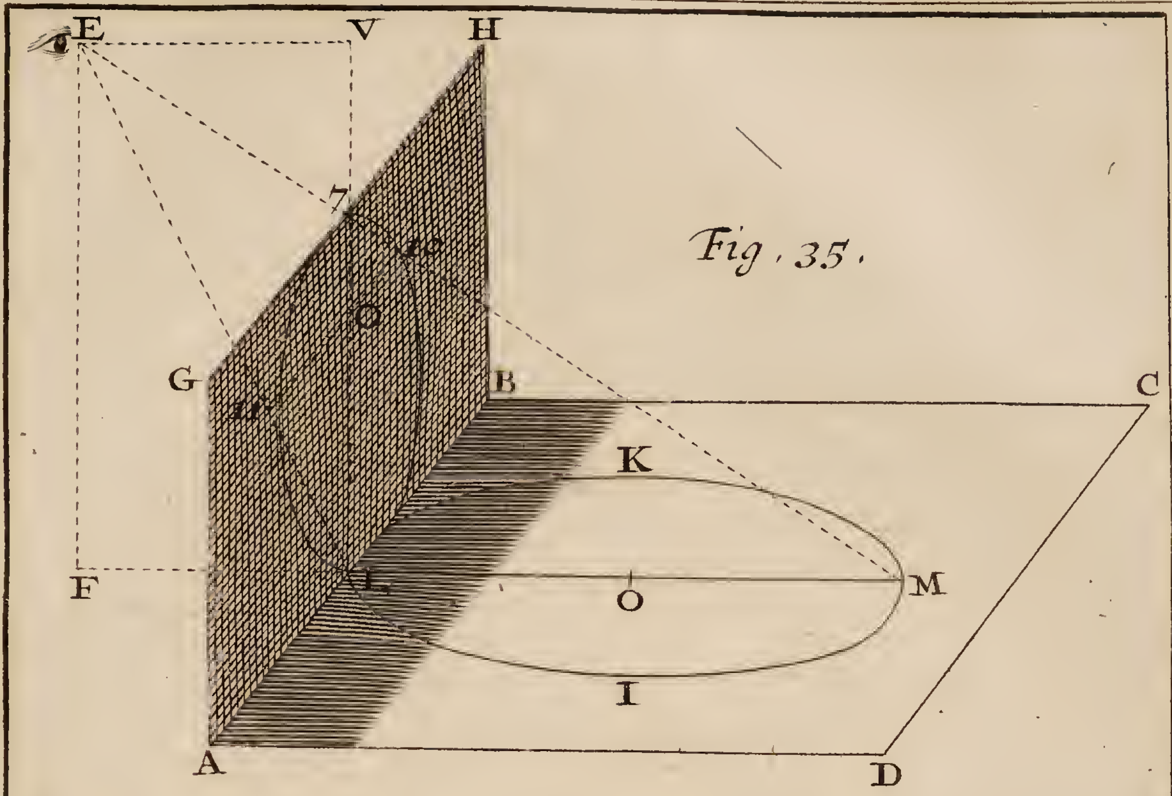
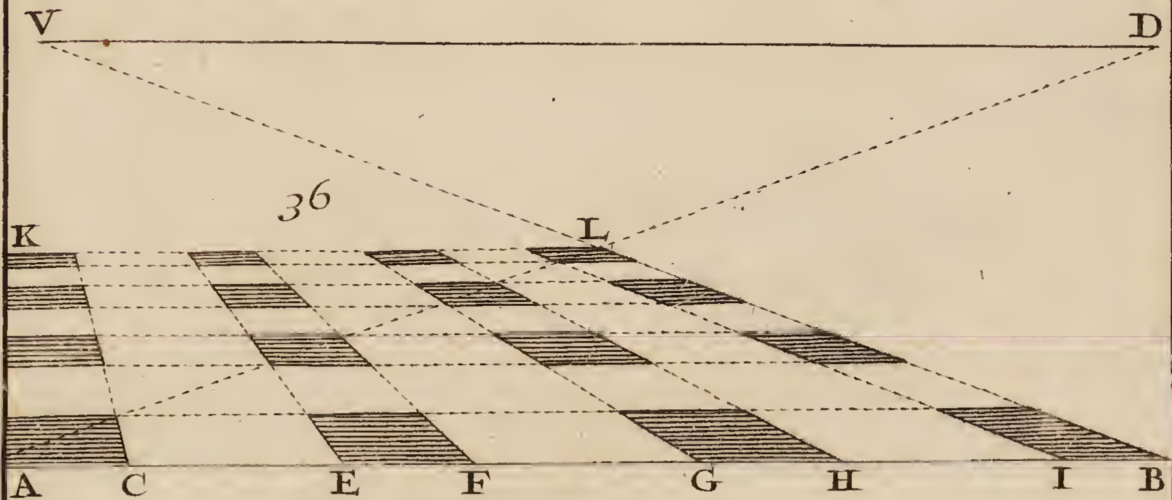


Fig. 35.



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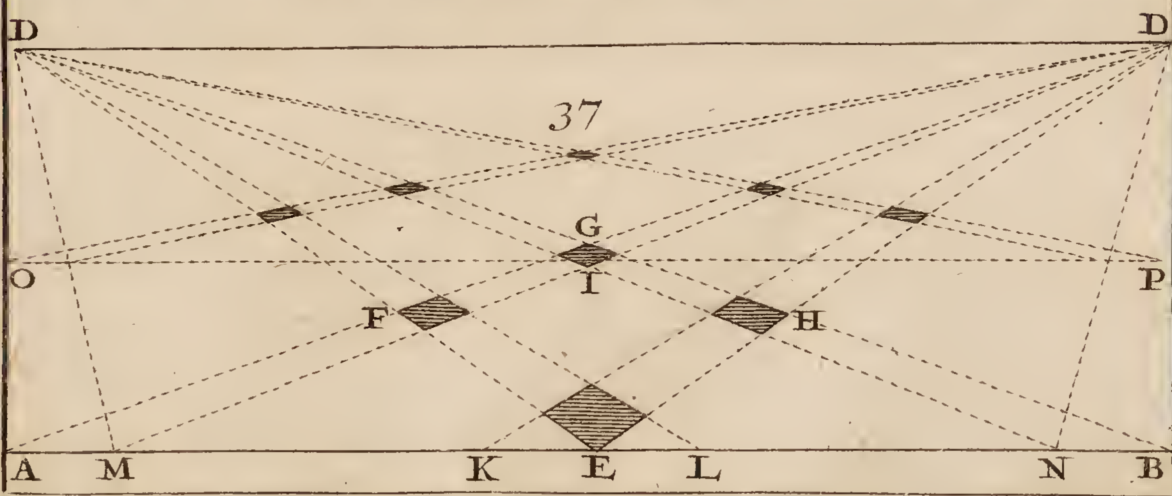

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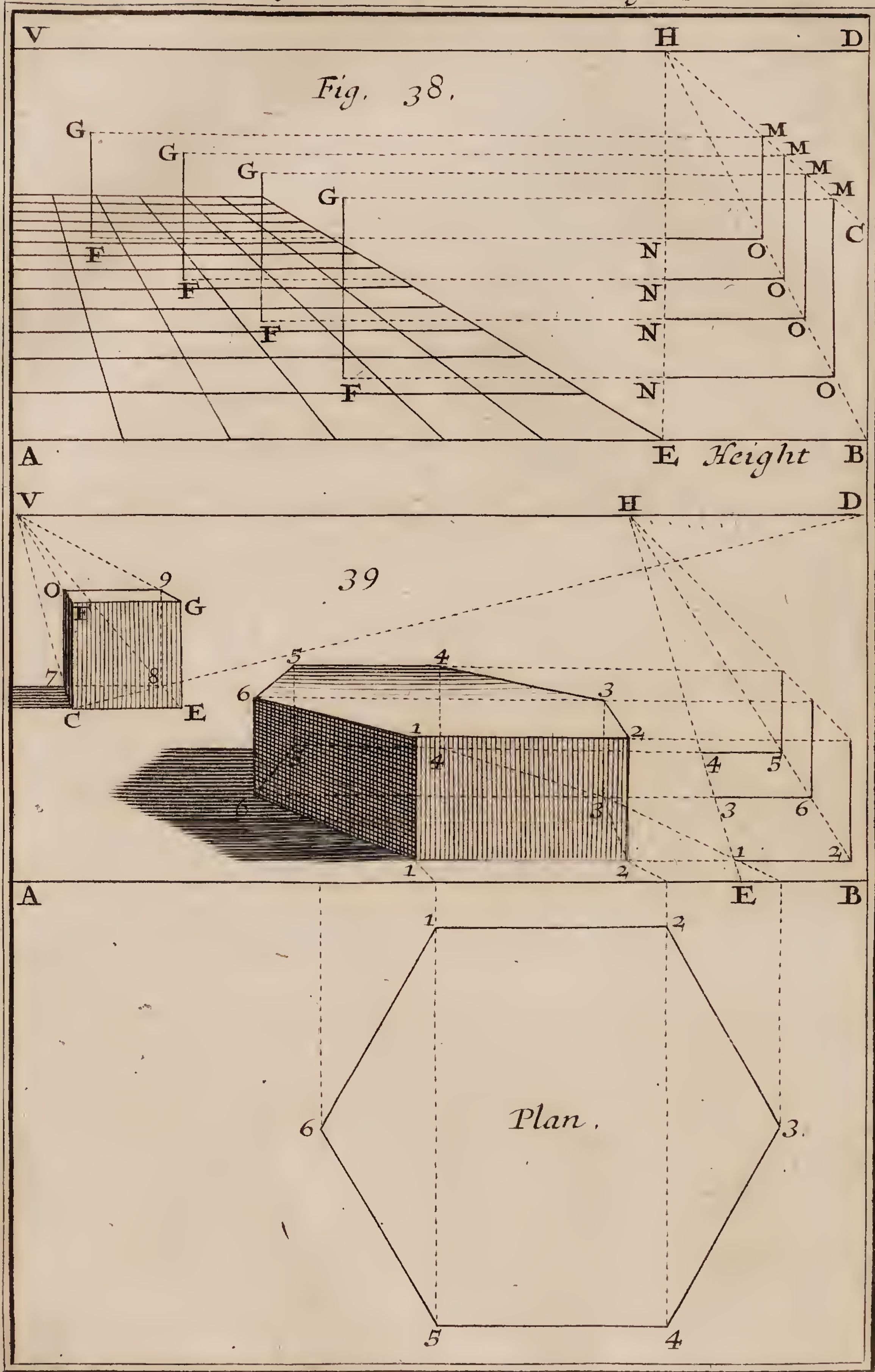
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Fig. 38.
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Square EL, because of the Triangle EFL, Rectangular at F: *Plate* 19
likewise if you add together the Two Squares EF, FM, you *Fig.* 35.
will have $aa + bb + 2bx + xx$ for the Square EM, because of the Rectangular Triangle EFM.

Because the Four Squares EL : L7 : : EL : LM are proportional, by reason of the Similar Triangles EL7, ELM, you will have in Analytical Terms in this Analogy, $aa + xx : \frac{aab}{bb + 2bx + xx} :: aa + bb + 2bx + xx : bb$, whose Two Consequents being divided by bb, you will have this, $aa + xx : \frac{aa}{bb + 2bx + xx} :: aa + bb + 2bx + xx : 1$, whose Two First Terms being multiplied by $bb + 2bx + xx$, you will have this last Analogy, $aabb + 2aabx + aaxx + bbxx + 2bx^3 + x^4 : aa :: aa + bb + 2bx + xx : 1$, and consequently this Equation $aabb + 2aabx + aaxx + bbxx + 2bx^3 + x^4 = a^4 + aabb + 2aabx + aaxx$, from which taking the Square $aabb + 2aabx + aaxx$, you will have this other Equation, $bbxx + 2bx^3 + x^4 = a^4$, and by the Square Root you will have this $bx + xx = aa$, from which we have drawn the foregoing Construction, according to the usual Method for resolving by Geometry the Equations of Two dimensions.

OPERATION XI.

How to represent in Perspective the Situations of several Cubes seen foreright, equal, and equally distant from one another, and in Rows, which end in the Point of Sight, without a Geometrical Plain.

If you wou'd have Four Rows of equal Squares at equal *Fig.* 36.
distances, mark upon the Ground-line AB, the Breadths AC, EF, GH, IB, of those Squares, in such manner that those Breadths may be equal to one another, as well as their distances CE, FG, HI; and draw thro' the point A,C,E,F,G, H,I,B, to the Principal Point V as many Right-lines, which will be cut by the line AD, (which line must be drawn from the Point A to the opposite Point of Distance D) in points, thro' which you must draw as many Lines parallel to the Ground-line AB, the last of which will be KL, and if you wou'd have more of 'em, draw thro' the point K to the Point of Distance D, a Second Line, *&c.*

OPER-

OPERATION XII.

How to represent in Perspective a Square seen Corner-wise, with Four other Little Squares seen also Corner-wise, and Situated at the Four Angles of the great Square, without a Geometrical Plain.

Fig. 37. LET us suppose the Angle of the Square, whose Appearance we wou'd have in the Picture ABDD, to touch the Ground-line at the point E; take from that point E, upon the Ground-line AB, the lines EA EB, each equal to the Diagonal of the Square, whose Appearance you wou'd have, and draw thro' the Two Points of Distance DD, to the Three points A, E, B, Right-lines, which by their intersections will give the Appearance or Representation EFGH of the Square requir'd.

Now to represent at the Angles of that Square EFGH, Four
Plate 19. other little Squares, which are as the Bases of Four Bays of
Fig. 37. Building, of Four Pavillions, or Four Pillars, &c. You must take upon the Ground-line AB, the lines EK, EL, AM, BN, each equal to the Diagonal of one of those little Squares, which are suppos'd to be about the Square of the Geometrical Plain; and make an end of the rest as has been taught, and as you see in the Figure.

If you wou'd have other Squares, you must draw thro' the point I, the line OP parallel to the Ground-line AB, and make an end of the rest as the Figure shews, without any need of a farther Explication.

O F

OF
ELEVATIONS,
OR,
SCENOGRAPHY.

HAving ſufficiently treated of the Repreſentation of Points, Lines and Plains; to go on Methodically and in Order, we muſt now ſpeak of Elevations, and Firſt of Upright Bodies, and afterwards of Leaning or Inclin'd Bodies, as you will ſee in the following Problems.

OPERATION XIII.

How, from a Point given in the Picture, to raiſe a Perpendicular to the Ground-line, of a length given in Repreſentation.

I Suppoſe the point F to be given in the Picture ABDV, from which you muſt raiſe a Perpendicular to the Ground-line AB ſuch as FG here, which is to repreſent (*for Example*) the Height of Two Foot. We have here given this point F in Four different places of the Picture, to ſhew that the Way to determine the Height FG is every where the ſame, as you will ſee. *Plate* 20. *Fig.* 38.

Since a Height of Two Foot is requir'd in Repreſentation, a real length of Two Foot is to be mark'd ſomewhere on the Ground-line AB, as from B to E; and thro' the Two points E, B, and the point H taken at pleaſure upon the Horizontal Line VD, are to be drawn the lines HE, HB, which will ſerve as a Scale, which *Deſargues* calls *Fleeing Scale*, whoſe Uſe follows.

Draw thro' the given Point F, the line FO parallel to the Ground-line AB, and the part NO comprehended in the Fleeing Scale will repreſent Two *Perſpective-Feet*, which *Deſargues* calls *Feet in Front*, as the line HE is call'd the *Fleeing Line*, and the line SO, the *Line in Front*, which *Deſargues* calls alſo the *Scale in Front*, when it is divided into equal parts, as here, by Lines in Front which proceed from the Principal Point V, by the equal Diviſions of the Ground-line AB, which repreſent real Feet or Inches, *&c.* If then the line FG be made equal

equal to its correspondant line NO, it will represent the Height of Two Feet as it was propos'd.

SCHOLIUM.

Plate 20. Fig. 39. If in the Picture ABDV, you describe a Floor of Squares, whose sides are each of One Perspective Foot, which *Desargues* calls Fleeing Foot, when it loses it self upon a Fleeing Line, as has been taught in *Operation* 4. Those Squares will be a means to find out the length of the line FG, which must be equal to the length of Two Foot in Front, taken in any part of the line in Front FO.

But because it is too tedious to describe a Floor of Squares, and that there is not always room in the Picture to make a Fleeing Scale; that is, to set off upon the Ground-line AB, the Height given BE, learn this other Method, which may be practic'd without any confusion.

Raise from the point B taken at pleasure upon the Ground-line AB, the Perpendicular BC Two Foot long, or of such a length as you wou'd make FG appear to be, and draw from the point H taken at pleasure upon the Horizontal-line VD, to the Two points B, C, the lines HB, HC, between which you must determine the length of the Perpendicular FG, by drawing from the point O, (where FO the Line in Front meets the line HB,) the line OM parallel to the *Line of Elevation* BC, and that parallel OM will be the Height FG requir'd.

OPERATION XIV.

How to represent in Perspective a Right Prism.

Plate 20. Fig. 39. TO represent in the Picture ABDV, an upright Prism, whose Base or Situation is (*for Example*) an Hexagon; First describe the Sexangular Base 1, 2, 3, 4, 5, 6, in the Geometrical Plain over against the Ground-line AB, from which it must be distant in Proportion of the distance from the Prism to the Picture; and its Position in respect to the Ground-line AB, and to the Principal Point V must be according as that of the Prism that you wou'd represent in Perspective, is in respect to the Picture and the Eye.

Having made this Preparation, you must by, *Operation* 3. put in Perspective the Plan 1, 2, 3, 4, 5, 6, and from all the Angles of the Perspective-Hexagon raise lines perpendicular to the Ground-line AB, to which you must give a Height equal in Representation to that of the propos'd Prism, as has been taught; namely, by setting off that Height given on the Ground-line from B to E, or on its Perpendicular BC, *&c.*

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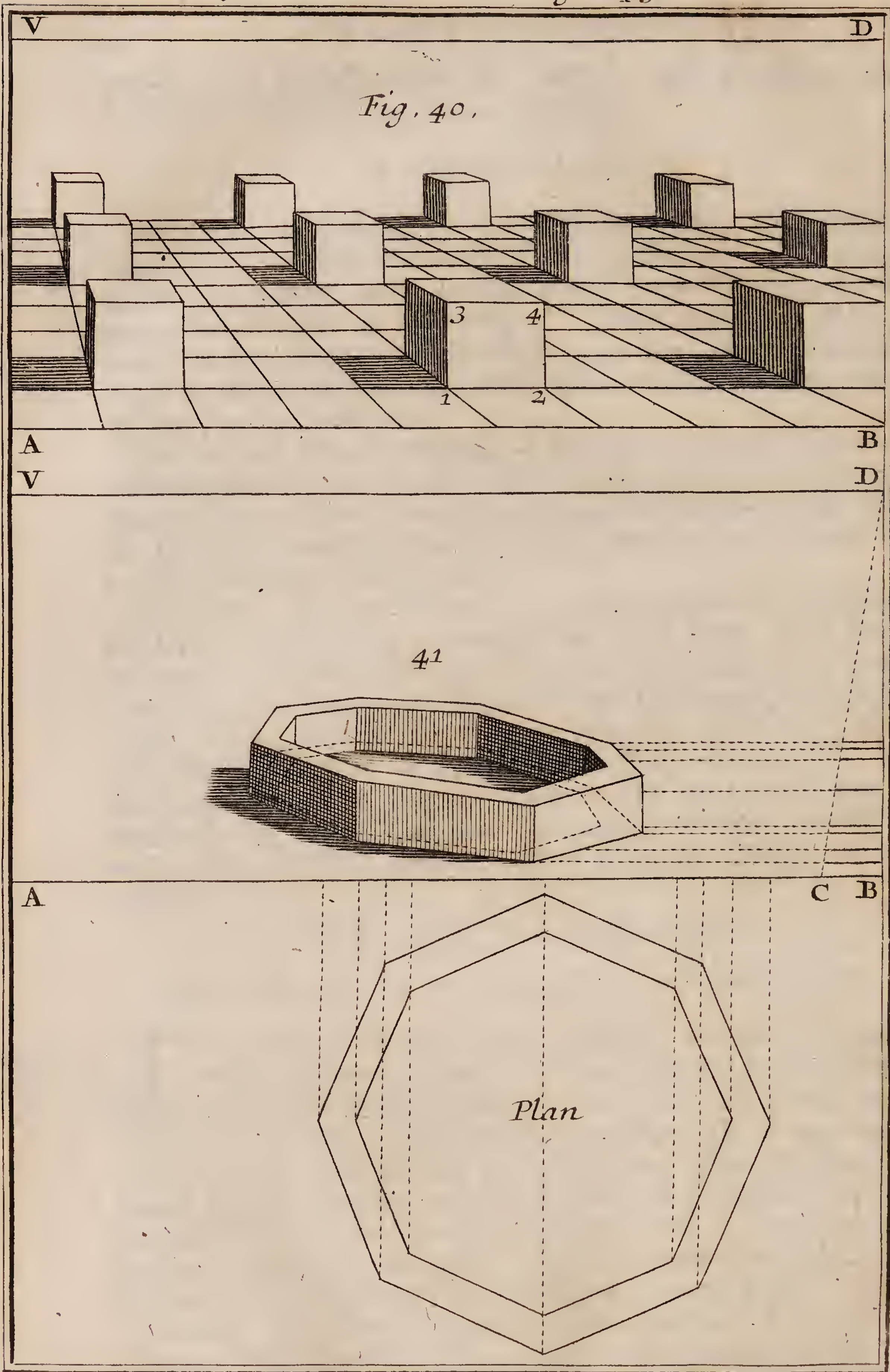
V
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Fig. 40.
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SCHOLIUM.

When you wou'd repreſent a Cube ſeen in Front, whoſe Side is equal in Repreſentation to a Line in Front, given in the Picture, as to the Line CE, you may make uſe of this ſhort Method.

Draw thro' the Two points E,C, to the Principal Point V, the Rays VE,VC, and thro' the Point C to the Point of Diſtance D, the Ray CD, which will upon the Ray VE give the point 8; thro' which you muſt draw to the given Side CE, or to the Ground-line AB, the Parallel 7, 8, which will be terminated at the point 7, by the other Ray VC, and the Perſpective Square 7CE8 will be the Baſe of the Cube to be deſcrib'd; then you need only to raiſe from the Two points E,C, the lines EG,CF, equal and perpendicular to the ſide CE, and likewiſe from the Two points 7, 8, the Two lines 70, 89, equal and perpendicular to the ſide 7,8, *&c.*

When you wou'd repreſent in Perſpective a Right Cylinder, whoſe Baſe is a Circle, deſcribe that Circle in Perſpective, *by Operation* 10. and from ſeveral points of that Perſpective Circle, raiſe Perpendiculars, Each equal in Repreſentation to the given Height of the Cylinder; then join the Upper End of all thoſe Perpendiculars by a Curve, and the whole Operation is perfected.

OPERATION XV.

How to repreſent in Perſpective ſeveral Upright Cubes equally diſtant from one another, and ſet in Rows parallel and perpendicular to the Picture.

FIRST you muſt deſcribe in the Picture the Situation of all thoſe Cubes, which are ſo many Perſpective Squares, as has been taught in *Operation* 11. Then from all the Angles raiſe Perpendiculars to the Ground-line AB, and Each equal to its Correſpondant Side which is parallel to the Ground-line AB, (as we have ſaid in the foregoing Scholium) as the Two Perpendiculars 13, 24, Each equal to its Correſpondant Side 12, *&c.* *Plate* 21. *Fig.* 40.

SCHOLIUM.

If inſtead of Cubes you wou'd have Square Pillars, you muſt work the ſame Way, except that the Height of each Pillar wou'd not be equal to its correſpondant Side; but it may be determin'd according as it is given, by the General Rule of *Operation* 13. But if inſtead of Square, you wou'd have Pillars or Cylinders, whoſe Baſes are Circles, you muſt *by Operation* 10. deſcribe the Repreſentations of thoſe Circles in

in the Little Perspective Squares, and the rest as has been taught in the foregoing *Scholium.*

OPERATION XVI.

How to represent in Perspective an Upright Concave Prism.

Fig. 41. IF you wou'd have the Base of a Concave Prism (*for Ex.*) an Octogon, describe in the Picture ABDV the Appearance of a double Octogon for that Base, and raise from all the Angles of the said Base as many Right-lines perpendicular to the Ground-line AB, Each equal in Representation to the Height, which you wou'd give your Prism, such as is here BC, and join the Ends of all those Perpendiculars by Right-lines, and the whole is ended.

OPERATION XVII.

How to represent in Perspective an Upright Body cut sloping.

Plate 22. Fig. 42. IF you wou'd have the Base of the Body be, *for Example*, an Hexagon, describe in the Picture ABDV, the Appearance of a double Hexagon, the inward being the Base or Situation of the Upper Hexagon, and the outward will be for the *Talu*, or Slope, which is known as well as the Height of the slop'd Body, by means of the *Profil*, call'd also the *Porfil*, which is the Section of a Body and a Vertical Plain (that is perpendicular to the Horizon) as EFGH, where the Height of the Body propos'd is represented by the Line GK, or HI, and its *Talu* or Slope by the Part KF or EI, terminated by the Lines HI,GK, perpendicular to the Base EF, *&c.*

Having then describ'd the Perspective Plain of the Body propos'd, raise from all the inward Angles Perpendiculars to the Ground-line AB, Each equal in Representation to the Height of the Body propos'd, which is found in the Profil, *viz.* HI, or GK, or its equal BC, and join the ends of all those Perpendiculars by Right-lines to have the Upper Hexagon, whose Angles must also be join'd to the Correspondant Angles of the *Talu* by Right-lines, as the Angle 1 to the Angle 2, the Angle 3 to the Angle 4, the Angle 5 to the Angle 6, *&c.*

OPER-

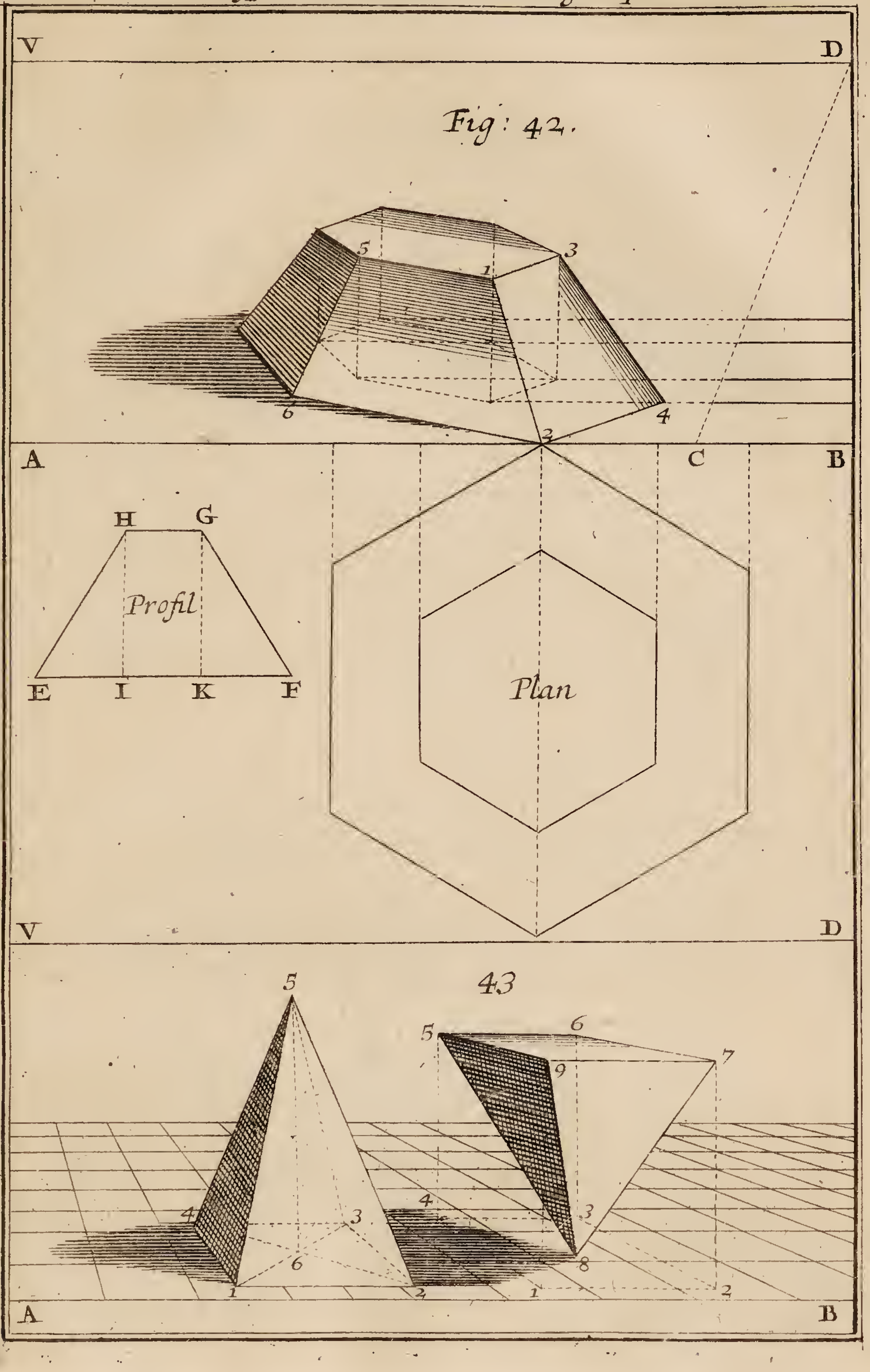
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Fig: 42.
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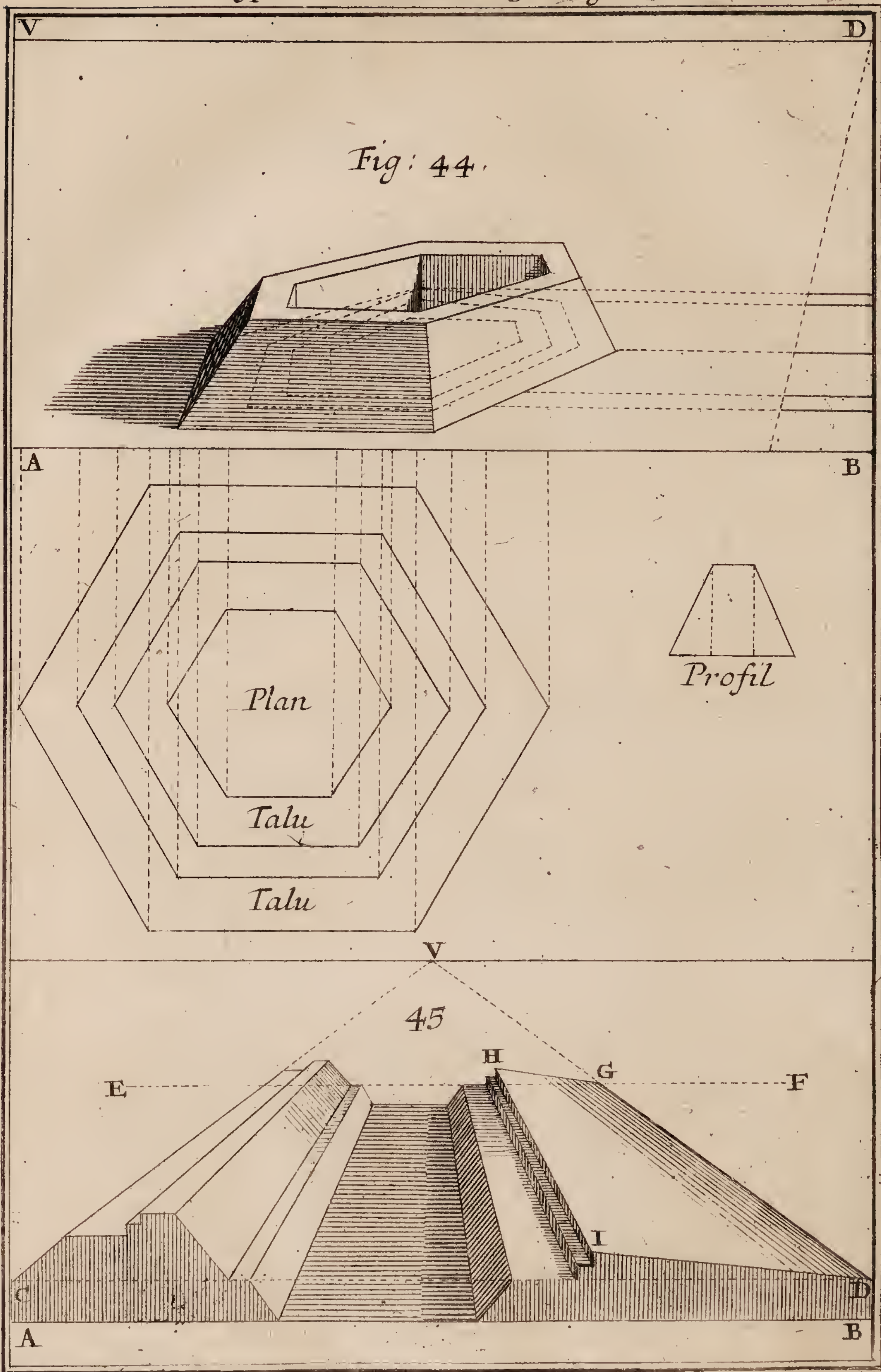
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OPERATION XVIII.

How to represent in Perspective Two Pyramids, of which the One stands upon its Base, and the Other is rais'd upon its Vertex.

FIRST to find the Appearance of a Pyramid ſtanding upon its Baſe, deſcribe that Baſe in Perſpective, as 1,2,3,4, raiſe from its Center 6, the Line 6, 5, perpendicular to the Ground-line AB, and equal in Repreſentation to the given Height of the Height of the Pyramid, to have at the point 5, the *Vertex* of the Pyramid; then make an end of the reſt as you ſee in the Figure. *Fig.* 43.

Secondly, To find the Appearance of a Pyramid reſting on its *Vertex*, having (as before) deſcrib'd the Perſpective Plan 1, 2, 3, 4, deſcribe upon that Plan the Priſm 4, 5, 6, 7, 2, 1, whoſe Height is equal in Repreſentation to that of the propos'd Pyramid; and take the Upper Plan 5,6,7,9, for the Baſe of the inverted Pyramid, and the Center 8 of the lower Baſe for the Vertex of the Pyramid, &c.

OPERATION XIX.

How to represent in Perspective an Upright Concave Body cut sloping both within and without.

IF you wou'd have the Baſe of the Upright Body cut ſloping within and without, to be, *for Example*, an Hexagon; deſcribe in the Picture ABVD the Appearance of a double Hexagon, according to the Thickneſs that you will give to the Sides of the Concave Body, or according as the Profil (if there be one) will give it you: And round about this double Hexagon deſcribe within and without, Two other Hexagons, parallel in Repreſentation to the Two former; and nearer or farther off according to the Breadth that you will find in the Profil of the inward or outward *Talu*. From all the Angles of the double Octogon, which is in the middle of the Two others, raiſe Perpendiculars to the Ground-line AB, and equal in Repreſentation to the Height of the Body propos'd, which you will find in the Profil, and end the reſt as we have taught in *Operation* 18. *Plate* 23. *Fig.* 44.

OPER-

OPERATION XX.

How to represent in Perspective the Profil of a Piece of Fortification.

Plate 23. Fig. 45. HAving drawn in the Picture the Profil which you wou'd represent in Perspective, in its Natural Dimensions, in such manner, that CD the Level of the Field may be parallel to the Ground-line AB, draw from all the Angles of that Profil to the Principal Point V, Rays which you will terminate thus. Draw at pleasure a Second Line EF for the Level of the Field parallel to the First CD, and that Line EF will terminate the First Ray DG at the point G, thro' which you must draw to the line DI the parallel GH, which will at H terminate the Second Ray HI, *&c.* This may also be done another Way, but the Figure is too small to shew what cou'd be said more concerning it.

SCHOLIUM.

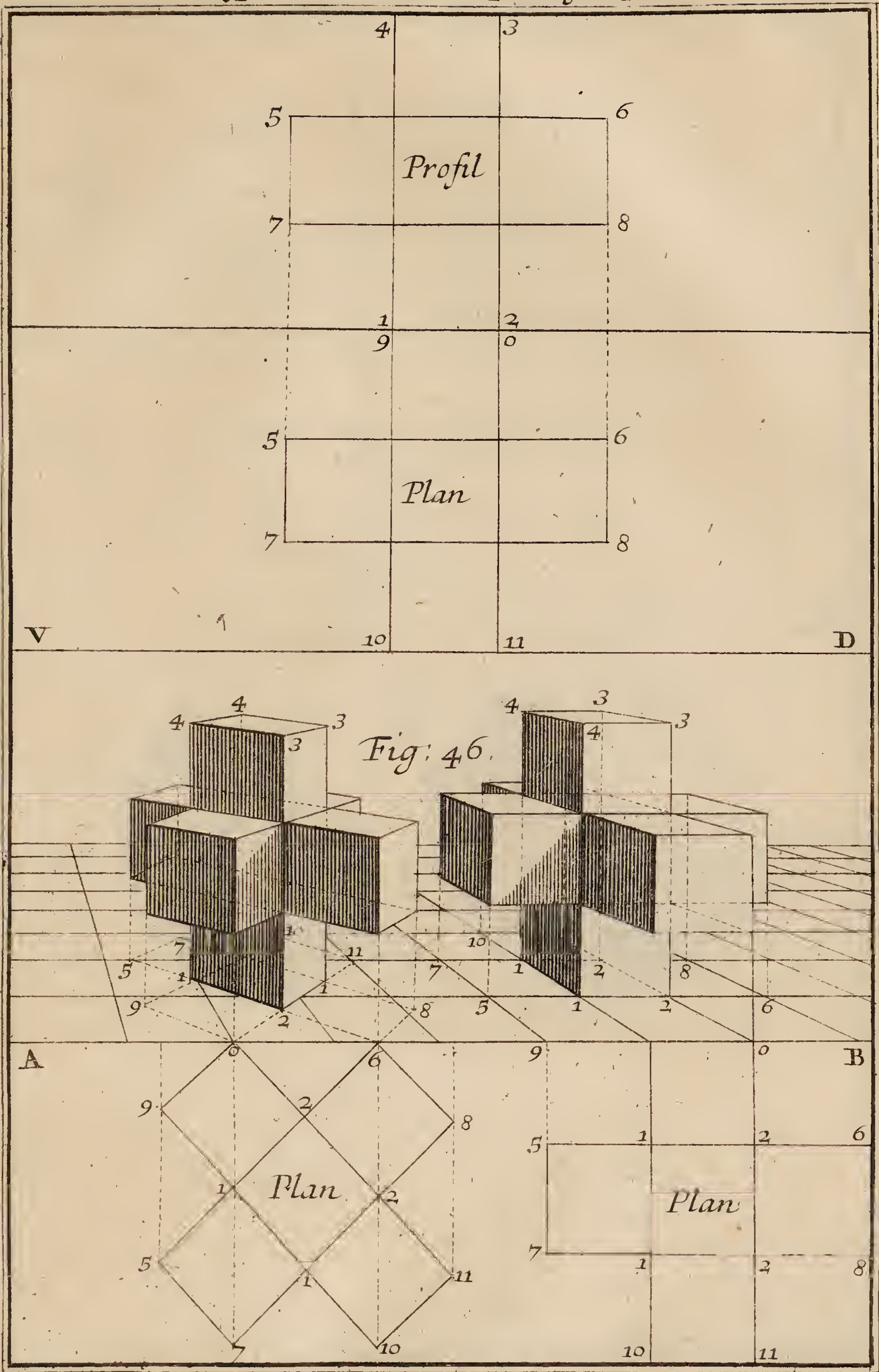
As all the Bodies which we have hitherto describ'd, had all their parts of the same Height, except the foregoing Profil, we have represented them only by means of their Plan or Base. But when they have different Heights, you must have the Profil as well as the Plan; because that Profil will shew what Heights are to be given to the several parts of the Body which you wou'd represent in Perspective. Wherefore in such Cases, we shall make Use both of the Plan and of the Profil, as you will see in the following Operation.

OPERATION XXI.

How to represent in Perspective a double Cross rais'd at Right Angles upon the Geometrical Plain.

Plate 24. Fig. 46. HAving drawn both the Plan and the Profil of the Cross which you wou'd represent in Perspective, place this Plan in the Geometrical Plain over against the Ground-line, according to the Situation that you have a mind to give to the Cross; and when you have put this Plain in Perspective, raise from all its Angles Perpendiculars to the Ground-line AB, to set upon it the Heights of the Correspondant Parts of of the Cross, (such as you see 'em in their true Dimensions in the Profil, which you must shorten by the Rules of *Operation* 13.) and to join the Ends of those Perpendiculars by Right-lines, as those of the Profil are join'd.

SCHO-


Profil
Plan
V
D
Fig: 46.
A
B
Plan
Plan

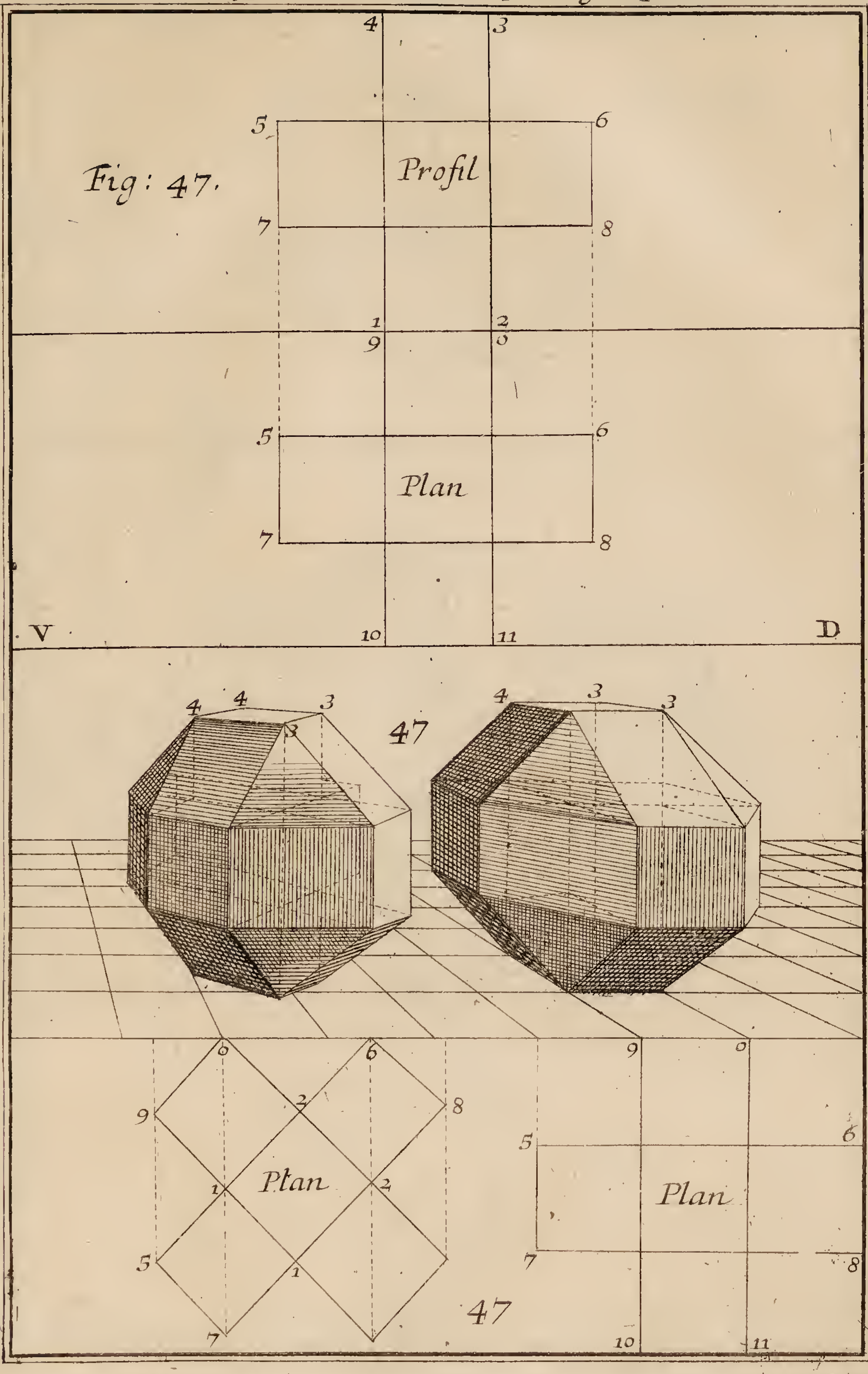


Fig: 47.
4
3
5
6
Profil
7
8
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9
0
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Plan
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11
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4
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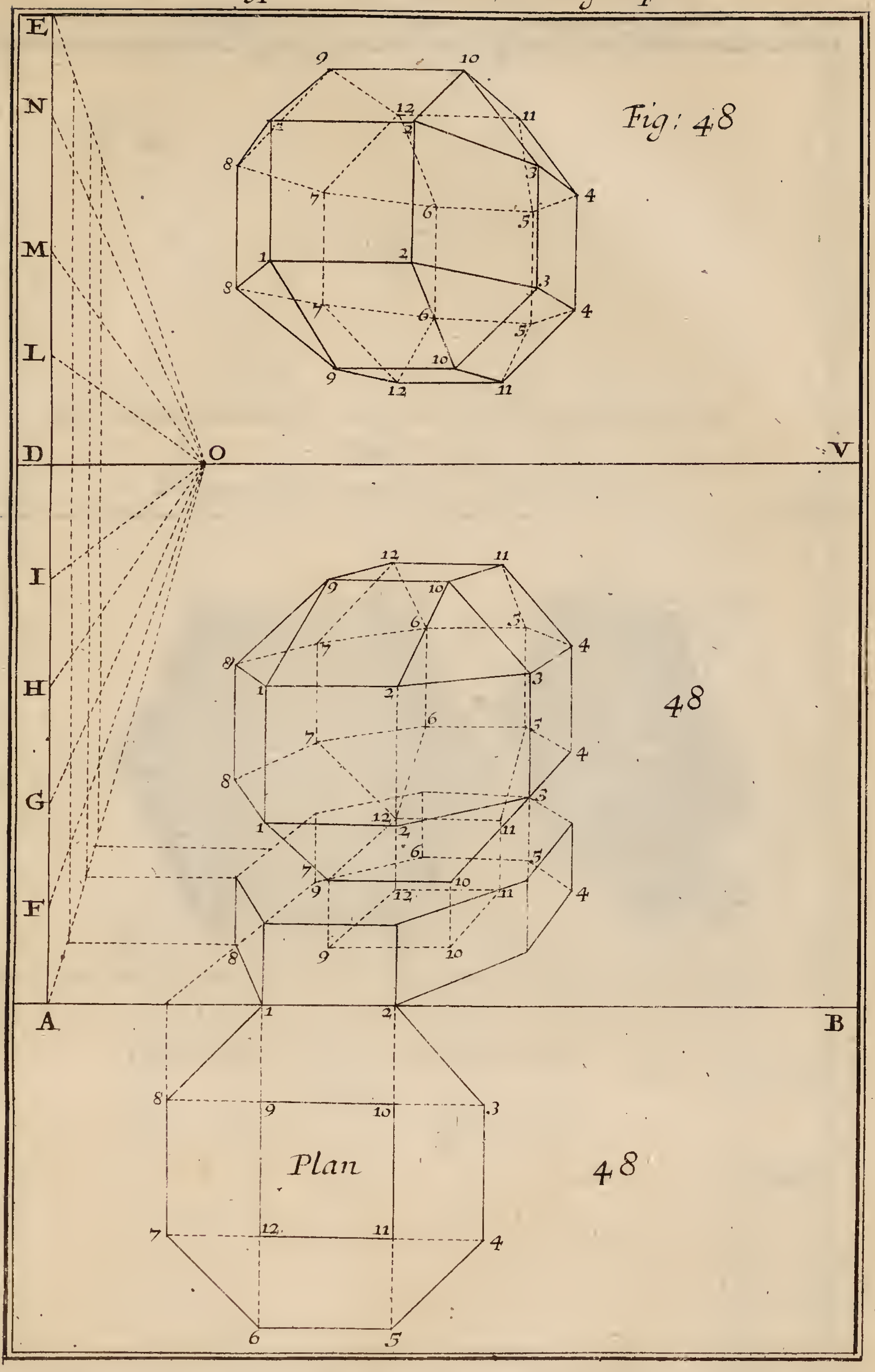
Fig: 48
E
N
M
L
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O
V
I
H
G
F
A
B
48
Plan
48

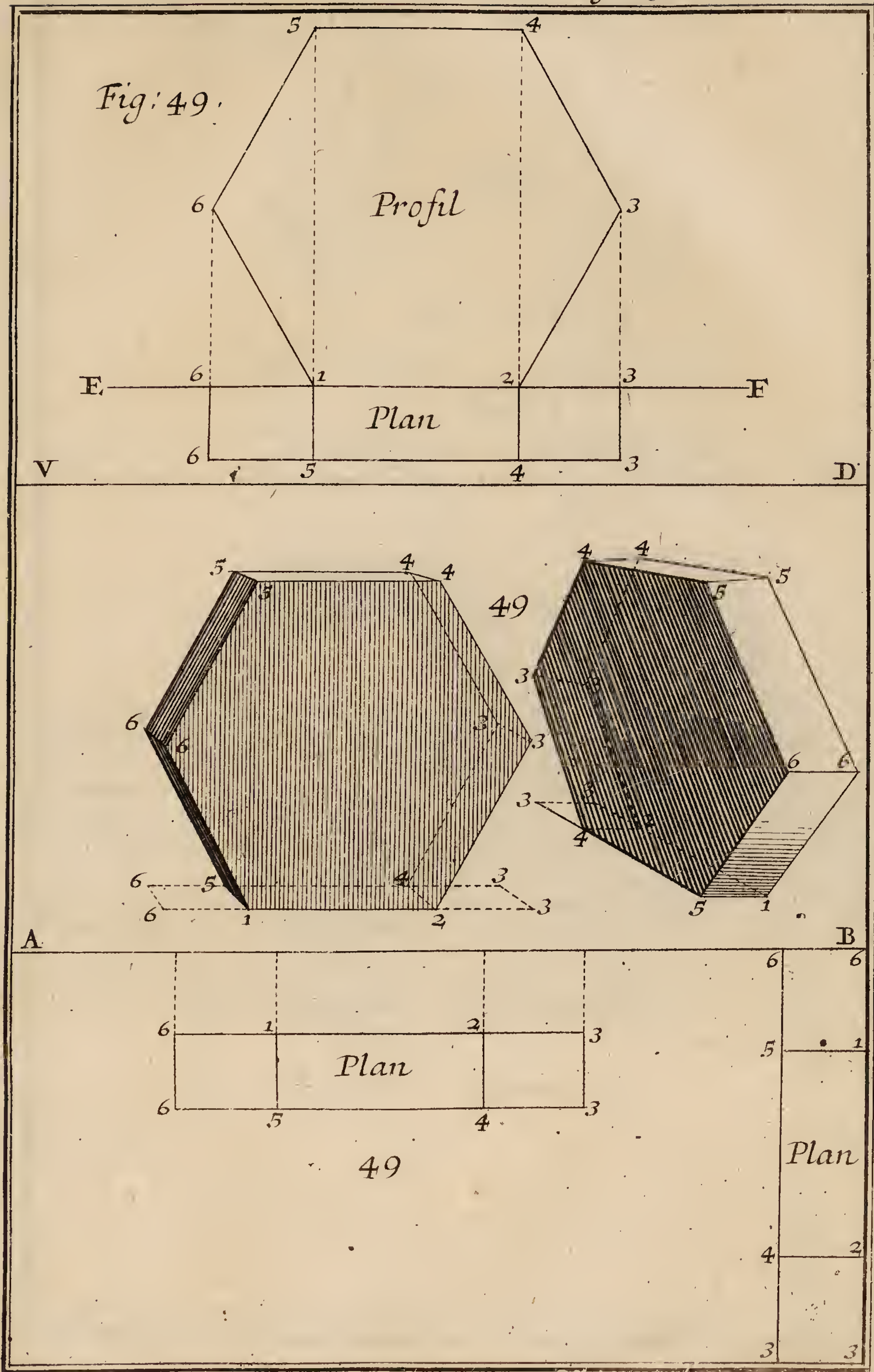
Fig: 49.
Profil
Plan
E
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V
D
49
A
B
Plan
49
Plan
1
2
3
4
5
6

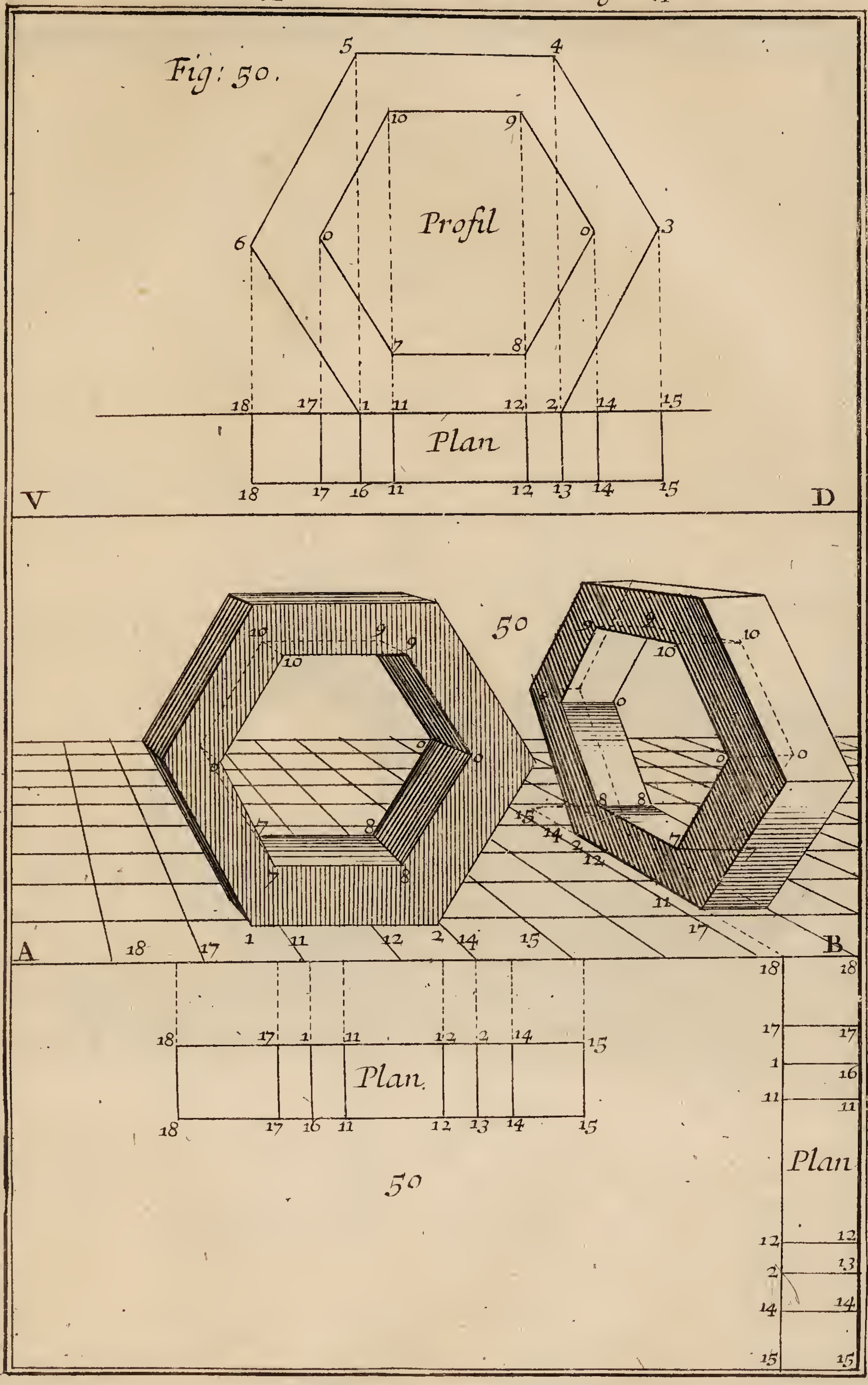
Fig: 50.
Profil
Plan
V
D
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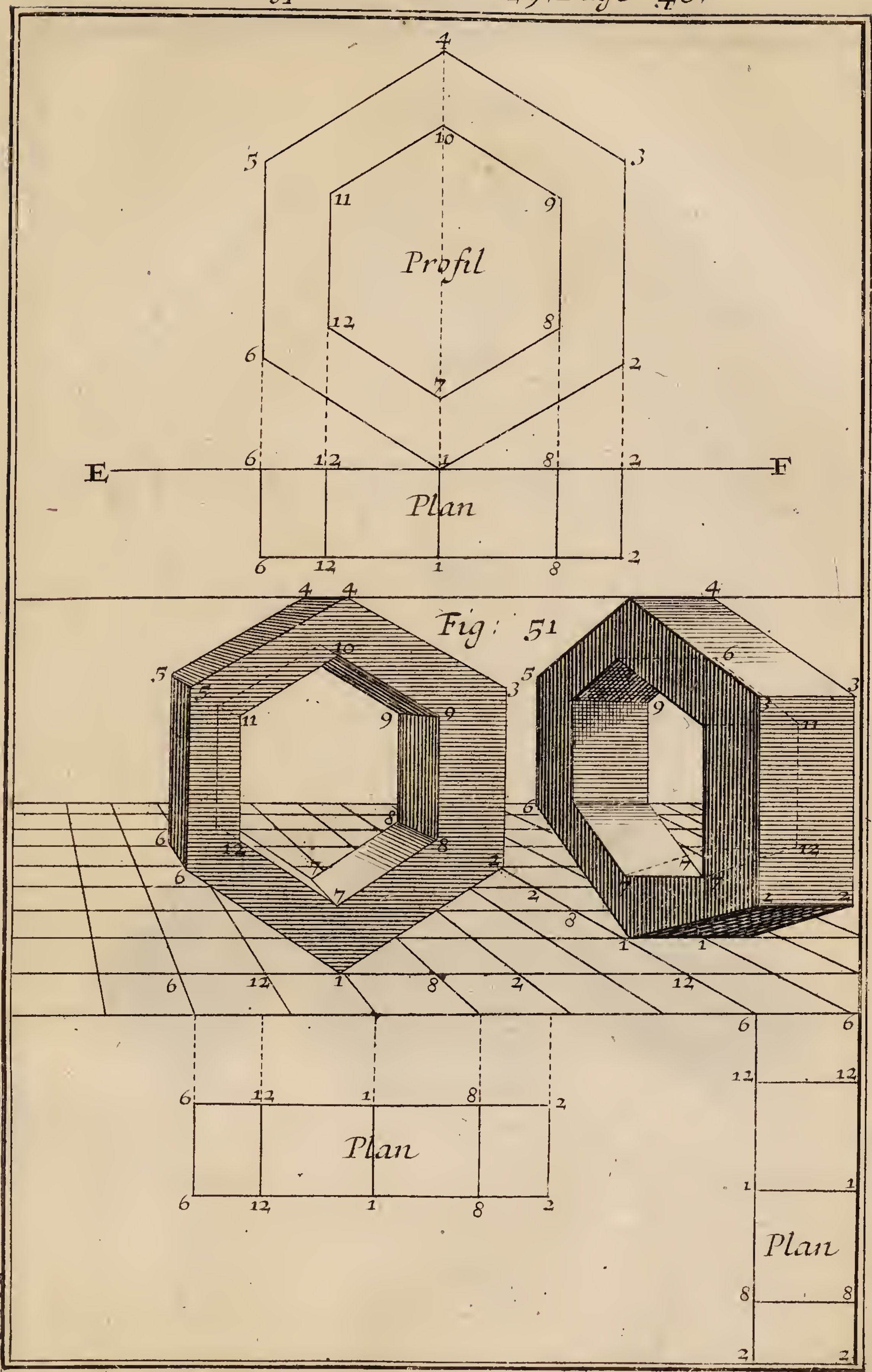
A
B
Plan
50
Plan


Profil
E
F
Plan
Fig: 51
Plan
Plan

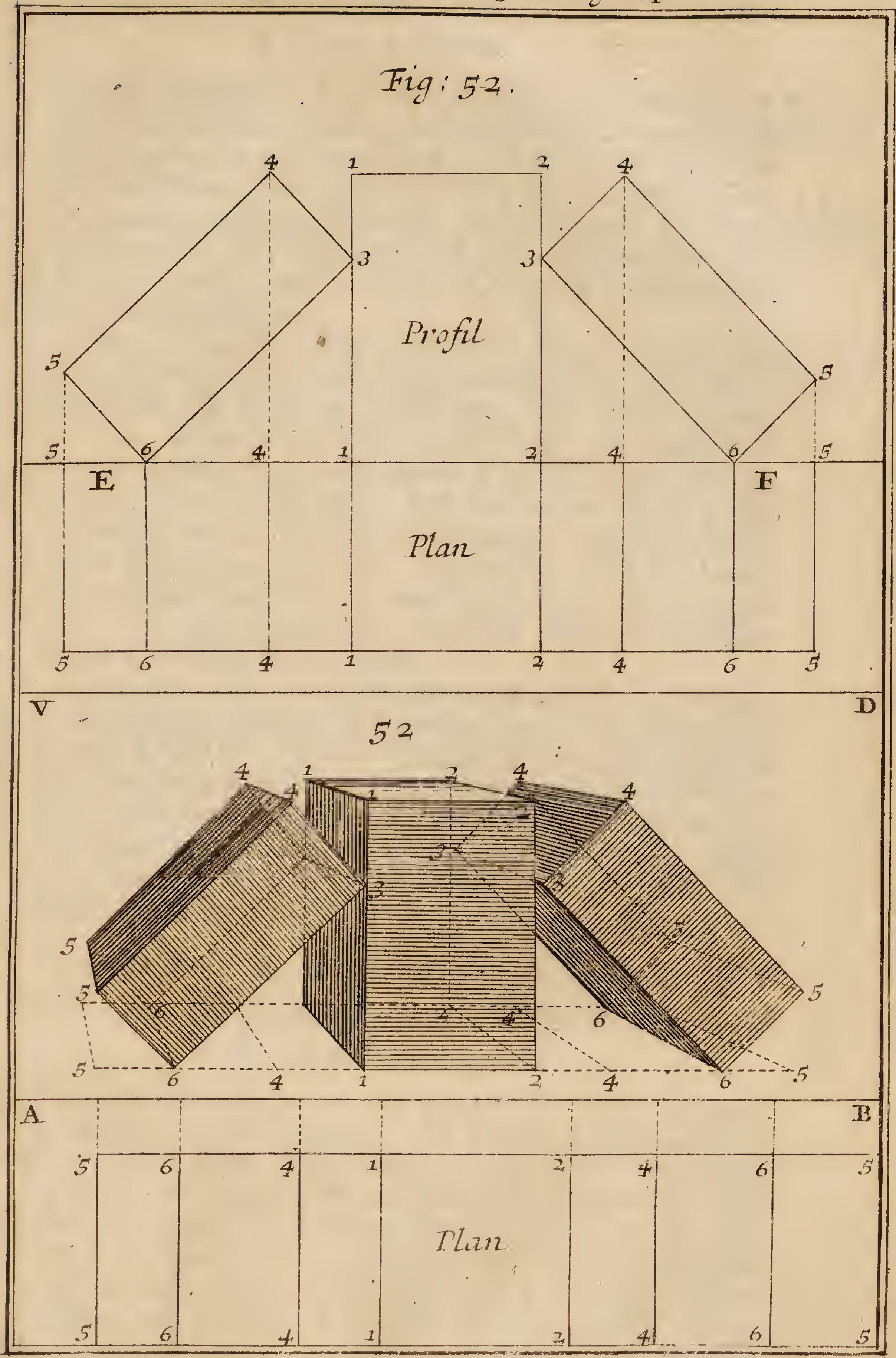


Fig: 52.
Profil
Plan
E
F
V
D
52
A
B
Plan

SCHOLIUM.

If a double Equilateral Croſs be deſcrib'd, and the Ends of its Four Arms and the Tree be join'd by Right-lines, you will have the Appearance of a *Polyhedrum*; that is, of a Body made up of ſeveral Plains, and † *inſcriptible in* a Sphere, whoſe Center will conſequently be the Center of the Sphere. Of thoſe Plains, which are 26 in Number, 18 will be Squares and Equal to one another, and 8 will be Triangles Equal to one another, and Equilateral. *Plate* 25. *Fig.* 47.

† *Fit to be inſcrib'd in.*

But there is another Way to deſcribe ſuch a *Polyhedrum*, by ſeeing the *Figure* 48. where you muſt obſerve, that AF is the Height of the Firſt rais'd Octogon, and of the Square 9, 10, 11, 12, which is the Baſe of the *Polyhedrum* upon the Pedeſtal, or the Firſt rais'd Octogon: That AG is the Height of the Second rais'd Octogon; AH that of the Third; and AI the Height of the Second rais'd Square, when the Polyhedrum is higher than the Eye: And laſtly, That the Lines FG, HI, are Each equal to the Line 8,9, or to the Line 1,9, of the Plan of Situation, and the Line GH equal to the Side 1,2, of the ſame Plan of Situation. *Plate* 26. *Fig.* 48.

But when the *Polyhedurm* is higher than the Eye, the Line DL is the Height of the Firſt Square 9, 10, 11, 12, above the Horizontal Line DV; DM the Height of the Firſt Octogon of the *Polyhedrum* above the ſaid Horizontal Line; DN the Height of the Second Octogon; and DE the Height of the Second Square, the Lines LM, EN being likewiſe each equal to the Line 8,9, of the Plan of Situation; and the Line MN to the Side 1,2, of the ſaid Plan of Situation.

OPERATION XXII.

How to repreſent in Perſpective a Right Priſm rais'd upon One of its Oblique Plains.

HAving deſcrib'd upon the Line EF, which I ſuppoſe parallel to the Horizon, the Profil of the propos'd Priſm, with its Plan, which will be terminated by Lines drawn at Right Angles upon the Line EF, from all the Angles of the Profil; this Plan muſt be deſcrib'd in Perſpective, by putting it over againſt the Line AB in the Geometrical-plain, according to the Situation that you wou'd give that Priſm; and from all the Angles of that Perſpective Plan, Lines muſt be drawn perpendicular to the Ground-line AB, and equal in Repreſentation to the Height of thoſe Points which anſwer to 'em in the Profil: And joining thoſe points together, which belong to the ſame ſide (as the Profil will direct you) the Problem will be ſolv'd.

SCHOLIUM.

Plate 28. *Fig.* 50. If you wou'd represent this Prism with such a Hole thro' as may give passage to the Light, draw likewise its Profil and Plan, to end the rest as has been taught, and as you may see in *Fig.* 50.

Plate 29. *Fig.* 51. After this Manner also may be represented in Perspective a Right Prism bearing upon One of its Sides, Namely, by describing upon the Line EF parallel to the Horizon the Profil of that Prism; and below the said Line EF, its Plan, whose Breadth 6, 6, or 2, 2, represents the Thickness of the Prism, or the Length of that Side on which it rests; and making an end of the rest as before, and as you may see in *Fig.* 51.

OPERATION XXIII.

How to represent in Perspective a Prism Inclin'd to the Horizon, resting on one side, and sustain'd by another Prism, which is Upright.

Plate 30. *Fig.* 52. IF you describe upon the Line EF parallel to the Horizon, the Profil of the Inclin'd Prism, and of the Right-One against which it leans, with their Plans of Situation, it will be as easy to represent those Bodies in Perspective as the foregoing; therefore I think a sight of the Figure is enough, in Imitation of which, and by help of what has been said, it will be easy to represent in Perspective a Body resting upon one of its Solid Angles, if you have its Plan and Profil: And thus you may likewise represent what you will in the Picture, without my giving any more Examples.

SCHOLIUM.

What has been said hitherto supposes the Picture Right, or Perpendicular to the Horizon, because it is usually so: Nevertheless, as in some Cases it may be Inclin'd, (as when you wou'd paint upon the Surface of a vaulted Roof) we shall here occasionally give the Method to represent in a Picture Inclin'd to the Horizon a Prism perpendicular to the Horizon, by means of the Profil and Plan of the Picture, which must be prepar'd thus:

Plate 31. *Fig.* 53. First to describe the Profil of the Picture, draw the indefinite Line EF, which you must take for the Line of Station; and raise upon its End E the Perpendicular EG, equal to the Height of the Eye above the Geometrical-plain, to have at G the place of the Eye, and at E its Situation. Then draw thro' the point H (distant from E by the Distance from the Situ-

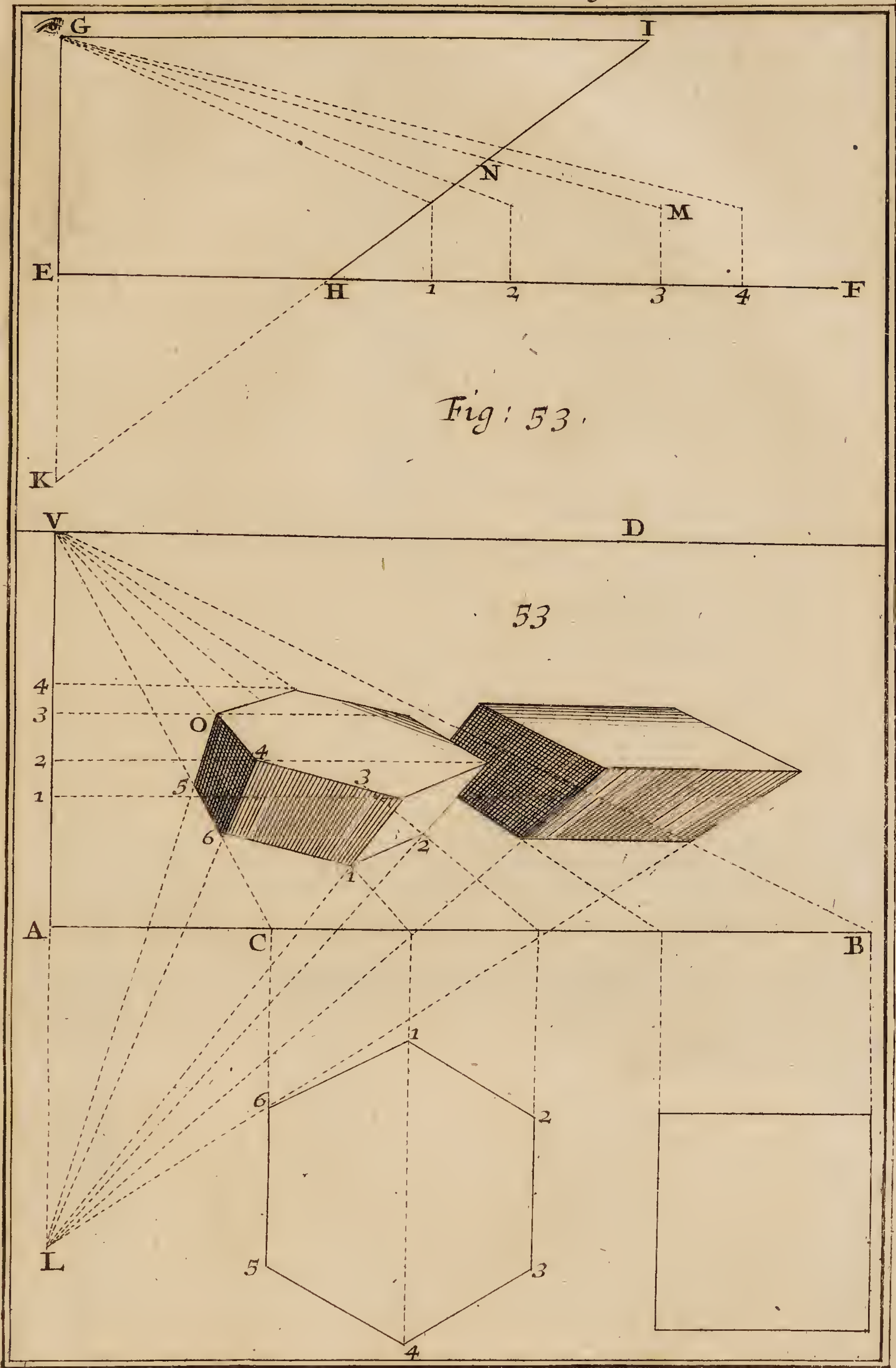
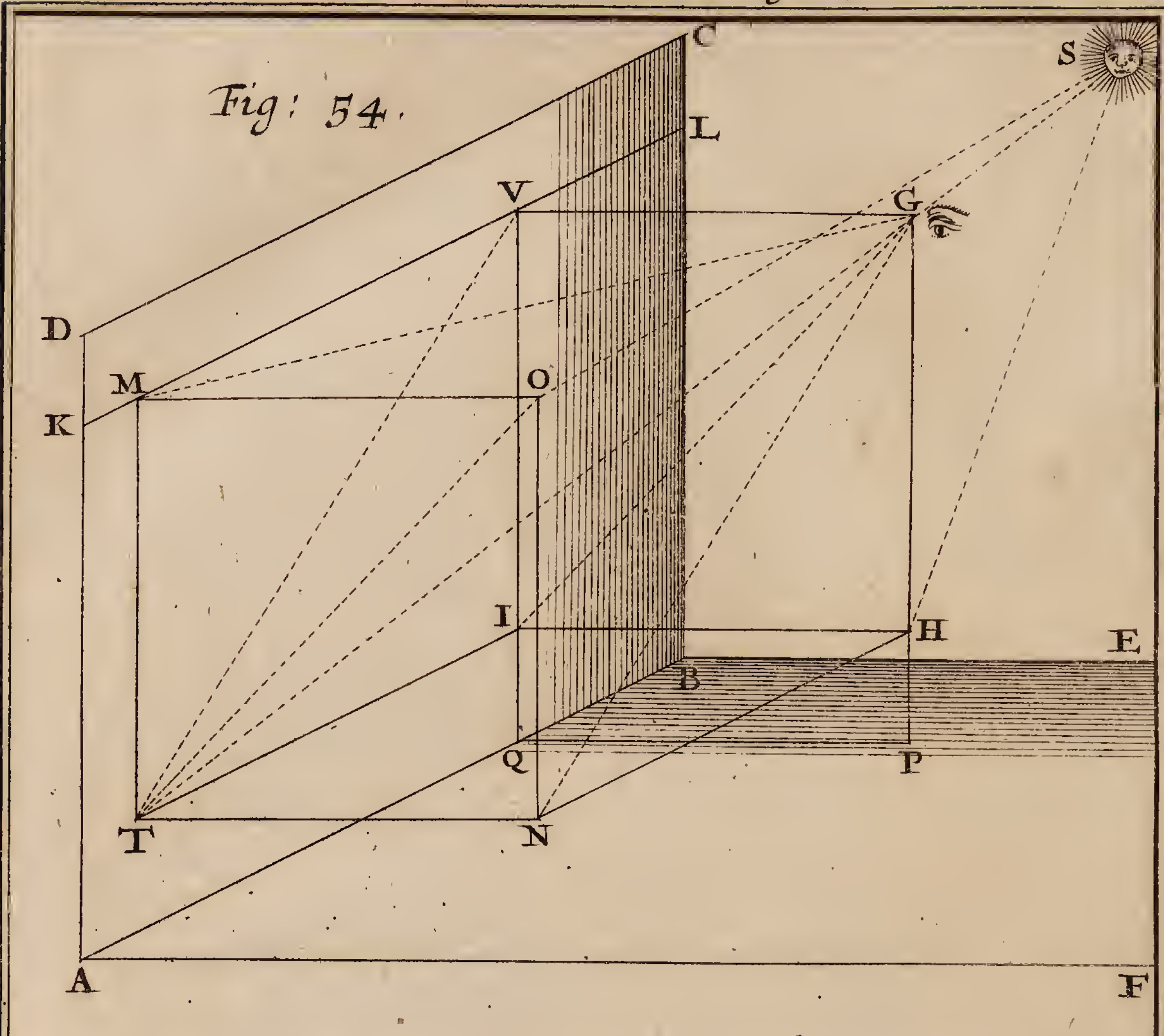
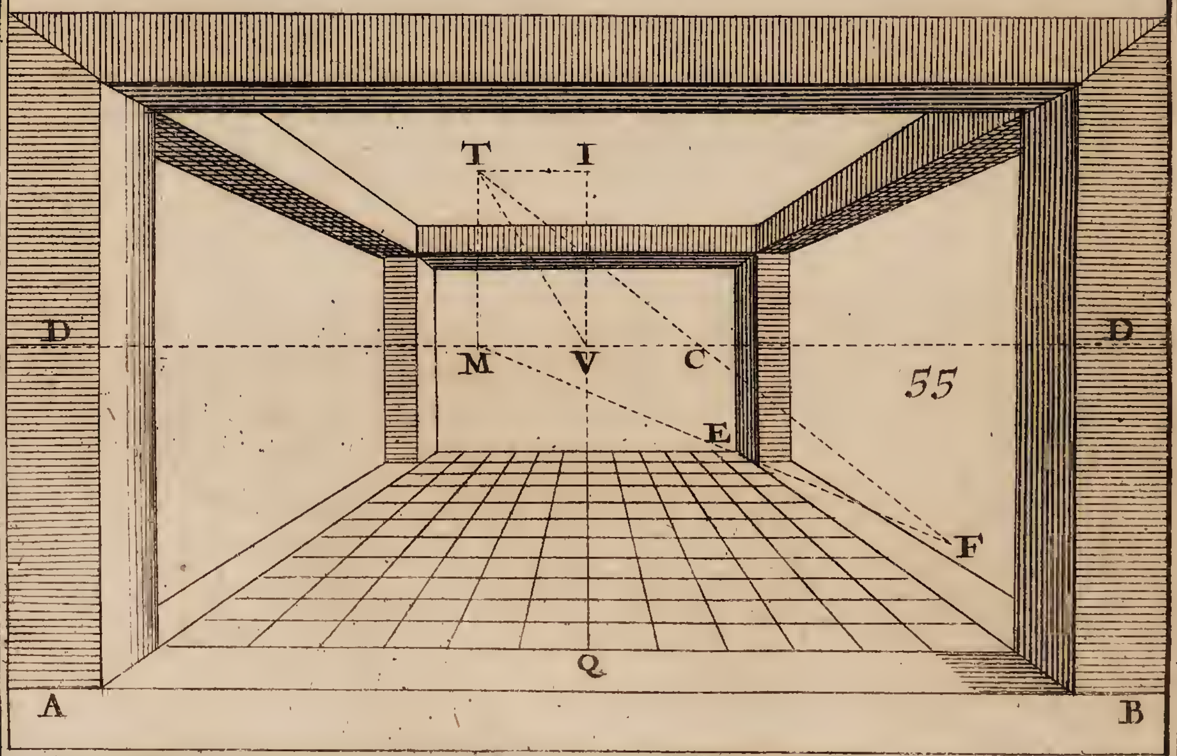


Fig: 53.

Fig: 54.





Situation of the Eye to the Picture) the indefinite Line HI Inclin'd to the Line of Station EF, as much as you wou'd have the Picture Inclin'd to the Horizon, and that Line HI must be taken for the Vertical Line and for the Picture. Draw again from the Eye plac'd at G the Line GI parallel to the Line of Station EF, and this Line GI which represents the principal Ray, will upon the Vertical Line HI, give the Point I, which will represent the Point of Sight. Lastly, produce the Lines EG, HI, till they cut one another in a point, as K, which we shall call the *Accidental Point of the Picture*, where the Appearances of all the Lines which are perpendicular to the Horizon being produc'd must end, *by Theor.* 8. (which, as well as all the others, agrees as well with the Inclin'd as with the Vertical-picture) and the Profil of the Picture is ended. Plate 31. Fig. 53.

To describe the Plan of the Picture, draw in another place of your Paper, the Horizontal-line VD, and Ground-line AB, parallel to the Horizontal VD, and distant from the said Horizontal by the Length of HI the Vertical-line of the Profil, in such manner, that the Perpendicular VA may be equal to that Vertical HI: And having taken the point V for the Point of Sight, draw VD equal to the principal Ray GI, to have at D the Point of Distance. Lastly, take upon VA the Perpendicular or Vertical-line produc'd, the line AL equal to the Hypotenuse HK of the Profil, to have at L the Accidental-point of the Picture, by means of which you must represent in this Picture a Right Prism in the following manner.

Having made this Preparation, place the Base of the Prism to be represented in Perspective, in the Geometrical-plain over against the Ground-line AB as usually, nearer or farther off, according to the Situation that you wou'd give the said Prism; as the Hexagon 1, 2, 3, 4, 5, 6, which must be put in Perspective, as if the Picture was Right; but instead of drawing from the Angles of the Perspective Hexagon Lines perpendicular to the Ground-line AB, (as you ought to do if the Picture was perpendicular to the Horizon) you must draw them from the Accidental-point L, and terminate the Height of the Prism thus.

To determine, *for Example*, the apparent Height of the point 5, which is in the Picture; draw from the point 5, which is in the Geometrical-plain, 5C Perpendicular to the line AB, whose Length must be set off upon the line EF of the Profil from H to 3, from which you must raise 3M perpendicular to the line EF, and equal to the given Height of the Prism. Then draw from the Eye G, thro' the Point M the Ray GM, which will upon the Line HI give the Point N, and carry the Distance HN upon AV the Vertical Line of

Plate 31. Fig. 53. the Picture, from the point A to the point 3, thro' which you must draw to the Ground-line AB the parallel to 3O, which will give upon the line LO (drawn from the Accidental-point L, thro' the point 5 of the Picture) the point O, and the line 5O will be equal in representation to the given Height of the Prism, *&c.*

Of Shadows.

Plate 32. Fig. 54. SHadows make all the Beauty of a piece of Perspective, because they distinguish the parts of a Body, which are from the Light, and serve in some measure to encrease the Splendor of the enlightned Parts; and these parts may be either enlighten'd by the Sun, whose Rays must be consider'd as parallel, because they come from a point at an Infinite Distance from the Picture; or by a small Light, as a Torch, whose Rays cannot be parallel, because they come from a Point not very far Distant from the Picture.

As in Perspective those Three Plains are chiefly consider'd, which are perpendicular to one another, *viz.* the Plain of the Picture ABCD, which we always suppose perpendicular to the Horizon; the Vertical-plain PGVQ, which is perpendicular to the Picture; and the Geometrical-plain ABEF, to which the other Two are perpendicular: So likewise these Three following Lines are chiefly to be consider'd, which are perpendicular to one another, and Each to some of those Plains, *viz.* the principal Ray GV, and all its Parallels which are perpendicular to the Picture; the Height of the Eye PG, and all its Parallels which are perpendicular to the Geometrical-plian; and last of all, all the Right-lines which are parallel to one another and to the Picture, and consequently perpendicular to the Vertical-plain, of which the Line which we shall consider chiefly goes thro' the Eye G, Namely, GO.

To determine the shadows of Bodies upon any Plain whatever, it is sufficient to know the Shadows of the Lines which are their Bounds, and as those Shadows are usually mark'd in the Sun-shine; we shall here give Rules to determine those Shadows, when the Sun is out of the Plain of the Picture, and when it is in the Plain of the Picture. For this End, let us suppose the Sun to be at S, and One of its Rays to be ST, which going thro' the Eye G, meets the Picture at the point T, which we shall call the *Place of the Sun in the Picture*; because, as we have suppos'd the Rays of the Sun parallel, their Appearance concurring at T, determines at the said point T the place of the Sun in the Picture.

This

This being ſuppos'd, draw from T, the place of the Sun, TM parallel to the Vertical Line VQ, which will be terminated at M by the Horizontal-line KL; and to the principal Ray VG, or Line of Station PQ, the Parallel TN, which you muſt terminate at the point N, thus. Draw thro' the ſaid point T, to the Ground-line AB, the Parallel TI, which will be terminated by the Vertical-line VQ at the point I; thro' which you muſt draw to the principal Ray VQ, or to the Line of Station PQ the Parallel IH, which will be terminated by the Height of the Eye GP, at the point H, thro' which you muſt again to the line TI draw the parallel HN, which will meet the line TN at the point N, from which you muſt draw the line NO equal and parallel to the line TM; and join MO, which will be parallel and equal to TN; and GO, which will be parallel and equal to HN, as alſo the line TI. Join alſo the Two lines OT, GI, which will be parallel and equal to one another. Laſtly join VT, and HS; and you will have the Two Plains MOGV, TNHI, parallel, to one another and to the Geometrical-plain ABEF, and conſequently perpendicular to the Plain of the Picture ABCD; and the Two Plains TNOM, IHGV, parallel to one another, and to the Vertical-plain, and conſequently perpendicular to the Picture; and laſtly, the Two Plains TIVM, NOGH, parallel to one another, and to the Picture, and conſequently perpendicular to the Vertical-plain. *Plate* 32. *Fig.* 54.

We ſhall call the point I, the *Point of Inclination of the Rays of the Sun*, (becauſe that point depends on the Height of the Sun above the Horizon; it being certain that it will be nearer to the Horizontal-line KL when the Sun is nearer to the Horizon) and the point M, the *Point of the Declination of the Rays of the Sun*, becauſe that point depends on the Declination of the Sun from the Vertical-plain; it being certain, that it wou'd be in the Vertical Line VQ, if the Sun was in the Vertical-plain, in which Caſe the Two points I, T, wou'd coincide, and alſo both of 'em coincide with the principal Point V, if the Sun was in the Interſection of the Vertical Plain, and of the Horizon; and they wou'd wholly vaniſh if the Sun was in the Zenith. This being Explain'd, let us proceeed to the Rules.

Rules for the Shadows caus'd by the Sun, suppos'd out of the Plain of the Picture.

I.

Plate 32. Fig. 54. THE Shadow that the principal Ray VG, which is perpendicular to the plain of the Picture, makes or casts upon the Picture is VT; and generally the Shadows which all the Lines parallel to the principal Ray VG, or perpendicular to the Picture, make either on the Picture or on Plains parallel to the Picture, will be also parallel to VT; because the Plain of the Shadow TVG, cutting the Picture by the Line TV, will cut all the Plains parallel to the Picture by Lines parallel to TV; and the other Plains of Shadow parallel to the Plain of Shadow TVG, will also cut the Picture and the other Plains which are parallel to it, by Lines parallel to TV.

II.

The Appearance of the Shadow which the said principal Ray VG makes upon the Horizontal-plain MOGV, tends to the point of Sight V; and generally the Appearance of the Shadows made (by Lines parallel to the principal Ray or perpendicular to the Picture) on the Geometrical-plain, or Plains parallel to it, and consequently to the Horizontal-plain, tend to the Point of Sight: Because the Plain of Shadow TVG, cutting the Horizontal-plain MOGV, (which is parallel to the Geometrical-plain ABEF,) by the line VG, which passes thro' the principal Point V; and the Plain TNHI, (which is also parallel to the Geometrical-plain) by the line TN, which is parallel to the principal Ray VG, and consequently perpendicular to the Picture, (because those Two plains MOGV, TNHI, are parallel to one another,) the Appearance of the line TN must also tend to the principal Point V, *by Theor.* 8. and as the Sections made by the Plains of Shadow parallel to the Plain TVG, with Plains parallel to one another, and to the Two foregoing, (that is, to the Geometrical or Horizontal-plain) are also parallel to the principal Ray VG, *by* 16.11. their Appearance must likewise tend to the principal Point V.

III.

The Appearance of the Shadow which the said principal Ray VG, and its parallels make upon the Vertical-plain GH-

GHIV, and its parallels, tends to the principal Point V; because the common Section of the Plain of Shadow TVG, and of the Vertical-plain GHVI, is VG, which passes thro' the principal Point V: And if any other Plain parallel to the Vertical-plain be cut by the said Plain of Shadow TVG, or by others parallel to it; the common Sections will be parallel to one another, and to the principal Ray VG, and their Appearances will *by Theor.* 8. tend to the said point of Sight V, which is their Accidental-point. *Plate* 32. *Fig.* 54.

IV.

The Shadow made by the line GO, which is perpendicular to the Vertical-plain, upon the Geometrical-plain ABEF, is parallel to TI, and consequently to the Ground-line AB; and generally the Shadows made upon the Geometrical-plain, or other Plains parallel to it, by Lines perpendicular to the Vertical-plain, are parallel to one another and to the Ground-line AB; because the lines GO, TI, (being the common Sections of the Plain of the Light or of Shadow GOS, and of the Two parallel Plains MOGV, TNHI,) are parallel to one another, *by* 16. 11. and consequently to the Ground-line AB; and what I say of those Two parallel Plains MOGV, TNHI, must also be understood of the Geometrical-plain ABEF, and of all the others which are parallel to it: And as the Appearances of the line TI, and of all its parallels, are parallel to the Ground-line AB, *by Theorem* 7. it follows that the Appearance of the Shadows of all the Lines perpendicular to the Vertical-plain, which fall upon the Geometrical-plain or its parallels, is parallel to the Ground-line AB.

V.

The Shadow which the said line GO makes upon ABCD, the Plain of the Picture, is parallel to the Horizontal-line KL or to the Ground-line AB, and generally the Shadows which all the Lines perpendicular to the Vertical-plain, make upon the Picture, are parallel to one another and to the Ground-line AB; because the Plain NOGH being parallel to the Plain of the Picture ABCD, the common Sections OG, TI, of those Two parallel Plains, and of the plain of Light or Shadow GOS, are parallel to one another *by* 16. 11. and consequently to the Ground-line AB; and what I may say concerning those Two parallel Plains NOGH, TMVI, must also be understood of all the others, which being parallel to the Picture, go thro' Lines parallel to the line GO, and are cut by a plain of Shadow or Light, parallel to the plain GOS. Whence it is easy to conclude, that all the Shadows which the Lines perpendicular to the Vertical-plain, cast on the

Plate 32. *Fig.* 54. Picture, or on Plains parallel to it, are parallel to one another and to the Ground-line AB, and *by Theor.* 7. that the Appearances of all those parallel Lines, are also parallel to the Groundline AB.

VI.

The Appearances of the Shadows which the said line GO, and its parallels cast on the Vertical-plain GPQV, and its parallels, concur at I, the point of the Inclination of the Sun's Rays; because the common Section of the Plain of Light GOS, and the Vertical-plain VGPQ, is the line GI, which goes thro' I the point of Inclination of the Sun's Rays; and if any other Plain parallel to the Vertical-plain be cut by the said Plain of Shadow GOS, or by others parallel to it, the common Sections will be parallel to one another and to the line GI, and their Appearances will, *by Theor.* 8. tend to the same point I, which is their Accidental-point.

VII.

The Appearance of the Shadow which the line GP, which is perpendicular to the Geometrical-plain ABEF, casts upon the said Geometrical-plain, ends at M, the Point of the Declination of the Sun's Rays, and commonly the Appearance of the Shadow which a Line perpendicular to the Geometrical-plain casts upon that Geomearical-plain, or Plains parallel to it concurs at M the Point of the Declination of the Sun's Rays; because the Plain of Light or Shadow GHS, cutting the Plain GVMO, which is parallel to the Geometrical ABEF, by the line GM, which goes thro' the point M of the Declination of the Sun's Rays) will cut the Geometrical-plain, and all those which are parallel to it, by Right-lines parallel to the line GM: And likewise if you suppose along other Lines perpendicular to the Geometrical-plain, Plains of Light to pass parallel to the plain of Shadow GHS, those plains will also cut the Geometrical-plain and its parallels, by Right-lines parallel to one another and to the Line GM: And as M is the Accidental-point of all those parallel Lines, it follows, *by Theor.* 8. that their Appearances ought to concur in the point M.

VIII.

The Shadow which the said line GP casts on the plain of the Picture, is parallel to TM, or perpendicular to the Ground-line AB; and generally the Appearance which the Shadow of a Perpendicular to the Geometrical-plain, makes on the Picture and all its parallel Plains is perpendicular to the Ground-line AB; because the plain of Light GHS cutting NOGH the plain in Front by the line GH, which is perpendicular to the Geometrical-plain, will cut all the other *Plains in Front*, that is, the Picture and its parallel Plains by Lines pa-

rallel

rallel to GH, and consequently perpendicular to the Geometrical-plain: And likewise those plains in Front will be cut by other plains of Light parallel to the plain GHS, by Lines also perpendicular to the Geometrical Plain, whose Appearances must, *by Theorem* 7. be perpendicular to the Ground-line AB. *Plate* 32. *Fig.* 54.

IX.

The Appearance of the Shadow which the said GP, and its parallels cast upon the Vertical-plain, or upon its parallels, is perpendicular to the Ground-line AB, because the plain of Light GHS cutting the Vertical-plain GVQP, by the line GP, which is perpendicular to the Vertical-plain; will also cut all the other *Plains of Profil*, (that is, all the plains parallel to the Vertical-plain) by lines parallel to GP; as also all the plains of Light, parallel to the plain of Shadow GHS, will cut the Vertical-plain and all the Plains of Profil, by Lines parallel to one another, and to the line GP, and consequently perpendicular to the Geometrical-plain, whose Appearances, *by Theorem* 7, are perpendicular to the Ground-line AB.

Thus you see, That when the Sun is out of the Plain of the Picture ABCD, the Appearance of the Shadow which a Line perpendicular to the Plain of the Picture, as GV, casts upon the said Picture or its parallels, and upon the Geometrical Plain ABCE, or its parallels; and upon the Vertical-plain or its parallels, tends to the principal Point V.

That the Appearance of the Shadow which a Line perpendicular to the Vertical-plain VG PQ, as GO, casts upon the Geometrical-plain ABEF, or its parallels, is parallel to the Ground-line AB; as also of that which it casts upon the Picture ABCD, or its parallels: And the Appearance of the Shadow of which it casts upon the Vertical-plain, or upon the Plains of Profil, tends to I the point of Inclination of the Rays of the Sun.

And lastly, That the Appearance of the Shadow of a Line perpendicular to the Geometrical-plain ABEF, as GP, casts upon the said Plain or its parallels, tends to M, the point of Declination of the Rays of the Sun: And the Appearance of that which it casts upon a plain in Front or the plain of the Picture ABCD; and also upon the Vertical-plain or a Plain of Profil, is perpendicular to the Ground-line AB.

Rules for the Shadows, when the Sun is suppos'd in the Vertical Plain and the Plain of the Picture.

SINCE the Sun S is suppos'd to be in the Picture ABCD, and in the Vertical-plain VGPQ, it must be necessarily in the *Zenith*, and then it will happen, that The

X.

Plate 32. Fig. 54. The Appearance of the Shadow which a Right-line perpendicular to the Picture, casts either upon that plain or its parallels will be Infinite perpendicularly downwards; that is, perpendicular to the Ground-line: It being certain, that the Shadow which the principal Ray VG, which is perpendicular to the Picture ABCD, casts upon the said Picture, is the Infinite Line VQ, which being perpendicular to the Geometrical-plain ABEF, its Appearance in the Picture must, *by Theorem* 7. be perpendicular to the Ground-line AB.

XI.

The Appearance of the Shadow which a Right-line perpendicular to the Plain of the Picture, makes upon the Vertical-plain or its parallels, is a Surface which extends it self perpendicularly downwards *ad infinitum* from that Perpendicular, whose apparent Breadth is equal to the apparent Length of the said perpendicular: It being certain that the Shadow which VG the principal Ray, which is perpendicular to the Picture ABCD, shou'd cast upon the Vertical-plain VGPQ, wou'd cover it *ad infinitum* by the Breadth VG, from VG perpendicularly downwards.

XII.

The Appearance of the Shadow, which a Line perpendicular to the Picture casts upon the Geometrical-plain, and its parallels, Ends at the point of Sight: It being certain that the Shadow which the principal Ray VG, which is perpendicular to the Picture ABCD shou'd make upon the Geometrical-plain ABEF, wou'd be PQ, which being perpendicular to the Picture, its Appearance in the Picture must, *by Theorem* 8. End in the principal Point V.

XIII.

The Appearance of the Shadow which a Line perpendicular to the Vertical-plain, throws upon the Geometrical-plain and its Parallels, is parallel to the Ground-line: It being certain that the Shadow which the line GO, which is perpendicular to the Vertical-plain VGPQ shou'd cast upon the Geometrical-plain ABEF, wou'd be NH, which being perpendicular to the Picture ABCD its Appearance in the Picture must, *by Theorem* 7. be parallel to the Ground-line AB.

XIV.

The Appearance of the Shadow which a Line perpendicular to the Vertical-plain, casts upon the Plain in Front, (in

(in which it is) is a Surface continu'd perpendicularly downwards *ad infinitum*, whoſe apparent Breadth is equal to the apparent Length of the ſaid Perpendicular: it being certain that the Shadow, which the line GO, perpendicular to the Vertical-plain VGPQ, ſhou'd caſt upon the plain NOGH, in which it is, wou'd be infinite perpendicularly downwards from OG, having OG for its Breadth. *Plate* 32. *Fig.* 54.

XV.

The Appearance of the Shadow which a Line perpendicular to the Vertical-plain, caſts upon that Plain, and its Parallels, is perpendicular to the Ground-line: it being certain that the Shadow which the line GO, which is perpendicular to the Vertical-plain VGPQ, ſhou'd caſt upon that plain, wou'd be GP, which being perpendicular to the Geometrical-plain ABEF, its Appearance muſt, *by Theorem* 7. be perpendicular to the Ground-line AB.

XVI.

The Appearance of the Shadow which a Line perpendicular to the Geometrical-plain, caſts upon that plain and its parallels is a Point, *viz.* the Appearance of the Point, where that Line meets the plain to which it is perpendicular: It being certain, that the Shadow which the line GP, which is perpendicular to the Geometrical-plain ABEF, ſhou'd caſt upon that plain wou'd be the point P, where it cuts the ſaid plain.

XVII.

The Appearance of the Shadow which a Line perpendicular to the Geometrical-plain, caſts upon a Plain in Front, is perpendicular to the Ground-line: It being certain, that the Shadow which the line GP, which is perpendicular to the Geometrical-plain ABEF, ſhou'd caſt upon the Plain in Front NOGH, in which it is, wou'd be the ſame Line it ſelf; and that which it ſhou'd caſt upon another Plain in Front, wou'd be parallel to the ſaid Line, *by* 16. 11. and conſequently perpendicular to the Geometrical-plain ABEF; wherefore *by Theorem* 7. its Appearance in the Picture muſt be perpendicular to the Ground-line AB.

XVIII.

The Appearance of the Shadow which a Line perpendicular to the Geometrical-plain, caſts upon the plain of Profil in which it is, is the Appearance of the ſame continued downwards from the Sun *ad infinitum*, and is conſequently perpendicular to the Ground-line AB: It being certain, that the Shadow which the line GP perpendicular to the Geometrical

Plate 32. *Fig.* 54. trical-plain ABEF, shou'd cast upon the Vertical-plain VGPQ in which it is, wou'd only be the same line GP continu'd downwards *ad infinitum*; which consequently being perpendicular to the Geometrical-plain ABEF, wou'd, *by Theorem* 7. have its Appearance in the Picture perpendicular to the Ground-line AB.

Thus you may see, That when the Sun is in the *Zenith*, the Appearance of the Shadow which a Line perpendicular to the Picture, as VG, casts upon a Plain in Front, is perpendicular to the Ground-line: And of that which it casts upon a Plain of Profil is an infinite Surface, which extends it self perpendicularly downwards, and whose apparent Breadth is equal to the apparent Length of that Perpendicular: And of that which it casts upon the Geometrical-plain, or its Parallels ends in the Point of Sight.

That the Appearance of the Shadow which a Line perpendicular to the Vertical-plain casts upon the Geometrical, or its Parallels, is parallel to the Ground-line: And of that which it casts upon a Plain of Profil is perpendicular to the Ground-line: And of that which it casts upon the Plain in Front in which it is, is an Infinite Surface which extends it self perpendicularly downwards, whose apparent Breadth is equal to the apparent Length of that Perpendicular.

And Lastly, That the Appearance, which the Shadow of a Line perpendicular to the Geometrical-plain casts upon that Plain, or its Parallels is a Point: And upon a Plain in Front, and a Plain of Profil, in which the said Line is, it is perpendicular to the Ground-line.

Rules for the Shadows, when the Sun is in the Plain of the Picture, and out of the Vertical Plain.

WE shall here suppose the Angle VTI to be equal to the Height of the Sun above the Horizon, in such manner that TU may be the Sun's Ray, which will determine the Height of the Sun: And then it will happen, that

XIX.

The Appearance of the Shadow which a Line perpendicular to the Picture casts upon the Plain in Front, ends in the Point of Sight: It being certain, that the Shadow which the principal Ray VG, which is perpendicular to the Plain of the Picture ABCD, casts upon that Plain is the Line TV continu'd *ad infinitum* from the principal Point V; and that the Shadow which the said Line GV, or its Parallels casts upon the Plains in Front, goes on *ad infinitum*, and is parallel

to

to the Line TV; whence, *by Theorem* 8. the Appearance of all those parallel Shadows tends to the principal Point V, which is their Accidental-point. *Plate* 32. *Fig.* 54.

XX.

The Appearance of the Shadow which a line perpendicular to the Plain of the Picture, casts upon the Geometrical-plain or its Parallels, ends in the Point of Sight: It being certain, that Shadow which the principal Ray VG, which is perpendicular to the Picture ABCD, casts upon the Geometrical-plain ABEF, is equal to it, and perpendicular to the Ground-line AB, wherefore, *by Theorem* 8. the Appearance of that Shadow tends to the principal Point V, as well as the Appearance of the Shadow of all the other Lines, which are perpendicular to the Picture.

XXI.

The Appearance of the Shadow, which a Line perpendicular to the Picture, casts upon a Plain of Profil, ends in the Point of Sight: It being certain, that the Shadow which the principal Ray VG, which is perpendicular to the Picture ABCD, casts upon a Plain of Profil as, *for Example*, upon the Plain TNOM, is a Line equal and parallel to VG, and consequently perpendicular to the Picture ABCD; wherefore, *by Theorem* 8. the Appearance of that Shadow must end in the principal Point V.

XXII.

The Appearance of the Shadow, which a Line perpendicular to the Vertical-plain, casts upon the Geometrical-plain, and its Parallels, is parallel to the Ground-line: It being certain that the Shadow which the line GO, which is perpendicular to the Vertical-plain VGPQ, casts upon the Geometrical-plain ABEF and its Parallels, is a line equal and parallel to GO, and consequently parallel to the Ground-line AB; whence, *by Theorem*. 7. the Appearance of such a Shadow is also parallel to the Ground-line AB.

XXIII.

The Appearance of the Shadow, which a Line perpendicular to the Vertical-plain, casts upon the Plain in Front, in which it is, is a Surface, whose Length is Infinite, and whose Breadth is terminated on either side by Rays parallel to that Plain which determines the Height of the Sun above the Horizon: It being certain, that the Shadow which the line GO, which is perpendicular to the Vertical-plain VGPQ, casts upon the Plain in Front NOGH, in which it is, is an In-

Plate 32. *Fig.* 54. Infinite Surface terminated by Two Rays parallel to the Ray VT, which determines the Height of the Sun above the Horizon.

XXIV.

The Appearance of the Shadow, which a Line perpendicular to the Vertical-plain, caſts upon a Plain of Profil, is perpendicular to the Ground-line: It being certain, that the Shadow which the line GO, which is perpendicular to the Vertical-plain VGPQ caſts upon a Plain of Profil, as upon the Plain TNOM, which it touches at the point O, is the line ON, which being perpendicular to the Geometrical-plain ABEF, its Apearance in the Picture muſt, *by Theor.* 7. be perpendicular to the Ground-line AB.

XXV.

The Appearance of the Shadow, which a Line perpendicular to the Geometrical-plain, caſts upon that Plain or its Parallels, is parallel to the Ground-line: It being certain, that the Shadow which the line GH, which is perpendicular to the Geometrical-plain ABEF, caſts upon the plain TNHI, which is parallel to the Geometrical-plain, is parallel to the Ground-line AB; whence, *by Theor.* 7. the Appearance of that Shadow is alſo parallel to the Ground-line AB.

XXVI.

The Appearance of the Shadow, which a Line perpendicular to the Geometrical-plain, caſts upon the Plain in Front, in which it is, is an Infinite Surface terminated (as to its Breadth) by Two Rays parallel to that Ray of the Sun which determines its Height above the Horizon: It being certain, that the Shadow which the line GH, which is perpendicular to the Geometrical-plain ABEF, caſts upon the Plain in Front, NOGH, in which it is, is an Infinite Surface terminated by Two Infinite Parallels, one of which, as GN, is parallel to the Ray TV, which determines the Height of the Sun above the Horizon.

XXVII.

The Appearance of the Shadow, which a Line perpendicular tothe Geometrical-plain, caſts upon a Plain of Profil, is perpendicular to the Ground-line; It being certain, that the Shadow which the line GH, which is perpendicular to the Geometrical-plain ABEF, ſhou'd caſt upon the Plain of Profil TNOM, wou'd be a Line equal and parallel to GH, and conſequently perpendicular to the Geometrical-plain ABEF; whence, *by Theor!* 7. the Appearance of the Shadow of ſuch a Line is perpendicular to the Ground-line AB. Thus

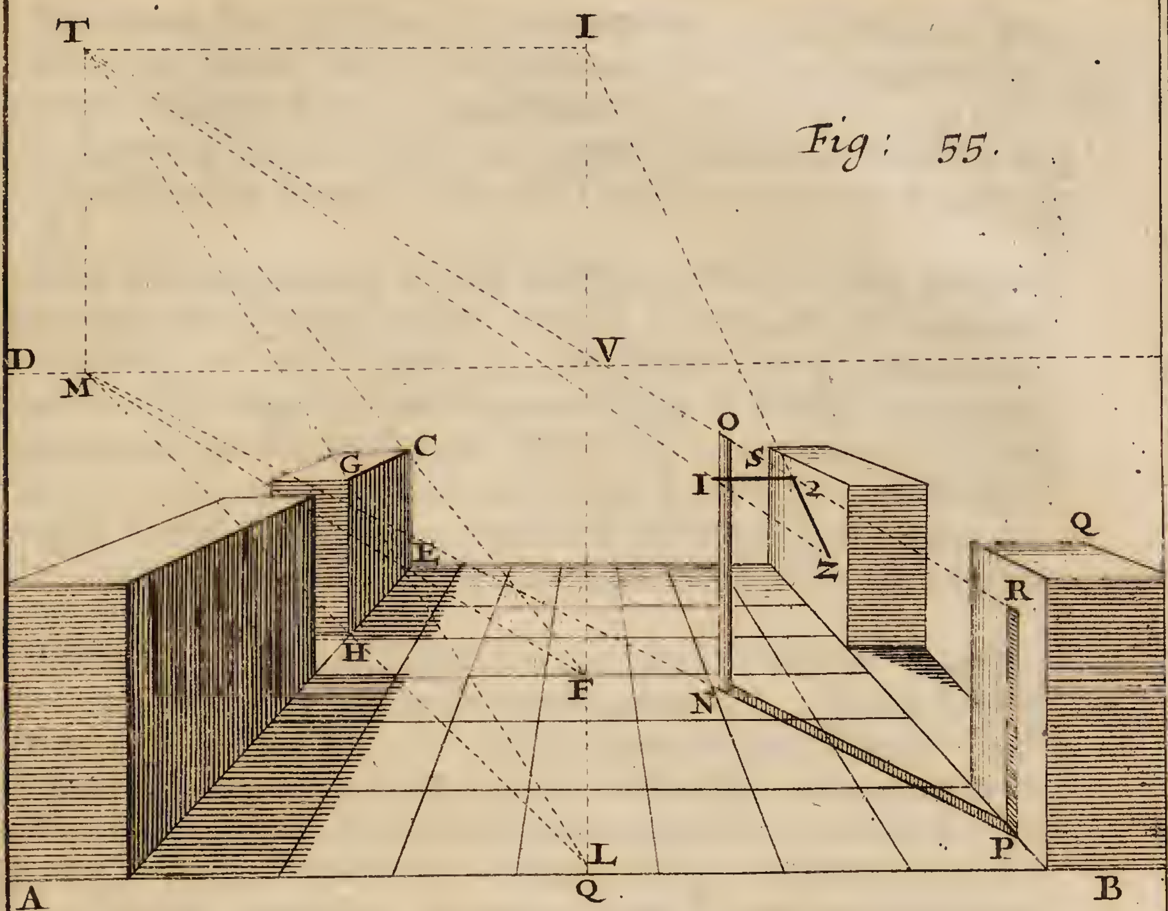
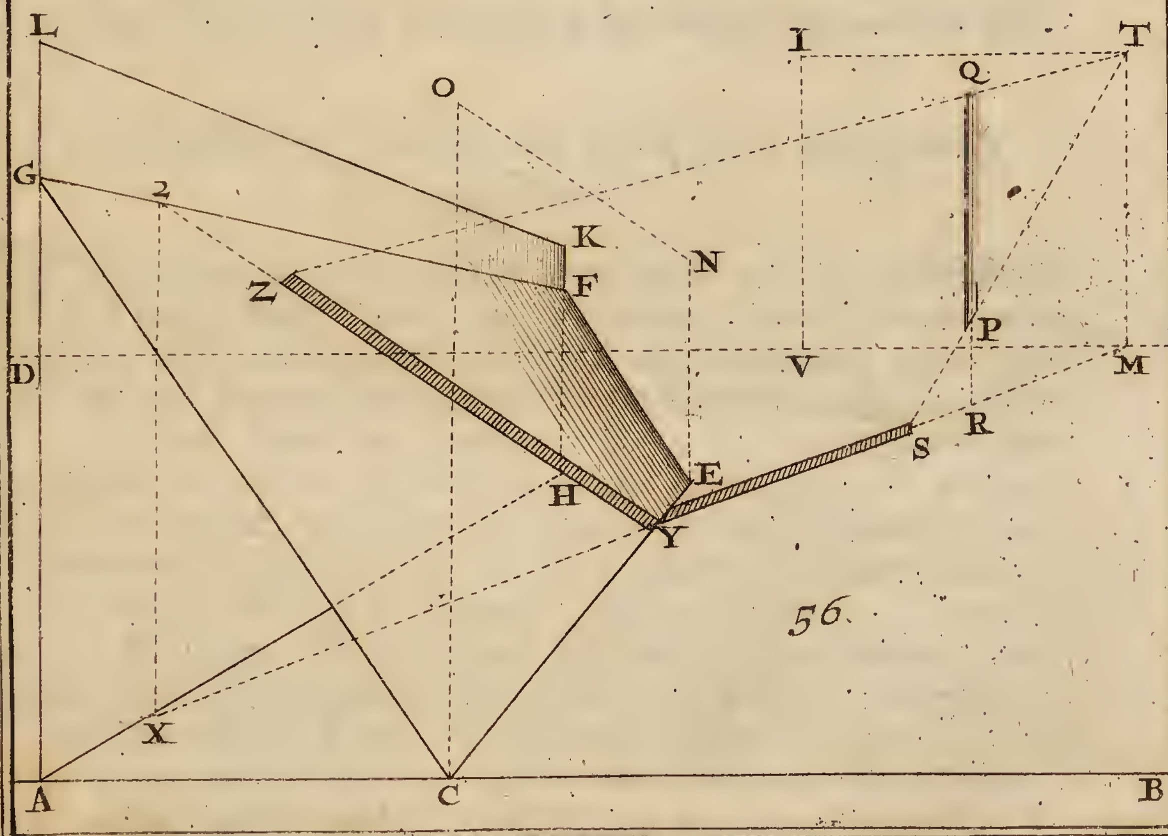


Fig: 55.



56.

Thus you see, That when the Sun is in the Plain of the Picture and out of the Vertical-plain, the Appearance of the Shadow, which a Line perpendicular to the Picture, as GV, casts upon a Plain in Front; or upon a Geometrical-plain, or its Parallels; or upon a Plain of Profil, ends in the Point of Sight. *Plate* 32. *Fig.* 54.

That the Appearance of the Shadow, which a Line perpendicular to the Vertical-plain, as GO, casts upon the Geometrical-plain, or its Parallels, is parallel to the Ground-line: And of that which it casts upon a Plain of Profil, is perpendicular to the Ground-line: And the Appearance of that which it casts upon a Plain in Front, is a Surface of Infinite Length, whose Breadth is contain'd between Two Lines parallel to that Ray of the Sun, which determines its Height above the Horizon.

And lastly, That the Appearance of the Shadow, which a Line perpendicular to the Geometrical-plain, casts upon that Plain, or its Parallels, is parallel to the Ground-line: And of that which it casts upon a Plain of Profil, is perpendicular to the Ground-line: and the Appearance of that which it casts upon that Plain in Front in which it is, is a Surface of an Infinite Length, contain'd between Two Lines parallel to that Ray of the Sun, which determines its Height above the Horizon.

The Practise of what has been said concerning Shadows.

TO come to the Practise, you must have in the Picture Four chief Points, the principal Point V, upon the Horizontal-line DD; the point M, on the Right, or on the Left of the Line of Declination of the Rays of the Sun, nearer to or farther from the Point of Sight V, according as the Sun declines more or less on the Right or Left of the Vertical-plain; and upon the Vertical Line VQ, the Point I, of the Inclination of the Rays of the Sun, above or below the Horizontal Line DD, and nearer to, or farther from it, according as the Sun will be before or behind the Picture, and higher or lower above the Horizon; and lastly, the point T of the place of the Sun in the Picture, which is also call'd the *Point of Concourse of the Rays of the Sun*; because the Rays of the Sun being suppos'd parallel to one another, that point T where the Picture is cut by a Ray drawn from the Center of the Sun, and thro' the Eye, is their Accidental-point, where their Appearance must concur, *by Theorem* 8. This point T, will be found upon the line MT perpendicular to the Horizontal-line DD, or to the Ground-line AB, and equal to VI, *Plate* 33. *Fig.* 55.

Plate 33. *Fig.* 55. or by drawing from the point I, the line IT parallel to the Horizontal-line DD.

These Four Points suppose the Sun to be out of the Vertical-plain, and out of the plain of the Picture; for when it is out of the plain of the Picture and in the Vertical-plain, you need only have Two points V, I, because in such a Case the Sun will not decline from that Plain: And when the Sun is suppos'd in the plain of the Picture, you need only have the point V, and the line VT, which is call'd the *Line of the Inclination of the Rays of the Sun*, because it makes with the Horizontal-line DD, the Angle MUT which shews the Height of the Sun above the Horizon.

By means of these Points and the foregoing Rules, it is easy to find upon one of those Three Plains which we have chiefly consider'd, (*viz.* the Geometrical-plain, and its Parallels, the Plain of the Picture and its Parallels or the Plains in Front, and the Vertical-plain and its Parallels, or the Plains of Profil) the Appearance of the Shadows of the Bodies put in Perspective; having found the Appearances of the Shadow of every Line that contains the said Shadows; which may be done by finding the Appearance of the highest or eminent Point of all those Lines, which are the bounds of the Bodies whose Shadow you wou'd represent.

As *for Example*, If you wou'd upon the Geometrical-plain find the Appearance of the point C, which answers perpendicularly to the point E upon the Geometrical-plain; draw thro' that point E, from M the Point of the Declination of the Rays of the Sun, the line EF, which *by Rule* 7. will be the Shadow of the line CE, which is perpendicular to the Geometrical-plain, and that Shadow will be terminated at F, which will consequently be the Shadow of the propos'd Point C, drawing thro' that point C, from T the Point of concourse of the Sun's Rays, or from the place of the Sun in the Picture, the Ray TCF. We shall explain this more particularly in the following Operations.

OPERATION XXIV.

How to find the Appearance of the Shadow of a Right-line put in Perspective, and perpendicular to the Geometrical-plain, or to the Picture, or to the Vertical-plain upon one of the said Plains, or its Parallels, when the Sun is out of the Plain of the Picture.

Plate 33. *Fig.* 55. WE shall here suppose T to be the place of the Sun in the Picture, where all its parallel Rays concur; M to be the point of Declination of its Rays upon the Horizon-

tal-

line DD; and I the point of Inclination of the said Rays, upon the Vertical Line VQ, which we will always mark with the same Letters, that we may not be oblig'd to say the same things over again. Plate 33. Fig. 55.

This being suppos'd, you must from the Point where the Line propos'd cuts one of the Plains draw the Line *directing the Shadow*, which must serve for the Appearance of the Shadow of the Line propos'd; and as often as that Line directing the Shadow meets with any of the said Plains, continue the Line of Shadow from the place where it first falls upon the new Plain, along that Plain, according to the foregoing Rules, and terminate this Line of Shadow by the intersection of a Ray drawn from the point T thro' the other end of the Line propos'd.

To find, *for Ex.* upon the Geometrical-plain the Appearance of the Shadow of the Line CE, which is perpendicular to the said Plain; draw thro' the point E where it meets the Geometrical-plain, and thro' M the point of Declination of the Rays of the Sun, the Line of Shadow EF, which you must terminate at F, by the Ray TF, drawn from the place of the Sun in the Picture T, thro' the End C, of the Line propos'd CE, whose shadow upon the Geometrical-plain must consequently be EF.

Likewise to find upon the said Geometrical-plain the Appearance of the Shadow of the Line GH which meets it at Right Angles at the point H; draw thro' that point H, and thro' the point M, the Line of Shadow HL, and terminate it at L by the Ray TL, drawn from the Point T, thro' the End G of the line GH, so that the line HL will be the Shadow of the line GH, and the line LF, consequently the Shadow of the line GC which being the Appearance of a Line perpendicular to the Picture, the Appearance FL of its Shadow must tend to the principal Point V, *by Rule* 2. whence may be drawn some shorter Method for Practice.

To find upon the Geometrical-plain, and upon the Plain of Profil PR of the solid Parallelepipedon BQ, the Appearance of the Shadow of the line NO, which is perpendicular to the Geometrical-plain; draw thro' the point N, where that Line cuts the Geometrical-plain, and thro' M the point of Declination of the Sun's Rays, the Line directing the Shadow NP, and from the point P, where this Line meets the Plain of Profil, raise upon the said Plain the line PR perpendicular to the Geometrical-plain, which you must terminate at R by the Ray TR drawn from T the place of the Sun in the Picture, thro' the end O of the Line propos'd NO, and the point R will be the Shadow of the point O, which must terminate that of the line NO; so that the

Plate 33. Fig. 55. Shadow of the line NO will be made up of the part NP upon the Geometrical-plain, and the part PR upon the Plain of Profil.

To find upon that Plain of Profil KS the Appearance of the Shadow of the line 12, which cuts it at Right Angles at the point 2, draw thro' that point 2, and thro' I the point of Inclination of the Rays of the Sun, the Shadow 2Z, which you must terminate at Z, by drawing from T the Point of the Sun's place in the Picture, thro' the end 1 of the Line propos'd 12, the Ray TZ, and the point Z will be the Shadow of that End 1, and the line 2Z will be the Shadow of the line LN, upon the Plain of Profil KS; and so of the rest.

OPERATION XXV.

How to find upon an Inclin'd Plain the Appearance of the Shadow of a point rais'd above the Geometrical-plain when the Sun is out of the Plain of the Picture.

TO find upon the Inclin'd Plain CEFG, which cuts the Geometrical-plain in the line CE, the Appearance of the Shadow of the point Q, whose Situation is R, raise the Two Plains of Profil CENO, AHKL, at what distance you please from one another, and from the Inclin'd Plain CEFG,
Fig. 56. so that they may cut it, in such manner that the Section may be (*for Example*) CE, FG. Draw from M the point of the Declination of the Sun's Rays thro' the point R, the line MR, which being produc'd will give here upon the line CE the point Y, and upon the line AH the point X, which not being in the Inclin'd Plain CEFG, you must from it raise the Perpendicular X2, which will upon the common Section FG, give the point 2, thro' which and thro' the point Y, you must draw the line Y2, which will be the line directing the Shadow of the line QR upon the Inclin'd Plain CEFG: Wherefore, if thro' the given point Q, and T the place of the Sun in the Picture, the Ray TZ be drawn, you will have at Z upon the Y2, (which is upon the Inclin'd Plain CEFG) the Appearance of the Shadow of the propos'd Point Q.

SCHOLIUM.

It is plain, That if the line MR did not meet the Bases AH, CE, of the Plains of Profil AHKL, CENO, or if the Ray TQ did not meet the line Y2 which directs the Shadow, the Shadow of the point Q wou'd not fall upon the Inclin'd Plain CEFG.

It

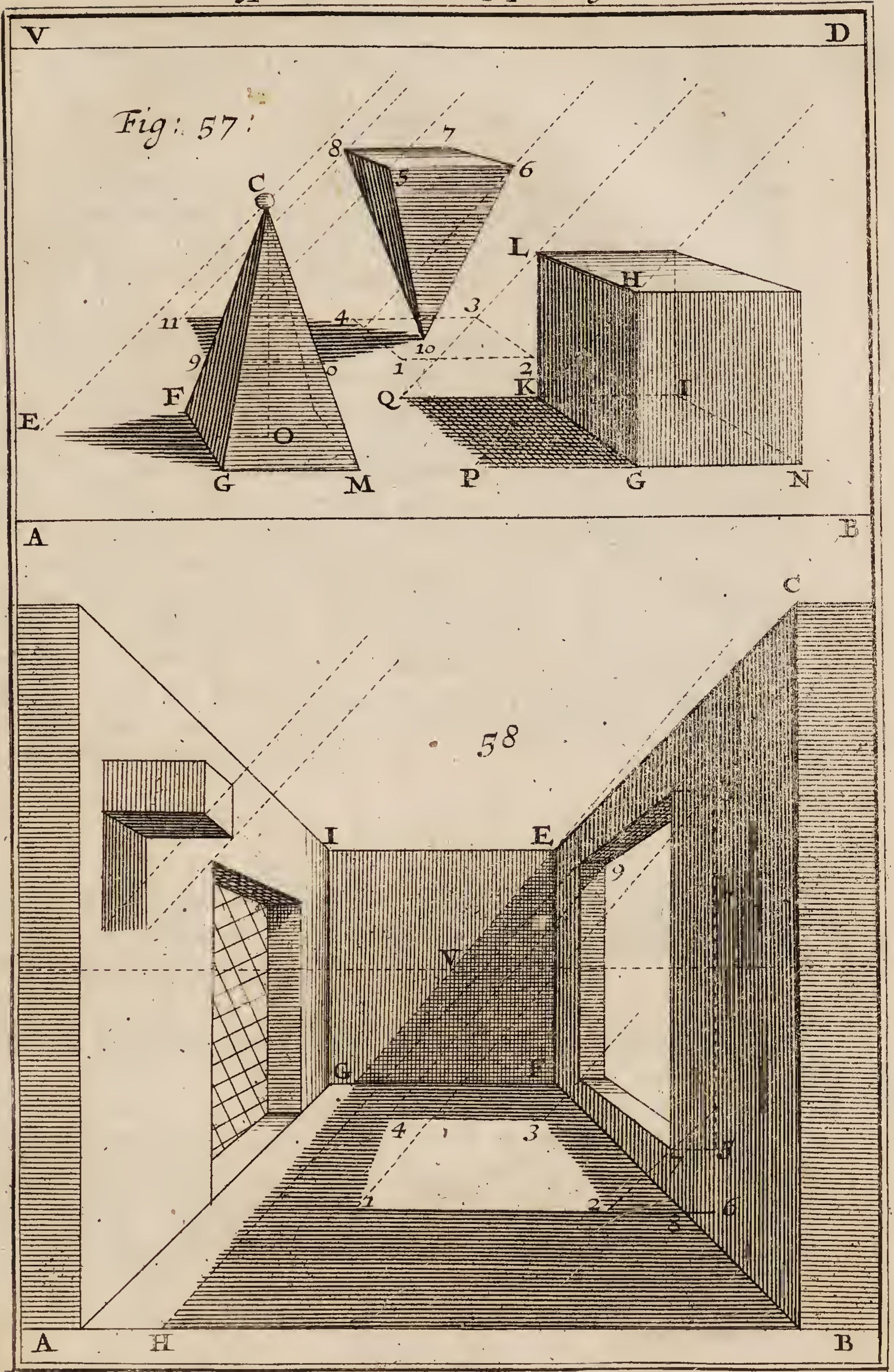

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Fig: 57.
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It is alſo evident, That the point S which is determin'd by the Ray TP, upon the line MR produc'd, is the Appearance of the Shadow of the point P upon the Geometrical-plain; and the line SY the Appearance of the Shadow of part of the line PQ upon the Geometrical-plain, as the line YZ is the Appearance of the Shadow of part of the ſaid line PQ upon the Inclin'd Plain CEFG. I ſaid *Part*, becauſe the Appearance of the Shadow of the point P falls out of the Inclin'd Plain CEFG, ſince the Ray TP cuts the line YS out of that Plain. *Plate* 33. *Fig.* 56.

Laſtly, it is evident, That by means of this and the foregoing Operation, One may eaſily find the Appearance of a Line Inclin'd, or Perpendicular; of a Right or Inclin'd Surface; of a Body Upright or Inclin'd to the Horizon upon any Plain, having only the Situation of that Line, or Surface, or Body: for by means of thoſe Situations you may find the Appearances of the Lines, (whether Right or Curve, Perpendicular to the Horizon, or Inclin'd, in the Air, or reſting on any Body) and conſequently of the Surfaces bounded by thoſe Lines, and of the Bodies bounded by the Surfaces; even tho' the Sun ſhou'd be in the Plain of the Picture; tho' in ſuch a Caſe the points M, T, I, vaniſh, as we ſhall ſhew more particularly by ſome Examples in the in the following Operations.

OPERATION XXVI.

How to find the Appearance of the Shadow of a Body on the Geometrical-plain, when the Sun is in the Plain of the Picture.

TO find the Appearance of the Shadow of the Pyramid FGMC, whoſe *Vertex* C has its Situation at the point O, draw thro' that point O the line OE, parallel to the Ground-line AB, and thro' the *Vertex* C the line CE parallel to the Ray of the Sun which determines its Height above the Horizon; and the point E, where that line CE meets the parallel OE, will be the Appearance of the Shadow of the *Vertex* C, upon the Geometrical-plain; and then you need only joyn EF, which will repreſent the Shadow of the Inclin'd Line CF; and likewiſe EG, which will be the Appearance of the Shadow of the Inclin'd Line CG, &c. *Plate* 34. *Fig.* 57.

To find the Appearance of the Shadow of the Pyramid 5, 6, 7, 8, 10, which ſtands upon its *Vertex* 10, and whoſe Situation is 1, 2, 3, 4, which anſwers perpendicularly to its Baſe 5, 6, 7, 8, you muſt draw from all the Angles, Lines

Plate 34. Fig. 57. parallel to the Ground-line AB, and from the Angles of the Base 5, 6, 7, 8, which is up in the Air, Rays parallel to that which determines the Sun's Height above the Horizon, and by the Intersection of those Lines you will have the Appearance of the Shadow of the Base 5, 6, 7, 8; wherefore you must draw from all the Points of that Shadow to the *Vertex* 10, which touches the Geometrical-plain, Right-lines which will represent the Shadow of the sides of the Pyramid, and the Work is done.

Likewise to find upon the Geometrical-plain the Appearance of the Shadow of the Cube GNHLK, whose Situation is the Perspective Square GNIK, you must draw from all the Angles of that Situation Parallels to the Ground-line AB, to terminate upon them the Shadow of the correspondent upper Parts, drawing from all their Angles Lines parallel to the Ray of the Sun, which determines its Height above the Horizon. Thus you will have at P, the Appearance of the Shadow of the point H; and at Q the Appearance of the Shadow of the point L, wherefore by joyning the line PQ, you will have the Appearance of the Shadow of the Line HL, and drawing the line PG, you will have the Appearance of the Shadow of the line GH, which touches the Geometrical-plain at G. And so of the rest.

SCHOLIUM.

You will by Practice find out several short Methods for the Representation of these Shadows: For if you suppose the Height of the Sun above the Horizon to be of 45 Degrees, (in which case the Shadow of a Line perpendicular to the Geometrical-plain will be equal to the same Line) you need only to make GP equal to the Height GH, to have at P the Appearance of the Shadow of H, and likewise the line KQ equal to the Correspondent Height KL, to have at Q the Representation of the Shadow of the point L, and so of the rest. And when you wou'd have the Sun to be more or less than 45 Degrees above the Horizon, having taken what Length you will for the Shadow of that Height which is nearest to the Ground-line, you may diminish the Shadows of the others which are more distant from the Ground-line, as we have diminish'd those Heights in *Oper.* 13.

OPER-

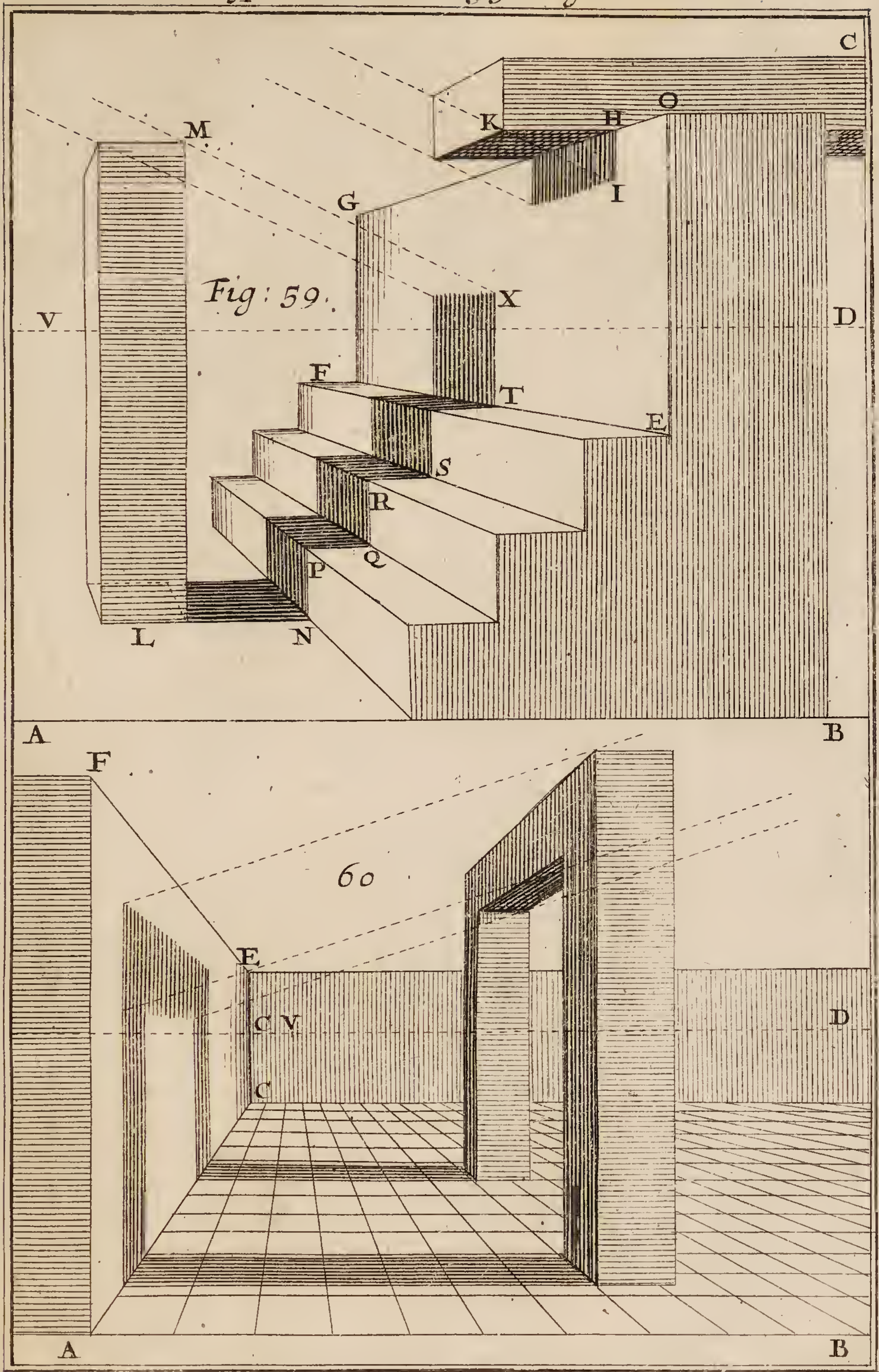
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Fig: 59.
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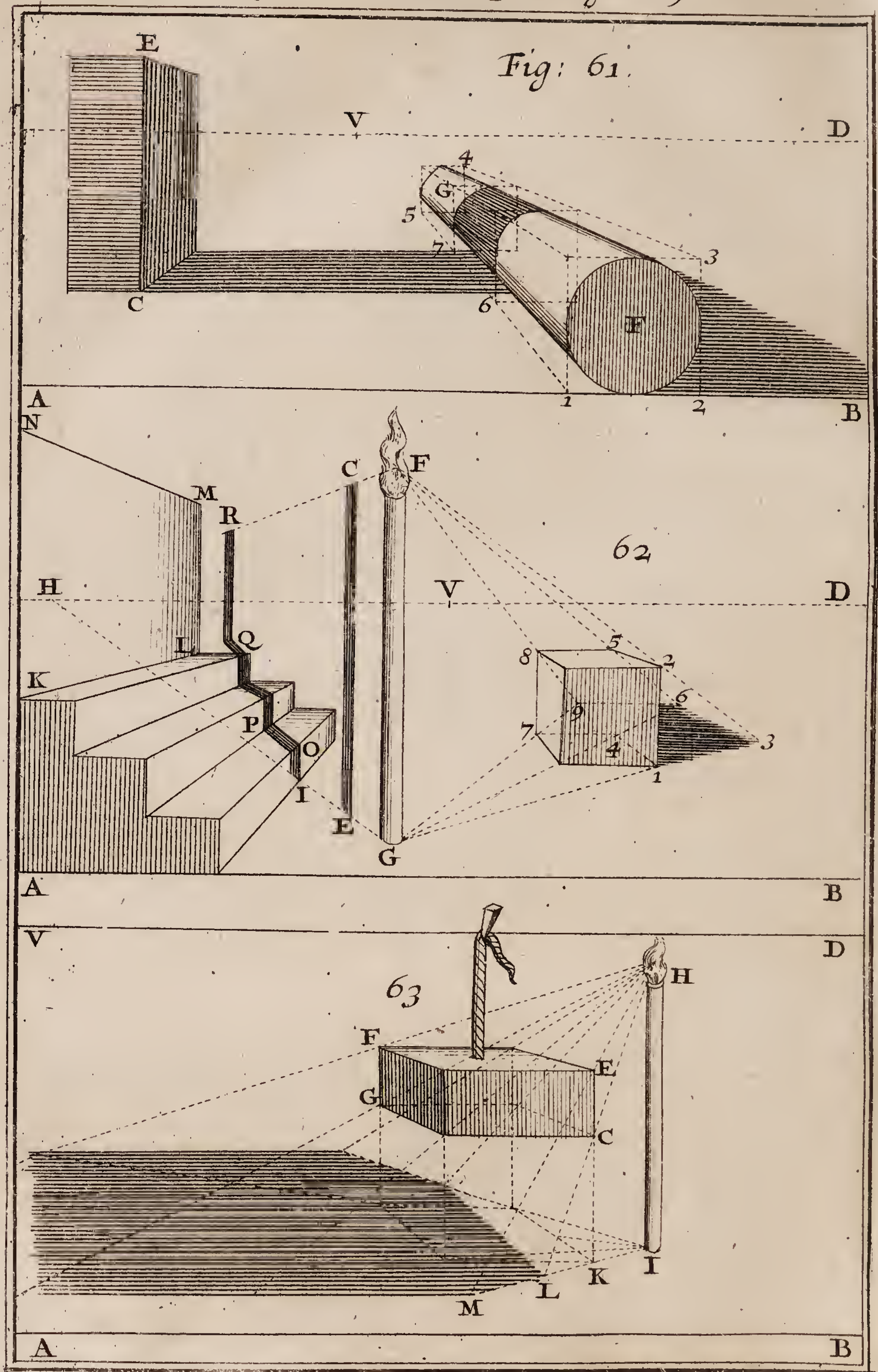
Fig: 61.
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OPERATION XXVII.

How to find upon the Geometrical-plain the Appearance of the Shadow of a Body, with ſuch an Hole in it as to let the Light Shine thro', when the Sun is in the Plain of the Picture.

TO find upon the Geometrical-plain the Figure 1,2,3,4, which is the Light of the Sun, which goes thro' the Window 5, 7, 9, of the Wall BCEF, whoſe Shadow upon the Geometrical-plain is BFGH, and EFG upon the Plain in Front EFGI; draw thro' the point 6, which is the Situation of the point 5, the line 6, 1, parallel to the Ground-line AB; and draw thro' the point 5, the Ray 5, 1, which being parallel to that of the Sun which determines its Height above the Horizon, will upon the Line in Front 6, 1, give the Appearance of the Shadow of the point 5 at 1. Likewiſe draw from the point 8, which is the Situation of the point 7, the Line in Front 8, 2, which you muſt terminate at the point 2, which will be the Shadow of the point 7, drawing thro' that point 7, a Ray parallel to the foregoing. And ſo of the reſt. *Plate* 34. *Fig.* 58.

OPERATION XXVIII.

How to repreſent in Perſpective ſuch Shadows as receive the Figures of the Plains on which they fall.

THIS ſort of Shadow is eaſy to find by means of what has been ſaid in *Oper.* 24. wherefore we ſhall now explain it in a few lines. Therefore to find the Shadow of the Beam CK upon the Plain of Profil EFGO; let fall from the point H where the Beam touches that Plain, the Perpendicular HI, which you muſt terminate at I, by the Ray KI, drawn from the end K thro' the place of the Sun in the Picture, (if the Sun is out of the Plain of the Picture) or parallel to the Line of the Inclination of the Rays of the Sun, (that is, the Line which determines its Height above the Horizon) if it be in the Plain of the Picture, as we ſuppoſe it here, and the point I will be the Appearance of the Shadow of the point K upon the Plain of Profil EFGO, *&c.* *Plate* 35. *Fig.* 59.

To find the Shadow of the Solid LM upon the Steps on the ſide of it, you muſt firſt mark its Shadow upon the Geometrical-plain, and from the point N, where it cuts the Baſis of the Firſt Step, you muſt raiſe the Perpendicular NP, and thro' the point P draw the Line in Front PQ: And likewiſe raiſe from the point Q the Perpendicular QR, to draw thro'

the point R the line SR, and ſo on, till you come to the Perpendicular TX, which will be terminated at X by the Ray of the Sun MX, *&c.*

Plate 35. *Fig.* 60. After the ſame manner you may upon the Wall ACEF, find the Appearance of the Shadow of a Portico, or of a Door, a Sight of the Figure being enough to make you underſtand the Way of it, always remembring that when you are to repreſent the Shadow of a Curve Figure, as of an Arch, you muſt find the Shadow of ſeveral of the Points of the Two Curves which are its Bounds, and the Number of thoſe Points cannot be too great for a juſt Operation, that the rounding of the Curve to which all the Points of Shadow belong may be exact: And this Method is to be us'd when you wou'd put ſuch a Shadow in Perſpective:

But it is ſomething more difficult to draw the Appearance of the Shadow of a Solid, which goes over a Column lying along the Geometrical-plain, which is done thus.

Plate 36. *Fig.* 61. To mark the Appearance which the Shadow of the Parallelepipedon CE caſts upon the Cylinder or Column FG, which lies along the Geometrical-plain, and whoſe Baſe F being ſeen in Front is repreſented by a Circle, *by Theorem* 2. having found upon the Geometrical-plain the Shadow of the Solid CE, and having deſcrib'd about the Cylinder FG, the Priſm or Parallelepipedon 1, 2, 3, 4, 5, (whoſe Two oppoſite Baſes 1, 3, and 4, 5, being circumſcrib'd about Two Circles, will be Two perfect Squares,) raiſe from the Two points 6, 7, where the Shadow of the Body EC cuts that Priſm, Perpendiculars to the Geometrical-plain, which you muſt terminate at the upper Surface of the circumſcrib'd Priſm, and upon which you muſt deſcribe Squares to inſcribe Circles, whoſe Circumferences will upon the Surface of the Cylinder FG, terminate the Shadow of the Solid CE, *&c.*

Operations on thoſe Shadows which are caus'd by a ſmall Light.

AS the Sun is infinitely greater than the Bodies here about us, and extremely diſtant from 'em, its Rays may be conſider'd as parallel to one another, and the Shadows of the Bodies which are very little in reſpect of that great Luminary, can undergo no ſenſible diminution, except when they are put in Perſpective. It is not the ſame of the Shadows caus'd by a ſmall Light, as a Candle, which is very little in proportion to the Objects, and near enough to cauſe a Shadow which encreaſeth the farther it goes. Such a Shadow may be eaſily deſcrib'd upon any Plain, by obſerving what has been ſaid concerning the Sun; wherefore we ſhall here only give a few Examples of it.

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OPERATION XXIX.

How to find the Appearance of the Shadow of a Point expos'd to a Candle.

TO find upon the Geometrical-plain the Appearance of the point 2 expos'd to the Candle F, whose Situation is G, which we shall call *Foot of Light*, draw from that Foot of Light G, thro' the Situation 1, of the given Point 2, the Line 1, 3, directing the Shadow, which Line you must terminate at the point 3, by the Ray F 3 drawn from the Light F thro' the given Point 2, whose Shadow must consequently be at the point 3, and the Line 1, 3, will be the Shadow of the Perpendicular 1, 2, which touches the Geometrical-plain at the point 1. *Plate 36. Fig. 62.*

Likewise to find upon the said Geometrical-plain, the Appearance of the Shadow of the point 5, whose Situation is the point 4, draw thro' that point 4, and thro' the Foot of Light G, the Line 4, 6, directing the Shadow, and terminate it at 6, by the Line F 6, drawn from the Light F thro' the point 5, whose Shadow will be consequently at the point 6, and the Line 4, 6, will be the Appearance of the Shadow of the Perpendicular 4, 5; whence the Line 3, 6, is the Appearance of the Line 2, 5; and consequently the Surface 1, 3, 6, 4, is the Appearance of the Shadow of the Plain 1, 2, 5, 4. After the same manner, you may mark the Shadows of the other Plains, which are the Bounds of the Cube 1, 2, 8, 7, *&c.*

To find the Appearance of the Shadow of the point C, which is expos'd to the said Candle F; draw from the Situation G of the Light F, thro' the Situation E of the Point given C, a Line directing the Shadow, which you must continue along the Geometrical-plain, till it meets the Horizontal-line HD in some point, as H, in which case it must not be parallel to it, and when the said Line meets with a Plain perpendicular to the Geometrical-plain, (as here, where it meets the First Step at the point I) draw it upwards till having met with another Plain parallel to the Geometrical, (as here, the upper Part of the First Step at the point O) draw thro' that point O, to the said point H, the line OP till it meets the second Step at P, from whence you must raise a second Perpendicular, and so successively quite to the Perpendicular QR, which is here out of the Plain of Profil KLMN. Lastly, draw from the Light F thro' the Point given C, the Ray FC, which will upon the Perpendicular QR give the point R, which will be the Appearance of the Shadow of the given Point C upon the Plain KLMN, if it was continu'd, and the Line

EOIP, *&c.* will be the Appearance of the Shadow of the Perpendicular CE upon the Geometrical-plain, and upon the Steps.

OPERATION XXX.

How to find upon an Inclin'd Plain the Appearance of the Shadow of a Point expos'd to a ſmall Light.

Plate 33. *Fig.* 56. TO mark upon the Inclin'd Plain CEFG, which cuts the Geometrical-plain by the Line CE, the Appearance of the Shadow of the point Q, which is expos'd to the Light ſuppos'd at the point T, whoſe Foot is M; conſider that Foot M as the Point of the Declination of the Sun's Rays, and the Light T as the place of the Sun in the Picture; then this Problem will be ſolv'd as in *Oper.* 25.

OPERATION XXXI.

How to find the Shadow of a Body rais'd in the Air, caus'd by the Light of a Candle.

Plate 36. *Fig.* 63. TO find upon the Geometrical-plain, the Appearance of the Shadow of the Body CEFG, which is ſuſpended in the Air, and which is enlightned by the Candle H, whoſe Foot or Situation is I; mark the Shadow of each of its Points or Solid Angles, as has been taught in *Oper.* 29. Thus to find the Appearance of the Shadow of the Point C, whoſe Situation is K; draw from the Light H, thro' the point C the Ray HL, and from the Foot of Light I, thro' the Situation K, the Line IK, and the point L where theſe Two Lines interſect will be the Appearance of the Shadow of the point C. Likewiſe to find the Shadow of the Point E, whoſe Situation is the ſaid point K; draw thro' the point E, and thro' the Center of the Light H, the Ray HM, and from the Foot of Light I, thro' the Situation K the Line IM, and the point M, where thoſe Two Lines interſect, will be the Shadow of the point E, and the line LM will conſequently be the Shadow of the Line CE. And ſo of the reſt.

SCHOLIUM.

Having ſaid ſo much of common Perſpective, we ſhou'd here treat of *Curious Perſpective*; which teaches to make a Figure, which ſeems deform'd, appear in its juſt Proportions when ſeen from a certain Point, or on the Surface of a Cylinder, or of a Cone: But as that kind of Perſpective depends on *Catoptricks*, which treats of Reflection, and even *Dioptricks*, which treats of Refraction; ſince we have not given the Principles of thoſe Sciences in this Courſe of Mathematicks, we ſhall not ſpeak of it here, but in our Mathematical and Phyſical Recreations.

FINIS.

THE CONTENTS OF THE *PERSPECTIVE.*

Theor.

PROBLEMS.

PER-

PERSPECTIVE PRACTICAL.

Oper.

Of ELEVATIONS, Or SCENOGRAPHY.

A Table of the TERMS Explain'd in the *PERSPECTIVE.*

FINIS.